Mathematical Model of Face-Milling Spiral Bevel Gear with Modified Radial Motion (MRM) Correction

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Abstract—Conventional machine setting calculation for the cutting of face-milling spiral bevel and hypoid gears is based on the local synthesis at the mean point. The bias of contact path and motion curve are usually coupled with each other when applying commonly-used tooth flank corrections such as the helical motion, the modify roll method, and the cutter tilt. In this paper, we propose a method to determine the machine settings with modified radial motion (MRM) correction at specified contact point with predetermined motion curve and contact path bias on the pinion tooth surface. Parameters of MRM correction are calculated according to the equations of meshing and relationship between mating curvatures at the specified contact point. As shown by the numerical examples, the bias of contact pattern and the motion curve were controlled separately. The proposed methodology could be used to gain more control on the contact pattern and the motion characteristics. © 2005 Elsevier Ltd. All rights reserved.

Keywords—Spiral bevel gear, Hypoid gear, Kinematic correction, Machine settings.

NOMENCLATURE

\( a \) the major axis of the contact ellipse
\( a_{ij} \) coefficients in the equation of the principal curvatures (\( i = 1 \sim 6 \))
\( A_m(M) \) cone distance of predetermined contact point \( M \) on gear blank
\( f_i \) equation of meshing between two mating surfaces, \( i = 1, 2 \)
\( i \) tilt angle of the universal hypoid generator
\( j \) swivel angle of the universal hypoid generator
\( \sigma_i^{(j)} \) the principal directions on \( \Sigma_j \), \( i = s, q, f, \) and \( h \)
\( E_m \) vertical offset of the universal hypoid generator
\( L_{ij} \) the upper-left \( 3 \times 3 \) submatrix of \( M_{ij} \)
\( M_{ij} \) the \( 4 \times 4 \) homogeneous transformation matrix from the coordinate system \( S_y \) to the coordinate system \( S_i \)
\( n_1 \) surface normal vector of the work-gear

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1. INTRODUCTION

The calculation of machine setting for the cutting of spiral bevel and hypoid gears is based on the local synthesis at the mean contact point. The idea of local synthesis was proposed by Wildhaber [1] and then applied by Baxter [2] and Litvin [3-11] for hypoid gears and spiral bevel gears. However, the bias of contact pattern is coupled with the profile modification (i.e., the shape of the motion curve) when applying the well-known tooth flank correction methods such as helical motion, cutter tilt, or modify roll method. Therefore, a tooth flank correction method that will provide more freedom to modify tooth bias and motion curve separately is desired.

There are several bevel gear softwares available in the market [12] such as GAGE and CAGE by Gleason Works, CDS by Oerlikon Geartec, Kimos by Klingelberg Söhne, and HyGears by Professor Gosselin. All these softwares are capable of doing closed-loop manufacturing of bevel gears. The contact pattern and the motion curve of the bevel gear set can be simulated in these softwares before real manufacturing process. However, the basic mathematical models used in these softwares are similar. All calculations are based on the so-called universal hypoid generator as shown in the textbook written by Litvin [8]. The radial distance from cutter spindle to cradle spindle of the universal hypoid generator is assumed constant. The spiral angle of the imaginary generating gear is kept constant at defined cone distance during the generating process. Therefore, higher order tooth flank correction is required to modify the motion characteristics of the gear set. There are several well-known flank correction methods such as modified roll method, helical duplex method, cutter tilt, and spherical blade method. However, the head cutter and cutter tilt primarily determine the lengthwise curvature of the tooth flank while the profile curvature could be altered by these auxiliary flank correction methods. Therefore, the contact pattern bias and motion characteristics are coupled together. It is very difficult to design a reliable mechanism to continuously control the radial distance between cutter spindle and cradle spindle on the cradle-
type machine in the past. The assumption of constant radial distance made in the so-called universal hypoid generator is quite reasonable as long as the cradle-type generating machine is considered.

In recent years, the Gleason Works developed Cartesian-type hypoid generator [14], which raises a brand new concept of hypoid gear making. Taking advantages of the new CNC technology, Goldrich [14], Fong [15], and Stadtfeld [16,17] developed the higher-order flank modifications to reduce the tooth mesh impact with the free-form machine. However, the flank correction motion applied in these researches complied with existing flank corrections, i.e., helical motion, modified roll, and cutter tilt. However, the flank correction motions are evolved from linear functions to higher order functions. Six-order flank corrections are possible in the software provided by the Gleason Works or by the Klingelnberg Söhne but the actual flank correction effect and operating procedure are not clearly shown in the operation manual. Litvin [9] proposed a third-order flank correction with modified roll motion. Stadtfeld [17] illustrated a third-order flank correction with helical motion in his book. The generating theory of hypoid and spiral bevel gears is no longer restricted by the cradle machine.

In this paper, we used the local synthesis technique to calculate machine settings for the modified radial motion (MRM) machine settings at specified contact point. Taking advantages of Cartesian-type free-form generating machine, the radial distance and roll ratio of hypoid generator are used as variables during the tooth cutting in the MRM correction. With the MRM correction, the gear designer can specify an initial contact point and bias on the pinion tooth surface and its corresponding second-order motion curve. As shown by the numerical examples, the bias of contact pattern and the motion curve were able to control separately. The proposed method could be used to get more control on the contact pattern and the motion characteristics.

2. MATHEMATICAL MODEL OF MODIFIED UNIVERSAL HYPOID GENERATOR

2.1. Applied Coordinate Systems

The universal hypoid generator is based on the cradle type machine as shown in Figure 1 ([15]). The coordinate systems $S_t(x_t, y_t, z_t)$ and $S_l(x_l, y_l, z_l)$ are rigidly attached to the head cutter and the work gear, respectively. The coordinate system $S_t(x_t, y_t, z_t)$ is rigidly attached to the machine frame. As shown in the reference [15], there are five potential auxiliary motions can be used to modify the tooth flank: the cutter rotation angle $\phi_d$ (the flank correction of spiroflex method by Oerlikon Co.), the cradle rotation angle $\phi_c$ (the modify roll method by Gleason Works), the cradle radial setting $S_R$, the vertical offset $E_m$ (traditional discontinuous multicuting position method for the lengthwise crowning) and the sliding base setting $\Delta B$ (helixform, helical method by the Gleason Works). We used the cradle radial setting $S_R$ and the cradle rotation angle $\phi_c$ as the extra variables to modify the tooth flank in this paper.

2.2. Head-Cutter Surface

The normal section of head cutter blades can be designed with two circular-arc fillets and two straight edges (as shown in Figure 2). The straight edge and the fillet of the cutter blade, represented in the coordinate system $S_t(X_t, Y_t, Z_t)$, are shown in equations (1) and (2), respectively,

$$\begin{bmatrix} x_t^e \\ y_t^e \\ z_t^e \\ 1 \end{bmatrix} = \begin{bmatrix} (c_1 + u \sin \psi \cos \beta) \\ (c_1 + u \sin \psi \sin \beta) \\ c_2 - u \cos \psi \\ 1 \end{bmatrix},$$

$$\begin{bmatrix} x_t^f \\ y_t^f \\ z_t^f \\ 1 \end{bmatrix} = \begin{bmatrix} (c_1 + \rho_1 \sin \psi \cos \beta) \\ (c_1 + \rho_1 \sin \psi \sin \beta) \\ \rho_1 (\cos \psi - 1) \\ 1 \end{bmatrix},$$
where

\[ c_1 = \mp \frac{W}{2} \mp \rho_1 \left( \frac{\sin \psi - 1}{\cos \psi} + \cos \psi \right) + \rho_m, \quad c_2 = -\rho_1 (1 - \sin \psi). \]

Symbols \( \psi \), \( \rho_1 \), and \( W \) are the pressure angle, tip fillet radius, and point width of cutter blade, respectively, symbols \( u \) and \( \beta \) are the surface coordinates of the cutter blade. Parameter \( \rho_m \) is the mean radius of the head cutter. The unit normal vector \( n_i^z \) of the straight edge of the cutter is shown as follows,

\[ n_i^z = \frac{\partial_u r_i^z \times \partial_\beta r_i^z}{|\partial_u r_i^z \times \partial_\beta r_i^z|} = \begin{bmatrix} \pm \cos \psi \cos \beta \\ \pm \cos \psi \sin \beta \\ \sin \psi \end{bmatrix}. \]
The unit normal vector \( \mathbf{n}_t \) of the straight edge of the cutter can be derived from equation (2) and is omitted here. The "±" sign in equations (1)–(3) should be regarded as "+" sign for the outer blade, and the "−" sign for the inner blade.

The effective tool surface \( \Sigma_c \) is a cone surface and its principal curvatures are shown in equation (4),

\[
\kappa_s^{(c)} = \frac{\cos \psi}{u \sin \psi - c_1}, \quad \kappa_q^{(c)} = 0. \tag{4}
\]

The unit principal vectors \( \mathbf{e}_s^{(c)} \) and \( \mathbf{e}_q^{(c)} \) of the generating surface \( \Sigma_c \) are represented in the fixed coordinate system \( S_5 \) as follows,

\[
\mathbf{e}_s^{(c)} = [L_{5t}] \begin{bmatrix} \mp \cos \beta \sin \psi & \mp \sin \beta \sin \psi & -\cos \psi \end{bmatrix}^T, \quad \mathbf{e}_q^{(c)} = [L_{5t}] \begin{bmatrix} \pm \sin \beta & \mp \cos \beta & 0 \end{bmatrix}^T,
\tag{5}
\]

where \( L_{5t} \) is the upper-left \( 3 \times 3 \) submatrix of \( \mathbf{M}_{5t} \) as shown in equation (6).

### 2.3. Generated Gear Tooth Surface

The coordinate transformation matrices are shown in reference [15] and the details are omitted here. The coordinate transformation matrix from cutter axis to machine frame is shown as follows,

\[
\mathbf{M}_{5t} (\phi_1) = \mathbf{M}_{54} \cdot \mathbf{M}_{43} \cdot \mathbf{M}_{32} \cdot \mathbf{M}_{2t}. \tag{6}
\]

The position vectors and the unit normal vectors of cutter tool locus, represented in the fixed coordinate system \( S_5 \), are shown as follows,

\[
\mathbf{r}_s^{(c)} = [L_{5t}] \mathbf{r}_t, \quad \mathbf{n}_s^{(c)} = [L_{5t}] \mathbf{n}_t, \tag{7}
\]

where \( L_{5t} \) is the upper-left \( 3 \times 3 \) submatrix of \( \mathbf{M}_{5t} \).

Angular velocities of cradle \( \omega^{(c)} \) and work-gear \( \omega^{(1)} \) can be represented in the fixed coordinate system \( S_5 \) as follows,

\[
\omega^{(c)} = \omega^{(c)} - \omega^{(1)}, \quad \omega^{(c)} = \begin{bmatrix} 0 & 0 & -\gamma_c \end{bmatrix}^T, \quad \omega^{(1)} = \begin{bmatrix} -\cos \gamma_m & 0 & -\sin \gamma_m \end{bmatrix}^T. \tag{8}
\]

Symbol \( \gamma_c \) indicates the instantaneous roll ratio between the cradle and gear. Symbols \( \mathbf{v}_{tr}^{(c)} \) and \( \mathbf{v}_{tr}^{(1)} \) indicate the velocity of contact point in transfer motion represented in the fixed coordinate system \( S_5 \), respectively,

\[
\mathbf{v}_{tr}^{(c)} = \left( \omega^{(c)} \times \mathbf{r}_s \right) + \dot{\mathbf{R}}^{(c)}, \quad \mathbf{v}_{tr}^{(1)} = \left( \omega^{(1)} \times \mathbf{r}_s \right) + \left( \dot{\mathbf{O}}_5 \mathbf{O}_6 \times \mathbf{r}_s \right) + \dot{\mathbf{R}}^{(1)}.
\tag{9}
\]

where

\[
\dot{\mathbf{R}}_x^{(c)} = S_R \cos q + (\cos i \sin (j - q) (\sin \mu_y x_t + \cos \mu_y y_t)) + \cos (j - q) (-\cos \mu_y x_t + \sin \mu_y y_t), \quad \dot{\mathbf{R}}_y^{(c)} = -S_R \sin q + (-\cos i \cos (j - q) (\sin \mu_y x_t + \cos \mu_y y_t)) + \sin (j - q) (-\cos \mu_y x_t + \sin \mu_y y_t),
\]

\[
\dot{\mathbf{R}}_z^{(c)} = \sin i (\sin \mu_y x_t + \cos \mu_y y_t) \mu_y', \quad \dot{\mathbf{R}}_x^{(1)} = S_R \cos q + (\cos i \sin (j - q) (\sin \mu_y x_t + \cos \mu_y y_t)) + \cos (j - q) (-\cos \mu_y x_t + \sin \mu_y y_t), \quad \dot{\mathbf{R}}_y^{(1)} = -S_R \sin q + (-\cos i \cos (j - q) (\sin \mu_y x_t + \cos \mu_y y_t)) + \sin (j - q) (-\cos \mu_y x_t + \sin \mu_y y_t), \quad \dot{\mathbf{R}}_z^{(1)} = \sin i (\sin \mu_y x_t + \cos \mu_y y_t) \mu_y',
\]

\[
\mathbf{O}_5 \mathbf{O}_6 = [0 \ -E_m \ \Delta B]^T, \quad \dot{\mathbf{R}}^{(1)} = [0 \ -E_m' \ \Delta B']^T.
\]
Vectors $\mathbf{v}^{(c)}_t$ and $\mathbf{v}^{(l)}_t$ represent the linear velocities for the center of head cutter and the center of the work gear, respectively.

The equation of meshing $f_1$ is represented in the fixed coordinate system $S_3$ as

$$f_1 = n_5 \cdot \left( v^{(c)}_{tr} - v^{(l)}_{tr} \right) = 0. \quad (10)$$

The coordinate transformation matrix from cutting tool axis to the work-piece axis is shown as follows,

$$M_{1t}(\phi_1) = M_{17} \cdot M_{76} \cdot M_{56} \cdot M_{54} \cdot M_{43} \cdot M_{32} \cdot M_{2t}. \quad (11)$$

The position vector $r_1$ and unit normal vectors $n_1$ of generated tooth surface represented in the work gear coordinate system $S_1$, can be obtained by solving the simultaneous equations (10) and (12),

$$r_1 = [M_{1t}] r_t, \quad n_1 = [L_{1t}] n_t. \quad (12)$$

### 3. MATHEMATICAL MODEL OF THE UNIVERSAL EPG GEAR TESTER SIMULATOR

#### 3.1. Applied Coordinate System

The global tooth meshing condition can be simulated by a tooth contact analysis (TCA) computer program. TCA is developed to simulate the universal gear tester for decades. Pinion and gear are mounted on the virtual universal gear tester at the predetermined working condition. There are four degrees of freedom on the universal EPG tester: the offset movement perpendicular to the gear and pinion axes ($E$), the pinion axial movement ($P$), the gear axial movement ($G$), and the shaft angle ($\Sigma$). The position vectors and unit normal vectors of contact point $M$ on pinion surface and gear surface are denoted as $r^{(M)}_P$, $n^{(M)}_P$, $r^{(M)}_G$, $n^{(M)}_G$, respectively. As shown in Figure 3, the coordinate systems are arranged so that the rotation angles of pinion $\phi^P$ and gear $\phi^G$ are in the range of $[-\pi/2, \pi/2]$. The spatial relationship between the pinion and the gear is represented mathematically as follows,

$$\begin{align*}
\begin{bmatrix}
{x}^{(M)}_P \\
{y}^{(M)}_P \\
{z}^{(M)}_P
\end{bmatrix} &= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi^P & \sin \phi^P & 0 \\
0 & -\sin \phi^P & \cos \phi^P & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
{x}^{(M)}_P \\
{y}^{(M)}_P \\
{z}^{(M)}_P
\end{bmatrix}, \\
\begin{bmatrix}
{x}^{(M)}_f \\
{y}^{(M)}_f \\
{z}^{(M)}_f
\end{bmatrix} &= \begin{bmatrix}
\cos \Sigma & 0 & -\sin \Sigma & P \cos \Sigma \\
0 & -1 & 0 & 0 \\
-\sin \Sigma & 0 & -\cos \Sigma & -P \sin \Sigma
\end{bmatrix} \begin{bmatrix}
{x}^{(M)}_f \\
{y}^{(M)}_f \\
{z}^{(M)}_f
\end{bmatrix}, \\
\begin{bmatrix}
{x}^{(M)}_G \\
{y}^{(M)}_G \\
{z}^{(M)}_G
\end{bmatrix} &= \begin{bmatrix}
1 & 0 & 0 & G \\
0 & \cos \phi^G & -\sin \phi^G & -E \\
0 & \sin \phi^G & \cos \phi^G & 0
\end{bmatrix} \begin{bmatrix}
{x}^{(M)}_G \\
{y}^{(M)}_G \\
{z}^{(M)}_G
\end{bmatrix},
\end{align*}$$

where $\phi^P$ and $\phi^G$ are the rotation angle of pinion and gear, respectively. The position vectors and unit normal vectors of pinion and gear should be the same at the contact point. The contact path is the summation of instantaneous contact points. Therefore, the following system of equations should be observed at the contact point,

$$r^{(M^P)}_f = r^{(M^G)}_f, \quad n^{(M^P)}_f = -n^{(M^G)}_f. \quad (16)$$
We represent the angular velocities of pinion $\omega^P$ and gear $\omega^G$ in the coordinate system $S_f$ as follows,

$$\omega^P = \begin{bmatrix} -\cos \Sigma & 0 & \sin \Sigma \end{bmatrix}^T,$$

$$\omega^G = \begin{bmatrix} \eta_{GP} & 0 & 0 \end{bmatrix}, \quad \left| \omega^P \right| = 1. \quad (17)$$

The velocities of contact point $M$ in transfer motion of pinion $\Sigma_P$ and gear $\Sigma_G$ are represented as

$$v_{tr}^P = \omega^P \times r_f - E \times \omega^P,$$

$$v_{tr}^G = \omega^G \times r_f. \quad (18)$$

### 3.2. Relationship of Rotation Angles $\phi^G$ and $\phi^P$ of Mating Gears

Consider that the gear tooth surface $\Sigma_G$ and the pinion tooth surface $\Sigma_P$ are in tangency at contact point $M$ in fixed coordinate system $S_f$ as shown in Figure 4.

![Figure 4. Instantaneous axis of rotation of surfaces $\Sigma_P$ and $\Sigma_G$.](image-url)
According to the theory of gearing, the common unit surface normal $n^{(M)}_f$ at the contact point $M$ must pass through point $I$ which lies on the instantaneous axis of rotation and expressed as the following,

$$\frac{x_f^{(I)} - x_f^{(M)}}{n^{(M)}_{z_f}} = \frac{y_f^{(I)} - y_f^{(M)}}{n^{(M)}_{y_f}} = \frac{z_f^{(I)} - z_f^{(M)}}{n^{(M)}_{z_f}}.$$  

where

$$[r_f^{(I)}] = [x_f^{(I)} \ y_f^{(I)} \ z_f^{(I)}]^T = \begin{bmatrix} -\chi (\cos \Sigma + \eta_{GP}) \\ 0 \\ \chi \sin \Sigma \end{bmatrix}.$$ 

Here, $\chi$ is a position parameter of the pitch point $I$ along the instantaneous axis of rotation, and $\eta_{GP}$ is the instantaneous roll ratio between two mating gears. Equation (19) (equation of meshing $f_2$ between gear surface and pinion surface) could be expressed as

$$f_2 = -n^{(M)}_{yG'} \left( G \cos \phi' (E - \chi \sin \Sigma) \sin \phi' + y_f^{(M)} \right) + n^{(M)}_{zG'} \left( \cos \phi' (E - \chi \sin \Sigma) - G \sin \phi' + z_f^{(M)} \right) = 0.$$  

where

$$\chi = \frac{(-n^{(M)}_{yG'} \cos \phi' + n^{(M)}_{zG'} \sin \phi') x_G^{(M)} + n^{(M)}_{zG'} \left( G + y_f^{(M)} \cos \phi' - z_f^{(M)} \sin \phi' \right)}{(\cos \Sigma + \eta_{GP})(n^{(M)}_{yG'} \cos \phi' - n^{(M)}_{zG'} \sin \phi')}.$$ 

4. DIRECT RELATIONSHIP BETWEEN THE PRINCIPAL CURVATURES OF TWO MATING SURFACES

Consider that two rigid bodies ($i$ and $j$) are in contact with each other and perform a prescribed relative motion. Contacting surfaces $\Sigma_i$ and $\Sigma_j$ of the corresponding rigid body are in continuous tangency. Due to the continuous tangency of two mating surfaces, the first differential of the position vector and unit normal at the contact point must be the same on both contacting surfaces. The determination of the required relationships is based on the approach proposed by Litvin [4,5]. Conditions of continuous tangency of mating surfaces yield the following basic relationship,

$$\mathbf{v}_{tr}^{(i)} + \mathbf{v}_{tr}^{(j)} = \mathbf{v}_{tr}^{(i)} + \mathbf{v}_{tr}^{(j)}, \quad \mathbf{n}_{z}^{(j)} = \mathbf{n}_{z}^{(i)} + \left( \omega^{(ij)} \times \mathbf{n} \right), \quad \frac{d}{dt} \left( \mathbf{n} \cdot \mathbf{v}^{(ij)} \right) = 0.$$  

There are two cases of contact:

(i) the mating surfaces $\Sigma_i$ and $\Sigma_j$ are in line contact at every instant, and $\Sigma_i$ is the envelope to locus of $\Sigma_j$, and

(ii) surfaces $\Sigma_i$ and $\Sigma_j$ are in point contact at every instant (the contact of $\Sigma_i$ and $\Sigma_j$ is localized).

(i) SURFACES $\Sigma_i$ AND $\Sigma_j$ ARE IN LINE CONTACT. In case of line contact between surfaces, the relationship between the principal curvature of cutter surface $\Sigma_j$ and gear surface $\Sigma_i$, are represented as follows [4,5],

$$\tan 2\sigma^{(ij)} = \frac{2a_{13}a_{23}}{a_{23}^2 - a_{13}^2 + \left( \kappa_j^{(j)} - \kappa_q^{(j)} \right) a_{33}},$$

$$\kappa_j^{(j)} = \frac{1}{2} \left( \kappa_j^{(j)} + \kappa_q^{(j)} \right) + \frac{a_{13}^2 + a_{23}^2}{a_{33}} - \frac{a_{23}^2 - a_{13}^2 + \left( \kappa_j^{(j)} - \kappa_q^{(j)} \right) a_{33}}{a_{33} \cos 2\sigma^{(ij)}}.$$  

$$\kappa_q^{(j)} = \frac{1}{2} \left( \kappa_j^{(j)} + \kappa_q^{(j)} \right) + \frac{a_{13}^2 + a_{23}^2}{a_{33}} + \frac{a_{23}^2 - a_{13}^2 + \left( \kappa_j^{(j)} - \kappa_q^{(j)} \right) a_{33}}{a_{33} \cos 2\sigma^{(ij)}}.$$  


where

\[
\begin{align*}
a_{13} &= \omega^{(ji)} \cdot e_{s}^{(j)} - \kappa_{s}^{(j)} \left( v^{(ji)} \cdot e_{s}^{(j)} \right), \\
a_{23} &= -\omega^{(ji)} \cdot e_{s}^{(j)} - \kappa_{q}^{(j)} \left( v^{(ji)} \cdot e_{q}^{(j)} \right), \\
a_{33} &= -n^{(j)} \cdot \left[ \left( \omega^{(ji)} \times v_{tr}^{(i)} \right) - \left( \omega^{(ii)} \times v_{tr}^{(j)} \right) \right] \\
&+ \left\{ \left( \omega^{(ii)} \right)^2 \eta_{ji} \left( n \times k_{j} \right) \cdot r^{(j)} - R \right\} \\
&- n^{(j)} \cdot \left( \omega^{(ji)} \times v^{(j)} \right) + \kappa_{s}^{(j)} \left( v_{s}^{(j)} \right)^2 + \kappa_{q}^{(j)} \left( v_{q}^{(j)} \right)^2,
\end{align*}
\]

\begin{align*}
\omega^{(ji)} &= \omega^{(j)} - \omega^{(i)}, \\
v^{(ji)} &= v_{tr}^{(i)} - v_{tr}^{(j)}, \\
v_{s}^{(ji)} &= v^{(j)} \cdot e_{s}^{(j)}, \\
v_{q}^{(ji)} &= v^{(j)} \cdot e_{q}^{(j)},
\end{align*}

which unit vectors $e_{s}^{(j)}$ and $e_{q}^{(j)}$ represent the principal directions on $\Sigma_{j}$ at contact point $M$. $\kappa_{s}^{(j)}$ and $\kappa_{q}^{(j)}$ are the respective principal curvatures of $\Sigma_{j}$. Angle $\sigma^{(ij)}$ is formed between unit vectors $e_{s}^{(j)}$ and $e_{q}^{(j)}$ and is measured counterclockwise from $e_{s}^{(j)}$ to $e_{q}^{(j)}$. Symbols $v_{tr}^{(i)}$ and $v_{tr}^{(j)}$ indicate the velocity of contact point in transfer motion of gear surface $\Sigma_{i}$ and cutter surface $\Sigma_{j}$ in the fixed coordinate system, respectively. The differentiation of the roll ratio $\eta_{ji}$ is zero for the line contact case.

(ii) SURFACES $\Sigma_{i}$ AND $\Sigma_{j}$ ARE IN POINT CONTACT. In case of point contact of surfaces, as shown in Figure 5, the principal curvature $\kappa_{s}^{(i)}$ and $\kappa_{h}^{(i)}$ of surface $\Sigma_{i}$ can be determined if following parameters are assigned: the derivative of roll ratio $\eta_{ji}$, the ratio of the major axis of the contact ellipse and the surface separation $a/\delta$, the bias angle $\mu_{j}$ of the contact path at contact point $M$ on the surface $\Sigma_{j}$. If the bias angle $\mu_{j}$ is chosen, the components of the relative velocity, $v_{s}^{(i)}$ and $v_{q}^{(i)}$, are represented as

\[
\begin{bmatrix}
v_{s}^{(i)} \\
v_{q}^{(i)}
\end{bmatrix} = \frac{1}{a_{13} + a_{23} \tan \mu_{j}} \begin{bmatrix}
a_{33} - a_{23} \left( \tan \mu_{j} v_{s}^{(j)} - v_{q}^{(j)} \right) \\
a_{33} + a_{13} \left( \tan \mu_{j} v_{s}^{(j)} - v_{q}^{(j)} \right)
\end{bmatrix},
\]

The principal curvatures $\kappa_{s}^{(i)}$, $\kappa_{h}^{(i)}$ and directions angle $\sigma^{(ij)}$ between principal directions of two mating surfaces can be determined by the following equations.

![Figure 5. Definition of the contact path bias angle and contact ellipse.](image-url)
\[
\tan 2\sigma^{(i)} = \frac{2\left(a_{23}v_q^{(i)} + v_s^{(i)} \left(-a_{13} + v_s^{(i)} \kappa_S\right)\right)}{2\left(a_{23}v_q^{(i)} - a_{13}v_s^{(i)}\right) - \kappa_S \left(v_q^{(i)} \right)^2 - \left(v_s^{(i)} \right)^2} + \frac{g_2 \left(\left(v_q^{(i)} \right)^2 + \left(v_s^{(i)} \right)^2\right)}{2},
\]

where,
\[
\kappa_F^{(i)} = \frac{\kappa_S^{(i)} + g_i}{2},
\]
\[
\kappa_R^{(i)} = \frac{\kappa_S^{(i)} - g_i}{2},
\]

\[
\kappa_S = \frac{(a_{13}^2 + a_{23}^2) / \left(\left(v_q^{(i)} \right)^2 + \left(v_s^{(i)} \right)^2\right) - 4A^2}{(a_{23}v_q^{(i)} + a_{13}v_s^{(i)}) / \left(\left(v_q^{(i)} \right)^2 + \left(v_s^{(i)} \right)^2\right) + 2A},
\]

\[
|A| = \frac{\delta}{\alpha^2},
\]
\[
\kappa_S^{(j)} = \kappa_S^{(j)} + \kappa_q^{(j)},
\]
\[
\kappa_S^{(j)} = \kappa_S^{(j)} - \kappa_S,
\]
\[
g_i = \frac{\left(a_{13}v_q^{(i)} + v_s^{(i)} \left(a_{23} - \kappa_S v_q^{(i)}\right)\right) \cot \sigma^{(i)}}{\left(v_q^{(i)} \right)^2 + \left(v_s^{(i)} \right)^2}.
\]

5. DETERMINATION OF PINION MACHINE SETTINGS WITH MRM CORRECTION

Assume that the gear tooth surface is given by the existing machine setting calculation instruction such as Gleason SB, SGM. The local synthesis technique is applied at the specified contact point on the gear tooth surface to determine the machine settings with modified radial motion (MRM) correction. The blank dimension of the pinion is virtually the same as Gleason system. There are five machine settings \((S_R, S_R', q, \eta_P, \phi_1)\) of MRM correction and two surface parameters \((\mu, \beta)\) should be determined as shown in follows.

STAGE 1. DEFINITION OF THE SPECIFIED CONTACT POINT M. The specified contact point \(M\) on the pinion tooth surface on the projection plane, the \(X - \bar{R}\) plane, is defined by the following equations,

\[
\begin{align*}
\bar{X}^{(M)} &= x_P^{(M)}, \\
\bar{R}^{(M)} &= \sqrt{\left(y_P^{(M)} \right)^2 + \left(z_P^{(M)} \right)^2},
\end{align*}
\]

where \(r_P^{(M)} (x_P^{(M)}, y_P^{(M)}, z_P^{(M)})\) is the position vector of the specified contact point \(M\) on the pinion tooth surface. Iterative processes are required to determine the actual values of position vector \(r_G^{(M)} (x_G^{(M)}, y_G^{(M)}, z_G^{(M)})\) of contact point \(M\) on gear surface as follows.

Step 1. Choose initial cone distance \(A_m^{(0)}\) and radial \(R_m^{(0)}\) on gear blank according to the given data \(X - \bar{R}\), and

\[
\begin{align*}
A_m^{(i)} &= x_G^{(i)} \cos \Gamma_G + \sqrt{(y_G^{(i)})^2 + (z_G^{(i)})^2} \sin \Gamma_G, \\
R_m^{(i)} &= \sqrt{(y_G^{(i)})^2 + (z_G^{(i)})^2}, \quad i = 0, 1, 2, \ldots
\end{align*}
\]
Step 2. Determine parameters $w_2$, $\beta_2$, and $\phi_2$ from equation (10) and (27). The position vector $r_G^{(M)}$ and unit surface normals $n_G^{(M)}$ of contact point on pinion tooth surface thus are determined.

Step 3. Determine the meshing angle $\phi^G$ of gear from equation (20) while the meshing angle $\phi^P$ of pinion is set zero at the chosen contact point $M$. In equation (20),

$$\eta_{GP} = \frac{n_p}{n_G},$$  \hspace{1cm} (28)

where $n_p$ and $n_G$ are the tooth number of pinion and gear, respectively.

Step 4. Rearrange equations (12)-(14) as follows,

$$X^{(M)} = X^{(M)} \cos \xi - Z^{(M)} \sin \xi,$$

$$Z^{(M)} = Z^{(M)} \cos \xi - X^{(M)} \sin \xi,$$

$$\xi^{1(M)} \sin \phi_1 = \frac{Z^{(M)} \cos \phi_2 - Z^{(M)} \cos \phi_2}{Z^{(M)} \cos \phi_2 + Z^{(M)} \cos \phi_2},$$

and substitute into equation (26) to calculate $X^{(M)}$ and $R^{(M)}$. If the value of $X^{(M)}$ and $R^{(M)}$ differ from the designed ones, change parameters $\left( A_{m}, R_{m}^{(i)} \right)$ and restart the iterations.

The position vectors and the unit surface normals of pinion and gear tooth surfaces should coincide with each other at the contact point. Thus,

$$r^{(M)} = r^{(M)} \cos \xi - z^{(M)} \sin \xi,$$

$$n^{(M)} = n^{(M)} \cos \xi - n^{(M)} \sin \xi,$$

and substitute into equation (26) to calculate $X^{(M)}$ and $R^{(M)}$. If the value of $X^{(M)}$ and $R^{(M)}$ differ from the designed ones, change parameters $\left( A_{m}, R_{m}^{(i)} \right)$ and restart the iterations.

The position vectors and the unit surface normals of pinion and gear tooth surfaces should coincide with each other at the contact point. Thus,

$$r^{(M)} = r^{(M)} \cos \xi - Z^{(M)} \sin \xi,$$

$$n^{(M)} = n^{(M)} \cos \xi - n^{(M)} \sin \xi,$$

and substitute into equation (26) to calculate $X^{(M)}$ and $R^{(M)}$. If the value of $X^{(M)}$ and $R^{(M)}$ differ from the designed ones, change parameters $\left( A_{m}, R_{m}^{(i)} \right)$ and restart the iterations.

Stage 2. Determination of Principal Curvatures $\kappa^{(P)}$ and $\kappa^{(P)}$ of Pinion Tooth Surface. The cutter-tool surface $\Sigma_c$ and pinion surface $\Sigma_P$ is in line contact at every instant, i.e., one of the relative curvatures is infinite. Therefore,

$$[A] = \frac{\delta}{a^2} = 0, \quad \sigma^{(P)c} = \frac{1}{2} \cos^{-1} \left( \frac{-\left( \kappa^{(P)}_\Sigma - \kappa^{(c)}_\Sigma \right)^2 + g_P^2 + g_c^2}{2g_P g_c} \right),$$

where $\kappa^{(P)}_\Sigma = \kappa^{(P)}_f + \kappa^{(P)}_h, \quad \kappa^{(c)}_\Sigma = \kappa^{(c)}_s + \kappa^{(c)}_q$, and $g_P = \kappa^{(P)}_f - \kappa^{(P)}_h$.

Rearrange equations (22) and (23), the principal curvature $\kappa^{(P)}_f$ of pinion tooth is expressed as

$$\kappa^{(P)}_f = \frac{-1}{4a_23a_333\kappa_{ch}} \left( a_2^3 \left( g_c - \kappa_{ch} \right) + 2a_33 \left( g_c \kappa_{ch} + \kappa_{ch} \right) + 2a_33 \left( g_c^2 - \kappa^{(c)}_\Sigma \kappa_{ch} \right) + \right.$$

$$\left. -2a_2^3 \left( a_2^2 + 2a_33g_c \right) \left( g_c^2 - \kappa^{(c)}_\Sigma \right) + a_2^4 \left( g_c - \kappa_{ch} \right)^2 + \left( g_c + \kappa_{ch} \right) \right),$$

where

$$\kappa_{ch} = \kappa^{(c)}_\Sigma - \kappa^{(P)}_h, \quad \kappa^{(c)}_\Sigma = \kappa^{(c)}_s + \kappa^{(c)}_q, \quad g_c = \kappa^{(c)}_s - \kappa^{(c)}_q.$$
Table 1. Gear blank dimension sheet for the numerical example.

<table>
<thead>
<tr>
<th>Items</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Blank Data</td>
</tr>
<tr>
<td>Teeth Number</td>
<td>17</td>
<td>32</td>
</tr>
<tr>
<td>Modulus</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Pressure Angle</td>
<td>20 degree</td>
<td></td>
</tr>
<tr>
<td>Spiral Angle</td>
<td>35 degree</td>
<td></td>
</tr>
<tr>
<td>Shaft Angle</td>
<td>90 degree</td>
<td></td>
</tr>
<tr>
<td>Face Width</td>
<td>12.0 mm</td>
<td></td>
</tr>
<tr>
<td>Face Angle</td>
<td>32D 8M 24S</td>
<td>64D 24M 59S</td>
</tr>
<tr>
<td>Pitch Angle</td>
<td>27D 58M 46S</td>
<td>62D 1M 14S</td>
</tr>
<tr>
<td>Outside Diameter</td>
<td>37.99144 mm</td>
<td>65.06980 mm</td>
</tr>
<tr>
<td>Pitch Apex to Crown</td>
<td>30.93977 mm</td>
<td>15.99313 mm</td>
</tr>
</tbody>
</table>

Table 2. Original machine settings based on the Gleason SB, SGM.

<table>
<thead>
<tr>
<th>Items</th>
<th>Pinion I.B.</th>
<th>Pinion O.B.</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grind Wheel Specification</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point Dia. of Grind Wheel</td>
<td>74.422 mm</td>
<td>72.644 mm</td>
<td>73.0 mm</td>
</tr>
<tr>
<td>Blade Angle</td>
<td>22D 0M 0S</td>
<td>18D 0M 0S</td>
<td>I.B. 22D 20M 0S</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>O.B. 17D 40M 0S</td>
</tr>
<tr>
<td>Point Width</td>
<td>-</td>
<td>-</td>
<td>0.889 mm</td>
</tr>
<tr>
<td>Fillet Radius</td>
<td>0.18137 mm</td>
<td>0.18137 mm</td>
<td>I.B. 0.1 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>O.B. 0.1 mm</td>
</tr>
<tr>
<td>Original Machine Settings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Swivel Angle</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Tilt Angle</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Head Cutter Rotation Angle</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sliding Base</td>
<td>-0.23991 mm</td>
<td>0.27140 mm</td>
<td>0.08658 mm</td>
</tr>
<tr>
<td>Eccentric Angle</td>
<td>8D 53M 4S</td>
<td>8D 39M 5S</td>
<td>8D 47M 9S</td>
</tr>
<tr>
<td>Blank Position</td>
<td>0.55302 mm</td>
<td>-0.63106 mm</td>
<td>-0.10226 mm</td>
</tr>
<tr>
<td>Blank Offset</td>
<td>-0.34465 mm</td>
<td>-0.58474 mm</td>
<td>0.17489 mm</td>
</tr>
<tr>
<td>Cradle Angle</td>
<td>285D 7M 15S</td>
<td>287D 2M 27S</td>
<td>77D 8M 55S</td>
</tr>
<tr>
<td>Generating Cam Guide Angle</td>
<td>-1D -28M 0S</td>
<td>-1D -12M 0S</td>
<td>0D 20M 0S</td>
</tr>
<tr>
<td>Generating Cam Setting</td>
<td>177.8 mm</td>
<td>177.8 mm</td>
<td>152.4 mm</td>
</tr>
<tr>
<td>Generating Cam Pitch Radius</td>
<td>144.80794 mm</td>
<td>151.30018 mm</td>
<td>127.33528 mm</td>
</tr>
<tr>
<td>Index Interval</td>
<td>10</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Machine Root Angle</td>
<td>25D 35M 1S</td>
<td>25D 35M 1S</td>
<td>25D 51M 36S</td>
</tr>
<tr>
<td>Cradle rotation angle for pinion convex</td>
<td>0.468293φ₁ + 2.03407 × 10⁻³φ₁² + 2.57946 × 10⁻⁵φ₁³ +1.86084 × 10⁻⁴φ₁⁴ + 4.22193 × 10⁻⁶φ₁⁵ + 1.66451 × 10⁻⁸φ₁⁶</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cradle rotation angle for pinion concave</td>
<td>0.483281φ₁ + 1.75079 × 10⁻³φ₁² + 1.85905 × 10⁻⁵φ₁³ +1.70513 × 10⁻⁴φ₁⁴ + 3.2215 × 10⁻⁶φ₁⁵ + 1.62271 × 10⁻⁸φ₁⁶</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cradle rotation angle for gear</td>
<td>0.890650φ₁ - 1.72953 × 10⁻³φ₁² + 9.70415 × 10⁻⁶φ₁³ -5.71650 × 10⁻⁴φ₁⁴ + 5.83123 × 10⁻⁶φ₁⁵ -1.84461 × 10⁻⁸φ₁⁶</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Symbols κ₁(P) and κ₉(P) denote the principal curvatures of pinion ΣP which is solved by equation (22). Coefficients a₁₃, a₂₃, and a₃₃ are derived from equation (23) with η₀c = 0.
Table 3. The output MRM correction variables for Example 1.

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Pinion convex</th>
<th>Pinion concave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact point $M(X(M), Y(M))$</td>
<td>(24.60241, 13.56147)</td>
<td>(24.15448, 13.31456)</td>
</tr>
<tr>
<td>Derivative of roll ratio $G_{BP}$</td>
<td>$7.05 \times 10^{-3}$</td>
<td>$-4.029 \times 10^{-3}$</td>
</tr>
<tr>
<td>Major axis of the contact ellipse $a$</td>
<td>4 mm</td>
<td>4 mm</td>
</tr>
<tr>
<td>Bias angle $\mu_G$ of the contact path</td>
<td>62D 22M 40S</td>
<td>110D 8M 2S</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output Variables</th>
<th>Pinion convex</th>
<th>Pinion concave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cradle radial setting $S_R$</td>
<td>31.3794 mm</td>
<td>30.7469 mm</td>
</tr>
<tr>
<td>Derivative Cradle radial setting $S'_R$</td>
<td>0.005216</td>
<td>-0.03357</td>
</tr>
<tr>
<td>Initial cradle angle setting $\theta_c$</td>
<td>72D 26M 34S</td>
<td>79D 41M 6S</td>
</tr>
<tr>
<td>Differential cradle angle $\eta_{c,p}$</td>
<td>0.468097</td>
<td>0.485007</td>
</tr>
</tbody>
</table>

Table 4. The output MRM correction variables for Example 2.

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Pinion convex</th>
<th>Pinion concave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact point $M(X(M), Y(M))$</td>
<td>(24.6012, 13.5608)</td>
<td>(24.15704, 13.3160)</td>
</tr>
<tr>
<td>Derivative of roll ratio $G_{BP}$</td>
<td>$7.05 \times 10^{-3}$</td>
<td>$-5.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Major axis of the contact ellipse $a$</td>
<td>4 mm</td>
<td>4 mm</td>
</tr>
<tr>
<td>Bias angle $\mu_G$ of the contact path</td>
<td>35D 0M 0S</td>
<td>55D 0M 0S</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output Variables</th>
<th>Pinion convex</th>
<th>Pinion concave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cradle radial setting $S_R$</td>
<td>31.381326 mm</td>
<td>30.74654 mm</td>
</tr>
<tr>
<td>Derivative Cradle radial setting $S'_R$</td>
<td>-0.097631</td>
<td>-0.016353</td>
</tr>
<tr>
<td>Initial cradle angle setting $\theta_c$</td>
<td>72D 26M31S</td>
<td>79D 40M 55S</td>
</tr>
<tr>
<td>Differential cradle angle $\eta_{c,p}$</td>
<td>0.47095</td>
<td>0.484464</td>
</tr>
</tbody>
</table>

Stage 3. Determination of Principal Curvatures $\kappa^P_f$ and $\kappa^P_h$ of Gear Tooth Surface. The principal curvatures $\kappa^P_f$ and $\kappa^P_h$ are solved by equations (4) and (22) with $\eta_G = 0$. Equations (30), (31), (10), (25), and (33) yield seven independent scalar equations with seven variables: $u$, $\beta$, $S_R$, $S'_R$, $q$, $\eta_{c,p}$, and $\phi_1$ with modified radial motion (MRM) correction.

The cradle rotation angle $\phi_0$ and cradle radial setting $S_R$ are linear function of $\phi_1$ and are represented as,

$$q = \theta_c + \eta_{c,p} \phi_1, \quad S_R = S_R^{(0)} + S'_R \phi_1,$$

where symbol $\theta_c$ and $S_R^{(0)}$ denote the initial cradle rotation angle setting and the initial cradle radial setting, respectively.

6. NUMERICAL EXAMPLE AND DISCUSSION

A face milling hypoid generator without cutter tilted is used to verify the proposed mathematical model. The blank dimension sheet is listed in Table 1 while the corresponding machine-setting summary obtained by Gleason SB, SGM calculation instruction (Gleason #463) is listed in Table 2 for the numerical example. The TCA results by original Gleason SB, SGM system is shown in Figure 6.

The gear tooth surface is assumed to be given by Gleason SB, SGM summary while the tooth surface of pinion is generated by the proposed mathematical model. Two numerical examples are proposed to show the separate control abilities on the bias of contact pattern and the motion curve by the proposed methodology.
As shown in Example 1, parameters, the position of specified contact point \( M(\hat{X}^{(M)}, \hat{Y}^{(M)}) \) on pinion blank, the derivative \( \eta_{GP} \) of roll ratio, the ratio \( a/\delta \), the bias angle \( \mu_G \) at contact point \( M \) of the contact path, are assumed to be given and listed in Table 3. The calculated variables, \( S_{R}^{(0)}, S_{R}, \theta_c, \) and \( \eta_{cP} \), are listed in Table 3. The TCA results after MRM correction

<table>
<thead>
<tr>
<th>Gleason system (Pinion Convex)</th>
<th>Gleason system (Pinion Concave)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Zero position</strong></td>
<td><strong>Zero Position</strong></td>
</tr>
<tr>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
</tr>
<tr>
<td><strong>Mean</strong> (E=-0.04121, P=0.05401)</td>
<td><strong>Mean</strong> (E=-0.02185, P=0.021)</td>
</tr>
<tr>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
</tr>
<tr>
<td><strong>Toe</strong> (E=-0.19628, P=0.22757)</td>
<td><strong>Toe</strong> (E=0.12221, P=-0.10699)</td>
</tr>
<tr>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td><strong>Heel</strong> (E=0.17747, P=0.17857)</td>
<td><strong>Heel</strong> (E=-0.22682, P=0.19089)</td>
</tr>
</tbody>
</table>

Figure 6. The results of TCA by Gleason SB, SGM calculation.
for Example 1 is shown in Figure 7. The contact point $M$ is assigned to toe position on pinion convex and concave sides. The bearing ratios of the proposed MRM method are almost the same with the Gleason system with the same cutter geometry. As shown in Example 1, the proposed mathematical model could easily control the contact position and bearing ratio.
As shown in Example 2, the control parameters are the same as in Example 1 except the derivative \( \eta_{GP} \) of roll ratio and the bias angle \( \mu_C \) at contact point \( M \) of the contact path as listed in Table 4. On the pinion convex side, we try to increase the bias-in of the contact path.

<table>
<thead>
<tr>
<th></th>
<th>Mean (E=0.08515, P=-0.04113)</th>
<th>Mean (E=0.15493, P=0.20793)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toe</td>
<td>(E=-0.02662, P=0.025)</td>
<td>(E=0.01682, P=0.00451)</td>
</tr>
<tr>
<td>Heel</td>
<td>(E=0.23713, P=-0.13485)</td>
<td>(E=-0.41261, P=0.48495)</td>
</tr>
</tbody>
</table>

**Figure 8.** The results of TCA after MRM correction for Example 2.
while keeping the kinematical error same as Example 1. However, on the pinion concave side, we try to keep the bias of the contact path same as Example 1 but increase the kinematical error (profile-out). The TCA results after MRM correction for Example 2 is shown in Figure 8. Compare Figure 7 with Figure 8, the motion curve and contact bias are modified as we planned. On the pinion convex side, the bias-in of the contact path is increased while the kinematical error are the almost same as Example 1. On the pinion concave side, the bias of the contact path are the almost same as Example 1 but the kinematical error are increased. Example 2 illustrate the capability of the proposed methodology to control the bias and kinematic error independently.

6. CONCLUSION

A methodology is proposed to determine the machine settings with Modified Radial Motion (MRM) correction at specified contact position. By adding an extra degree of freedom—radial distance on the machine plane—there are enough D.O.F. to control on the bias and kinematical error separately. Based on the (MRM) correction, the contact position, the motion curve, and the contact path bias on the pinion tooth surface can be controlled independently. As shown by the numerical examples, the contact position, the bias of contact pattern, and the motion curve were modified as planned. The proposed MRM methodology is useful to gain more control on the contact pattern and the motion characteristics.

REFERENCES