Abstract—A direct reconstruction method for three-dimensional (3-D) electrical resistance tomography was introduced by using the factorization method. Compared with the traditional image reconstruction algorithms based on the sensitivity/Jacobian matrix, the conductivity distribution in any part of the 3-D region of interest can be obtained directly and independently. A new way to calculate the Neumann-to-Dirichlet map was also introduced by using the adjacent current pattern. Several phantoms were constructed for image reconstruction in three dimensions. The data were collected from 16 electrodes on a single cross section, which can be only used to produce two-dimensional images in the literature. Neither matrix inversion nor iteration was used in the process of image reconstruction. The reconstructed results validated the feasibility of the method.

Index Terms—Direct reconstruction method, electrical resistance tomography, factorization method, Neumann-to-Dirichlet map, three-dimensional (3-D) imaging.

I. INTRODUCTION

ELECTRICAL resistance tomography (ERT) can be used to produce images inside a pipe/vessel by reconstructing the associated electrical conductivity. It is an attractive technique for visual monitoring for its non-invasive, non-intrusive, and fast response. Usually, sinusoidal currents are applied into the volume through electrodes on the boundary, and the resulting voltages are measured on the electrodes. As a result, the internal conductivity/resistivity distribution can be computed based on the boundary data by using certain image reconstruction algorithm [1]. Many applications of this technology are developed for both medical and industrial uses [2]. Usually, 2-D ERT are applied to obtain the information of the cross sections of the region of interest. In two-dimensional (2-D) ERT, the images are reconstructed based on the assumption that the injected currents are confined to the 2-D electrode plane. However, most industrial and medical electrical imaging problems are fundamentally three-dimensional (3-D); the current spreads out in three dimensions as electric current passes through the object. Actually, a substantial amount of current flows out of the electrode plane, creating distortions in the resulting images [3]. The 2-D ERT is a simplification of the 3-D case, which gives more spatial information of the material distributions and more realistic [4]. For example, a system with several planes of electrodes must make measurements of voltage on all planes, even if currents are only driven on one plane at a time [5].

In the early stages of ERT in the beginning of 1980s, only cross-sectional 2-D images were considered [6]. The image reconstruction for 2-D ERT has been studied extensively in the literature [7]. Most of the reconstruction algorithms can be clarified into two categories, i.e., sensitivity matrix-based algorithms [8] and direct algorithms [9], [10]. The former usually results in solving an ill-conditioned linear equation, which means the values of all pixels in the sensing region have to be reconstructed simultaneously. The algorithms in the latter category were implemented through the calculation of the Dirichlet-to-Neumann map or Neumann-to-Dirichlet map, and the gray value at each pixel can be calculated directly and independently [11]–[14]. In recent years, 3-D ERT are used to monitor drug release three-dimensionally as a function of time [15], to serve as an adjunct modality for enhancing standard clinical ultrasound imaging of the prostate [16], to image the rapidly varying objects in the flow pipes [17], and to monitor sedimentation in the control and optimization of industrial sedimentation processes [18] and etc. Compared with the 2-D ERT, the 3-D ERT requires more spatial information; as a result, usually more electrodes are required. The algorithms for image reconstruction in the 3-D ERT are also more time consuming. Fully 3-D ERT has been discussed, for example in [3], [4], [19]–[21]. A direct sensitivity matrix approach for fast 3-D image reconstruction was proposed in [5], which used the boundary element method in the construction of this matrix [20]. A state estimation approach approximated in a low-order basis was proposed and compared with conventional Kalman filter approaches in a numerical 3-D EIT study [22]. Meanwhile, there also exist direct algorithms for the 3-D ERT. For example, the complex geometrical optics solutions to the conductivity equation can be used to reconstruct the conductivity distributions [23]. However, the electrode configuration
in [23] is not suitable to monitor the flows in the pipe, since electrodes point electrodes located at the Gaussian quadrature points on the sphere. The conductivity at the boundary can be recovered with reasonable accuracy using practically realizable measurements [24].

Recently, we proposed a direct reconstruction method for 3-D ERT with only 16 electrodes at one cross section, in which case, only 2-D images can be produced in the literature [25]. In this paper, the method was extended in details and applied to more cases of phantoms for 3-D ERT. Moreover, in the literature, for the N-electrode sensor, the Neumann-to-Dirichlet map can be constructed by using the inner product method or from the Dirichlet-to-Neumann map [7], [11]. However, these two methods cannot provide a direct physical interpretation of the Neumann-to-Dirichlet map. A new way to calculate the Neumann-to-Dirichlet map was also introduced by using the adjacent current pattern. The reconstructed images validate the feasibility of the proposed method.

II. METHODS

A. Fundamentals

The governing equation of the sensing field, denoted by Ω, in ERT is

\[ \nabla \cdot [\sigma(x, y, z) \nabla \varphi(x, y, z)] = 0 \]  (1)

where \( \sigma(x, y, z) \) and \( \varphi(x, y, z) \) are conductivity and electric potential at \( (x, y, z) \), respectively. If the conductivity distribution can be denoted by \( \sigma(x, y, z) \), the current density-to-voltage map, i.e., the Neumann-to-Dirichlet map, can be written as

\[ R_\sigma : \sigma(x, y, z) \frac{\partial \varphi(x, y, z)}{\partial n} \bigg|_{\partial \Omega} \rightarrow \varphi(x, y, z) \bigg|_{\partial \Omega} \]  (2)

where \( R_\sigma \) denotes the current density-to-voltage map when \( \Omega \) contains the conductivity distribution \( \sigma(x, y, z) \). Specially, \( R_1 \) denotes the current density-to-voltage map when \( \Omega \) contains a constant conductivity of one [14].

The Neumann-to-Dirichlet map can also be obtained from the Dirichlet-to-Neumann map [11]. The two maps are related as follows.

If the conductivity distribution is \( \sigma(x, y, z) \), denote the voltage-to-current density map, i.e., the Dirichlet-to-Neumann map, as

\[ \Lambda_\sigma : \varphi(z) \bigg|_{\partial \Omega} \rightarrow \sigma(z) \frac{\partial \varphi(z)}{\partial n} \bigg|_{\partial \Omega} \]  (3)

here, \( \Lambda_\sigma(\cdot) \) denotes the voltage-to-current density map when the sensing field \( \Omega \) contains the conductivity distribution \( \sigma(x, y, z) \). In special, \( \Lambda_1(\cdot) \) denotes the voltage-to-current density map when \( \Omega \) contains a constant conductivity of one. The functional \( \Lambda_\delta(\cdot) \) represents the difference between the two Dirichlet-to-Neumann maps \( \Lambda_\sigma(\cdot) \) and \( \Lambda_1(\cdot) \), namely

\[ \Lambda_\delta(\cdot) = (\Lambda_\sigma - \Lambda_1)(\cdot) \]  (4)

where \( \sigma = 1 + \delta \sigma \).

For the same sensor configuration, the Neumann-to-Dirichlet map \( R_\sigma(\cdot) \) and the Dirichlet-to-Neumann maps \( \Lambda_\sigma(\cdot) \) satisfies

\[
\begin{cases}
\Lambda_\delta R_\sigma(\cdot) = I(\cdot) \\
R_\sigma \Lambda_\delta(\cdot) = I(\cdot) - P(\cdot)
\end{cases}
\]  (5)

where \( I(\cdot) \) and \( P(\cdot) \) means the identity operator and a non-zero projection operator, respectively [26]. It should be noted that \( \Lambda_\delta(\cdot) \) and \( R_\sigma(\cdot) \) are not the inverse operator of each other, as the dimension of the operator is \( N - 1 \) if there exists \( N \) electrodes in a sensor. In the following sections, the symbols for the operators are rewritten for convenience, e.g., \( \Lambda_\sigma(\cdot) \) and \( R_\sigma(\cdot) \) are denoted by \( \Lambda_\sigma \) and \( R_\sigma \), respectively.

B. Construction of the Neumann-to-Dirichlet Map by Using the Adjacent Current Pattern

For the N-electrode sensor, the Neumann-to-Dirichlet map can be constructed by using the inner product method or from the Dirichlet-to-Neumann map [7], [11]. However, these two methods cannot provide a direct physical interpretation of the Neumann-to-Dirichlet map. Here, we provide a direct way to calculate the map.

When the adjacent current patterns are applied to the sensor with \( N \) electrodes, if the unit electrical current flows from the \( i \)th (\( 1 \leq i \leq N \)) electrode to the \( i + 1 \)th electrode, the electrical potentials on the \( k \)th (\( 1 \leq k \leq N \)) and \( k + 1 \)th electrodes satisfies

\[ U_{i,i+1}^{k,k+1} \]

\[ U_{i,i+1}^{k+1,k} \]

\[ R_{i,i+1}^{k,k+1} \]

here, \( R_{i,i+1}^{k,k+1} \) represents the mutual resistance when the unit electrical current flows from the \( i \)th electrode to the \( i + 1 \)th electrode and the electrical voltage is measured between the \( k \)th and \( k + 1 \)th electrodes. Meanwhile, the electrical potentials on the \( k \)th and \( k + 1 \)th electrodes are denoted by \( U_{i,i+1}^{k} \) and \( U_{i,i+1}^{k+1} \), respectively.

In particular, we have

\[ U_{i,i+1}^{k} - U_{i,i+1}^{k+1} = R_{i,i+1}^{k,k+1} \]  (6)

Then

\[ U_{i,i+1}^{k} = U_{i,i+1}^{k+1} - R_{i,i+1}^{k,k+1} = U_{i,i+1}^{k} - \sum_{m=0}^{N-k} R_{i,i+1}^{k+m,k+m+1} \]  (7)

Since only the electrical potential between each two neighboring electrodes is used in the measurement, without loss of generality, the ground can be selected to satisfy \( \sum_{k=1}^{N} U_{i,i+1}^{k} = 0 \), then

\[ \sum_{k=1}^{N} \left( U_{i,i+1}^{k} - \sum_{m=0}^{N-k} R_{i,i+1}^{k+m,k+m+1} \right) = 0. \]  (8)

Namely

\[ U_{i,i+1}^{k} = \frac{1}{N} \sum_{k=1}^{N} k R_{i,i+1}^{k,k+1}. \]  (9)
From (6) and (7), we can obtain that
\[ \sum_{k=1}^{N} R_{i,i+1}^{k,k+1} = 0. \] (11)

According to (8) and (11), the general form of \( U_{i,i+1}^{k} \) can be written as
\[ U_{i,i+1}^{k} = \frac{1}{N} \sum_{n=1}^{N-1} nR_{i,i+1}^{n+n+k+1} \] (12)

here, for simplicity of the symbol notation, we assume that \( R_{i,i+1}^{N+k,N+k+1} = R_{i,i+1}^{k,k+1} \).

As a result, the general form of the electrical potential on each electrode can be expressed as
\[
\begin{bmatrix}
  U_{1,2}^1 & U_{1,3}^1 & \cdots & U_{1,N}^1 \\
  U_{2,3}^2 & U_{2,4}^2 & \cdots & U_{2,N}^2 \\
  \vdots & \vdots & \ddots & \vdots \\
  U_{N,1}^{N-1} & U_{N,2}^{N-1} & \cdots & U_{N,N}^{N-1}
\end{bmatrix}
= \frac{1}{N}
\begin{bmatrix}
  0 & 1 & 2 & \cdots & N-1 \\
  N-1 & 0 & 1 & \cdots & N-2 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  1 & 2 & 3 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  R_{1,2}^{1,2} & R_{1,3}^{1,2} & \cdots & R_{1,N}^{1,2} \\
  R_{2,3}^{2,3} & R_{2,4}^{2,3} & \cdots & R_{2,N}^{2,3} \\
  \vdots & \vdots & \ddots & \vdots \\
  R_{N,1}^{N,1} & R_{N,2}^{N,1} & \cdots & R_{N,N}^{N,1}
\end{bmatrix}.
\] (13)

For simplicity of writing, we denote
\[
B = \frac{1}{N}
\begin{bmatrix}
  0 & 1 & 2 & \cdots & N-1 \\
  N-1 & 0 & 1 & \cdots & N-2 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  1 & 2 & 3 & \cdots & 0
\end{bmatrix}
\]

\[
R =
\begin{bmatrix}
  R_{1,2}^{1,2} & R_{1,3}^{1,2} & \cdots & R_{1,N}^{1,2} \\
  R_{2,3}^{2,3} & R_{2,4}^{2,3} & \cdots & R_{2,N}^{2,3} \\
  \vdots & \vdots & \ddots & \vdots \\
  R_{N,1}^{N,1} & R_{N,2}^{N,1} & \cdots & R_{N,N}^{N,1}
\end{bmatrix},
\]

\[
U =
\begin{bmatrix}
  U_{1,2}^1 & U_{1,3}^1 & \cdots & U_{1,N}^1 \\
  U_{2,3}^2 & U_{2,4}^2 & \cdots & U_{2,N}^2 \\
  \vdots & \vdots & \ddots & \vdots \\
  U_{N,1}^{N-1} & U_{N,2}^{N-1} & \cdots & U_{N,N}^{N-1}
\end{bmatrix}.
\]

From the definition of the Neumann-to-Dirichlet map in (3), the map for the conductivity distribution \( \sigma(x,y,z) \) can be approximated by an \( N \times N \) matrix \( R_{\sigma}^{N \times N} \), which satisfies
\[ U = BR = R_{\sigma}^{N \times N}(I_{adj}). \] (18)

Since
\[ B^{T}I_{adj} = \frac{1}{N}
\begin{bmatrix}
  1 & \cdots & 1 \\
  1 & \cdots & 1 \\
  \vdots & \ddots & \vdots \\
  1 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
  1 - N & 1 & 1 & \cdots & 1 \\
  1 - N & 1 & \cdots & 1 \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  1 & \cdots & 1 & 1 - N
\end{bmatrix}
\] (19)

here, the upper index \( T \) in \( B^{T} \) means the transpose operation of the matrix, it can be approved that the first \( n-1 \) eigenvalues of \( B^{T}I_{adj} \) are all \(-1\), while the \( n \)th eigenvalue is zero. By calculating the eigenvalues and eigenvectors of \( B^{T}I_{adj} \), it can be rewritten as
\[ B^{T}I_{adj} = P \{ \text{diag}([-1 -1 \cdots -1 0]) \}_{N \times N} \} P^{T} \] (20)

where \( \text{diag}(\cdot)_{N \times N} \) means the \( N \times N \) diagonal matrix, \( P^{T}P = I, P = [p_{1} \ p_{2} \ \cdots \ p_{N-1} \ p_{N}] \). For the convenience of writing, we denote that the eigenvector \( p_{i} \) is the \( i \)th \((1 \leq i \leq n-1)\) column of \( P \), \( B^{T}I_{adj}p_{i} = (-1)p_{i} \), and \( p_{N} \) is the eigenvector corresponds eigenvalue of zero, and \( B^{T}I_{adj}p_{N} = (0)p_{N} \). Since the mean value of each row in (19) is zero, \( p_{N} = (1/N)[1 \ 1 \ \cdots \ 1]^{T} \).

As a result, \( B^{T}I_{adj} \) can be rewritten as
\[ B^{T}I_{adj} = -I_{N \times N} + p_{N}p_{N}^{T} \] (21)

here, \( I_{N \times N} \) is the \( N \times N \) identity matrix.

Similarly, by calculating the eigenvalues and eigenvectors, \( R \) has the rank of \( N-1 \), it can also be written in the following form:
\[ R = QS_{R}Q^{T} \]
\[ = Q \{ \text{diag}([\lambda_{1} \ \lambda_{2} \ \cdots \ \lambda_{N-1} \ 0])_{N \times N} \} Q^{T} \] (22)

where \( S_{R} = \text{diag}([\lambda_{1} \ \lambda_{2} \ \cdots \ \lambda_{N-1} \ 0]_{N \times N}) \) is a diagonal matrix consisting of \( n \) eigenvalues of \( R \), \( Q \) is composed by the associated eigenvectors. \( Q^{T}Q = I, Q = [q_{1} \ q_{2} \ \cdots \ q_{N-1} \ q_{N}] \).
the eigenvector $q_i$ is the $i$th ($1 \leq i \leq n - 1$) column of $Q$, $Rq_i = \lambda iq_i$, and $q_N$ is the eigenvector corresponding eigenvalue of zero, and $Rq_N = (0)q_N$. Since the mean value of each row in (11) is zero, $q_N = (1/N)[1 \; 1 \; \cdots \; 1]_{1 \times N}$.

Hence

$$Q^T B^T I_c Q = Q^T (-I_{N \times N} + p_N R^T N) Q$$
$$= -I_{N \times N} + Q^T (q_N R^T N) Q$$
$$= \text{diag}([-1 \; -1 \; \cdots \; -1 \; 0])_{N \times N}. \quad (23)$$

According to (18) and (22), we have

$$\Sigma_R = Q^T B^T R^{N \times N} B^{-T} Q$$
$$\times \left\{ \text{diag}([-1 \; -1 \; \cdots \; -1 \; 0])_{N \times N} \right\}. \quad (24)$$

Since $\Sigma_R$ is a diagonal matrix, $Q^T B^T \Lambda_{N \times N} B Q$ is also a diagonal matrix, then

$$Q^T B^{-1} R^{N \times N} B^{-T} Q = -\Sigma_R. \quad (25)$$

When the conductivity distribution is denoted by $\sigma(x, y, z)$, the discrete approximation of the Neumann-to-Dirichlet map, i.e., $R^{N \times N}_\sigma$ can be written as

$$R^{N \times N}_\sigma = -BRBT. \quad (26)$$

Finally, the matrix $R^{N \times N}_\sigma$ can be uniquely determined. Similarly, any orthogonal set of current patterns could be used to calculate the Neumann-to-Dirichlet map in the proposed way.

### C. Direct Reconstruction Algorithm

The factorization method was implemented by using the range of the difference between two Neumann-to-Dirichlet maps. In the followings, the details of the factorization will be depicted. If the sub-domain, e.g., $\Omega_1$ contains a different material other than the background in the sensing region $\Omega$, then we have the following results: without loss of generality and for simplicity of writing, $p$ and $w$ are used to denote the coordinates of $(x, y, z)$ and $(x', y', z')$, respectively. $w \in \Omega_1$ if and only if $\Phi_w|_{\partial \Omega_1}$ is in the range of the operator $|R_{\sigma} - R_1|^{1/2}$, where $R_1$ is the Neumann-to-Dirichlet map when $\sigma(p) = 1$ and $\Phi_w$ is the solution of

$$\begin{cases}
\Delta \Phi_w(p) = d \cdot \nabla \delta_w(p) \\
\int_{\partial \Omega} \Phi_w(p) \, dp = 0
\end{cases} \quad (27)$$

where $d$ is a unit vector and $\delta_w(p)$ is the Dirac delta function at $w$, $\partial / \partial n$ means the normal gradient. In particular, for the unit ball in the three dimensions, the test function $\Phi_w$ can be written as

$$\Phi_w(p) = \frac{1}{\pi} \frac{(w - p) \cdot d}{|w - p|^3} \quad \text{for all } p \in \partial \Omega. \quad (28)$$

For a real ERT sensor with only a limited number of electrodes, $|R_{\sigma} - R_1|^{1/2}$ can be approximated and denoted by $A$. According to the Picard criterion that

$$\Phi_w|_{\partial \Omega_1} \in \text{Range}[A^{1/2}] \quad (29)$$

if and only if

$$f(w) := \sum_{k=1}^{\infty} \frac{\| (\Phi_w|_{\partial \Omega_1}, u_k) \|^2}{\lambda_k} \quad (30)$$

is in the range of the operator $|R_{\sigma} - R_1|^{1/2}$.

For the discrete approximation, i.e., $A \in C^{N-1 \times N-1}$, the singular value decomposition can be implemented

$$Av_k = \lambda_k u_k, \quad k = 1, 2, \ldots, N - 1. \quad (31)$$

Here, $\lambda_k$, $u_k$, and $v_k$ are the singular value and associated vectors in the singular value decomposition of $A$, respectively. $N$ denotes the number of electrodes, $A$ is of the rank of $N-1$ as the Neumann-to-Dirichlet map has only $N-1$ independent bases. With positive $\{x_k\} \subset R$ (sorted in descending order) and orthonormal bases $\{u_k\}, \{v_k\} \subset C^N$, the function $f(w)$ can be rewritten as

$$f(w) := \sum_{k=1}^{N-1} \frac{\| \Phi^*_w v_k \|^2}{\lambda_k} \quad (32)$$

where $\Phi_w \in C^N$ is the values of $\Phi_w|_{\partial \Omega_1}$ on each electrode.
In this paper, we use the following indicator function, i.e.,
\[ \text{Ind}(w) := \log(f(w))^{-1}. \]

Then, a reconstruction of the whole sensing region is then obtained by evaluating the values of \( f(w) \) at all pixels in the region of interest.

The implementation of the factorization method can be shown in Fig. 1. It can be summarized into three steps. First, the adjacent current excitation and measurement scheme was applied to the \( N \)-electrode sensor in order to obtain the \( N(N - 1)/2 \) independent measurements of mutual conductance changes. Second, the discretized form of the Neumann-to-Dirichlet map, i.e., \( R_{\sigma}^{N \times N} - R_{\Gamma}^{N \times N} \) was obtained. Finally, the value of the indicator function at each pixel was calculated for reconstructed images.

### III. RESULTS AND DISCUSSIONS

In order to test the proposed method, an ERT system with 16 electrodes at a single cross section was used. Four 3-D phantom was selected to validate the proposed reconstruction method, as shown in Fig. 2. The phantom was used to simulate a bubble in the water. The pipe has a radius of 100 cm, while the bubble centered in the pipe has two radii i.e., 20 cm and
The two kinds of bubbles were positioned in both center and near boundary of the pipe, as shown in (a)-(b) and (c)-(d), respectively. For convenience of writing, we denote the electrode plane, i.e., the cross section as the x-y plane, while the direction along the pipe is the $z$-direction. The bubbles in (c) and (d) both have a displacement of 20 cm in the $y$-direction, compared with the bubbles in (a) and (b), respectively.

The reconstructed images of phantom (a) were shown in Fig. 3. The 3-D results were depicted in the range of $[-1, 1] \times [-1, 1] \times [-0.6, 0.6]$. There exist $20 \times 20 \times 15$ pixels in each 3-D image. The slice plot of the 3-D reconstruction was shown in Fig. 3(a), which showed that an object can be found in the center. To improve the visualization appearance of the results, the isosurface plot of the reconstruction was provided.
in Fig. 3(b). In the literature, when the sensitivity matrix-based algorithms were applied for image reconstruction, a similar forward problem should be solved to obtain the sensitivity matrix, which may lead to a high potential of failure in validation of the algorithms. However, in the proposed factorization method, the data of the forward problem were calculated by using the finite-element software, while the inverse problem was solved through the factorization method based on a different theory, i.e., the eigenvalues/vectors of the Neumann-to-Dirichlet map. The factorization methods in 2-D and 3-D cases obey the same theory, the difference between them only exist in the form of the test function, e.g., (28). As a result, the factorization method in 2-D case can be used as a special case of the factorization method in 3-D case. To validate the effectiveness
of the simulated data, the 2-D reconstruction for the electrode cross section was depicted in Fig. 3(c) by using the factorization method in the 2-D case [11]. It can be seen that the projection of the ball on the cross section can be reconstructed with high contrast. The images in Fig. 3(d)–(f) show the x-y view, y-z view, and z-x view of the 3-D result in Fig. 3(a), respectively.

For comparison, the reconstructed images of phantoms (b), (c), and (d) were shown in Fig. 4. Images in (a), (c), and (e) are slice plotting for phantoms (b), (c), and (d), respectively, while the images in (b), (d), and (f) are isosurface plotting for phantoms (b), (c), and (d), respectively. Compared with the slice plotting in Fig. 4(a), the red region in Fig. 4(e) shows the displacement in the y-direction, which agrees well with the truth that the bubble in phantom (d) deviates 20 cm from the center. When the bubbles of different sizes, i.e., phantom (c) and phantom (d), were located in the same position, the bigger bubble actually leads to larger isosurface plotting, see Fig. 4(d) and (f). However, although 3-D images can be obtained from the single plane electrode configuration, the spatial resolution along the pipe is lower than those along the other two directions, i.e., in the electrode plane. As a result, the reconstructed bubbles shown in Fig. 3(b) and Fig. 4(b), (d), and (f) are shorter in diameter along the pipe, while the phantom is an ideal ball.

To test the robustness of the proposed method, one of the electrodes was assumed to fail. As a result, all the data associated with the electrode was not suitable for image reconstruction. In this case, only the other $N-1$ electrodes were used for reconstruction. The rank of the matrix form of the Neumann-to-Dirichlet map reduces to $N-2$. The reconstructed images of phantoms (b), (c), and (d) were shown in Fig. 5. It showed that the quality of the 3-D images almost remain the same as those in Fig. 4, particularly in the aspect of visual appearance.

The factorization method still can be attractive for its simplicity and fast speed. To implement the proposed method, the major computation time was spent on the equation (32). Once the number of the pixels in the sensing region is determined, the main computation is just to calculate the inner product in (32). As a result, although the results in the work are preliminary, it also shows that the proposed method is suitable for fast imaging. For example, it took about 2 s to produce one 3-D image of $20 \times 20 \times 10$ pixels in the Matlab environment on a computer equipped with Core 2 processor of 2.53 GHz and 2 Gbytes RAM. The real-time performance of the algorithm can be further improved by using a compiled language, e.g., C++ codes.

IV. CONCLUSION

In this paper, the factorization method was used for direct reconstruction in 3-D ERT. Reconstructed results proved that the proposed method was valid and effective. Compared to the iterative algorithms based on the sensitivity/Jacobi matrix, the proposed method is a direct algorithm and fast. The conductivity perturbation at any position in the region of interest can be calculated directly and independently. It is superb when only part of the sensing field is interesting as the distribution in any part of the sensing field can be directly obtained without information of the other parts. While in the iterative algorithms based on the sensitivity/Jacobi matrix, any part of the distribution of interest cannot be obtained unless the reconstruction of the whole distribution is completed. As for ERT, the excitation frequency is usually lower than 10 kHz, the measured data can agree well with the simulated data, if the hardware system was well constructed. Moreover, it is just a preliminary result in this manuscript to show the feasibility of the proposed method, though the spatial resolution along the pipe is not satisfactory.

In our further work, the phantoms will be constructed, and more electrodes will be added, in order to test the performance of the proposed method. The factorization method is simple and easy to implement. However, it also has its own limitations, e.g., only suitable for single modality imaging, particularly if the inclusion is in the center of the region. The proposed method has potential applications in the environmental control and life support sub-system in the space station. In the weightless/microgravity environment, the bubble in the water tends to be only one ball-shaped phantom. It can be used to reconstruct one 3-D object in the sensor, as it is easy to be implemented on the hardware system for its simplicity. As a result, the void fraction measurement and alarm of gas-liquid two-phase flow in microgravity can be performed in a visible way in the manned spacecraft.

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