On Geometric Aggregation Over Interval-Valued Intuitionistic Fuzzy Information

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Abstract

The notion of interval-valued intuitionistic fuzzy set (IVIFS) was introduced by Atanassov and Gargov [1] as a generalization of an intuitionistic fuzzy set [2]. The fundamental characteristic of IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Some operators have been proposed for aggregating intuitionistic fuzzy sets. However, it seems that there is little investigation on aggregation techniques for dealing with interval-valued intuitionistic fuzzy information. In this work, we develop some interval-valued intuitionistic fuzzy geometric operators, such as the interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator, and interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator, etc., which are the generalizations of the geometric aggregation operators based on intuitionistic fuzzy sets [3]. Then we apply the developed operators to solve a multiple attribute decision-making problem involving the prioritization of a set of information technology improvement projects.

1. Introduction

Atanassov and Gargov [1] introduced the notion of the interval-valued intuitionistic fuzzy set (IVIFS), which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers. After that, some authors have investigated the IVIFSs from different points of view. Atanassov [4] gave some relations and operations over IVIFSs, and studied their basic properties. Bustince and Burillo [5] defined the concepts of correlation and correlation coefficient of IVIFSs, and introduced two decomposition theorems of the correlation of IVIFSs in terms of the correlation of interval-valued fuzzy sets and the entropy of the intuitionistic fuzzy sets, and the correlation of intuitionistic fuzzy sets. Hong [6] generalized the concepts of correlation and correlation coefficient of IVIFSs in a general probability space, and generalized the results of Bustince and Burillo [5]. Hung and Wu [7] proposed a method to calculate the correlation coefficient of IVIFSs by means of “centroid”. The method can reflect not only the strength of relationship between the IVIFSs, but also whether the IVIFSs are positively or negatively related. Xu [8] developed a new approach to deriving the correlation coefficients of IVIFSs. The prominent characteristic of the approach is that it can guarantee that the correlation coefficient of any two IVIFSs equals one if and only if these two IVIFSs are the same, and can relieve the influence of the unfair arguments on the final results. Mondal and Samanta [9] defined the topology of IVIFSs and studied some of its properties. They showed that the category of topological spaces of IVIFSs and continuous functions form a topological category. Deschrijver and Kerre [10] established the relationships between intuitionistic fuzzy sets, L-fuzzy sets [11], interval-valued fuzzy sets [12] and IVIFSs. Liu et al. [13] extended the entropy and subseothood from intuitionistic fuzzy sets to general IVIFSs. However, in many real-life problems, we need to aggregate the given interval-valued intuitionistic fuzzy arguments into a combined one. In such cases, some information fusion techniques are necessary. Up to date, it seems that there is little investigation on the aggregation of IVIFSs [14]. In this paper, we try to extend some geometric aggregation operators [3] developed for intuitionistic fuzzy sets to IVIFSs and study their properties, and apply the developed operators to solve a multiple attribute decision-making problem with interval-valued intuitionistic fuzzy information.

2. Basic Concepts and relations

Let \( X \) be a universe of discourse. In [2], Atanassov introduced the intuitionistic fuzzy set to deal with vagueness, which was given by

\[
A = \{x, \mu_A(x), \nu_A(x) | x \in X \}
\]

where the functions \( \mu_A : X \rightarrow [0,1] \) and \( \nu_A : X \rightarrow [0,1] \) determine the degree of membership and the degree of non-membership of the element \( x \in X \), respectively, and for every \( x \in X \):

\[
0 \leq \mu_A(x) + \nu_A(x) \leq 1
\]

However, sometime it is not approximate to assume that the membership degrees for certain elements of \( A \)
are exactly defined, but a value range can be given. In such cases, Atanassov and Gargov [1] defined the notion of IVIFS as below:

An IVIFS $\tilde{A}$ over $X$ is an object having the form:
\[
\tilde{A} = \{x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) : x \in X \}
\]
(3)
where $\mu_{\tilde{A}}(x) \subseteq [0,1]$ and $\nu_{\tilde{A}}(x) \subseteq [0,1]$ are intervals, and for every $x \in X$:
\[
\sup \mu_{\tilde{A}}(x) + \sup \nu_{\tilde{A}}(x) \leq 1
\]
(4)

Especially, if each of the intervals $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ contains exactly one element, i.e., if for every $x \in X$:
\[
\mu_{\tilde{A}}(x) = \inf \mu_{\tilde{A}}(x) = \sup \mu_{\tilde{A}}(x), \quad \nu_{\tilde{A}}(x) = \inf \nu_{\tilde{A}}(x) = \sup \nu_{\tilde{A}}(x)
\]
then, the given IVIFS $\tilde{A}$ is transformed to an ordinary intuitionistic fuzzy set.

Atanassov [4] further developed some operations of IVIFSs, shown as follows:

**Definition 2.1** [4]. Let $\tilde{A}$ and $\tilde{B}$ be two IVIFSs, then:
1) $\tilde{A} \cap \tilde{B} = \{x, \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)), \min(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x)) : x \in X \}$
2) $\tilde{A} \cup \tilde{B} = \{x, \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)), \max(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x)) : x \in X \}$
3) $\tilde{A} + \tilde{B} = \{x, \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) + \min(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x)) : x \in X \}$
4) $\tilde{A} - \tilde{B} = \{x, \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) - \min(\nu_{\tilde{A}}(x), \nu_{\tilde{B}}(x)) : x \in X \}$

For convenience of computation, we introduce another two operations over IVIFSs as follows:
5) $\tilde{A}^\lambda = \{x, [1-(1-\inf \mu_{\tilde{A}}(x))^\lambda, 1-(1-\sup \nu_{\tilde{A}}(x))^{\lambda^\lambda}] : x \in X \}$
6) $\tilde{A}^{\lambda} = \{x, [(\inf \nu_{\tilde{A}}(x))^\lambda, (\sup \mu_{\tilde{A}}(x))^{\lambda^\lambda}] : x \in X \}$, $\lambda > 0$.

Based on IVIFS, Xu [14] defined the notion of interval-valued intuitionistic fuzzy number (IFVN) and introduced some operations of IVIFNs:

**Definition 2.2** [14]. Let $\tilde{A} = \{x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) : x \in X \}$ be an IVIFS, then we call the pair $(\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x))$ an IFVN.

For convenience, we denote an IVIFN by $[a, b], [c, d]$, where
\[
[a, b] \subseteq [0, 1], \quad [c, d] \subseteq [0, 1], \quad b + d \leq 1
\]
(5)
and let $\Theta$ be the set of all IVIFNs.

**Definition 2.3** [14]. Let $\tilde{A}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{A}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFNs, then some operations of $\tilde{A}_1$ and $\tilde{A}_2$ can be defined as:
1) $\tilde{A}_1 \cap \tilde{A}_2 = ([\min(a_1, a_2), \min(b_1, b_2), [\max(c_1, c_2), \max(d_1, d_2)])$
2) $\tilde{A}_1 \cup \tilde{A}_2 = ([\max(a_1, a_2), \max(b_1, b_2), [\min(c_1, c_2), \min(d_1, d_2)])$
3) $\tilde{A}_1 \oplus \tilde{A}_2 = ([a_1+a_2-a_2, a_2, [c_1+c_2-c_2, c_2, d_1-d_2, d_2])$
4) $\tilde{A}_1 \ominus \tilde{A}_2 = ([a_1-a_2-a_2, b_1-b_2, [c_1+c_2-c_2, d_1-d_2, d_2])$
5) $\tilde{A}_1 \odot \tilde{A}_2 = ([1-(1-a_1)^4, 1-(1-b_1)^4], [c_1^4, d_1^4], \lambda > 0$
6) $\tilde{A}_1^\lambda = ([a_1^\lambda, b_1^\lambda], [1-(1-c_1)^4, 1-(1-d_1)^4], \lambda > 0$

Chen and Tan [15] introduced a score function to measure an intuitionistic fuzzy number. Later, Hong and Choi [16] defined an accuracy function to evaluate the accuracy degree of an intuitionistic fuzzy number. The score function and accuracy function are, respectively, defined as the sum and difference of the membership function and non-membership function of an intuitionistic fuzzy set. Xu [14] developed a score function and an accuracy function to measure an IVIFN, respectively.

**Definition 2.4** [14]. Let $\tilde{A} = ([a, b], [c, d])$ be an IVIFN, then we call
\[
s(\tilde{A}) = \frac{1}{2} (a - c + b - d)
\]
the score function of $\tilde{A}$, where $s(\tilde{A}) \in [-1, 1]$. The greater the value of $s(\tilde{A})$, the greater the IVIFN $\tilde{A}$. Especially, if $s(\tilde{A}) = 1$, then $\tilde{A} = ([1, 1], [0, 0])$, which is the largest IVIFN; if $s(\tilde{A}) = -1$, then $\tilde{A} = ([0, 0], [1, 1])$, which is the smallest IVIFN.
Definition 2.5 [14]. Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, then we call

$$h(\tilde{\alpha}) = \frac{1}{2}(a + b + c + d)$$  (7)

the score function of $\tilde{\alpha}$, where $h(\tilde{\alpha}) \in [0,1]$. 

Just as pointed out by Hong and Choi [16], the relation between the score function and accuracy function is similar to the relation between mean and variance in statistics. It is well known that an efficient estimator is a measure of the variance of an estimate's sampling distribution in statistics, the smaller the variance, the better the performance of the estimator. Motivated by this idea, it is meaningful and appropriate to stipulate that the greater the accuracy degree $h(\tilde{\alpha})$, the better the IVIFN $\tilde{\alpha}$. Based on Definitions 2.4 and 2.5, Xu [14] developed a method for the comparison between two IVIFNs:

Definition 2.6 [14]. Let $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ be any two IVIFNs, then

1) If $s(\tilde{\alpha}_1) < s(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$;

2) If $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$, then

i) If $h(\tilde{\alpha}_1) = h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ represent the same information, denoted by $\tilde{\alpha}_1 = \tilde{\alpha}_2$;

ii) If $h(\tilde{\alpha}_1) < h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$.

3. Interval geometric aggregation operators

In [3], Xu and Yager introduced various geometric aggregation operators to deal with intuitionistic fuzzy information. Based on these operators and Definition 2.6, Xu [14] gave an interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator:

Definition 3.1 [14]. Let $\tilde{\alpha} = ([a_j, b_j], [c_j, d_j])$ (j = 1, 2, ..., n) be a collection of IVIFNs, and let $IIFWG: \Theta^n \rightarrow \Theta$, if $IIFWG_n(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \tilde{\alpha}_1^w \odot \tilde{\alpha}_2^w \odot \cdots \odot \tilde{\alpha}_n^w$  (8)

where $w = (w_1, w_2, ..., w_n)^T$ is the weight vector of $\tilde{\alpha}_j$ (j = 1, 2, ..., n), with $w_j > 0$ and $\sum_{j=1}^n w_j = 1$, then $IIFWG$ is called the interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator. Especially, if $w = (1/n, 1/n, ..., 1/n)^T$, then the IIFWG operator is reduced to the interval-valued intuitionistic fuzzy geometric (IIFG) operator, which is defined as follows:

$$IIFG(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = (\tilde{\alpha}_1 \odot \tilde{\alpha}_2 \odot \cdots \odot \tilde{\alpha}_n)^{1/n}$$

Theorem 3.1 [14]. Let $\tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j])$ (j = 1, 2, ..., n) be a collection of IVIFNs, then their aggregated value by using the IIFWG operator is also an IVIFN, and satisfies

$$IIFWG_n(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = ([\prod_{j=1}^n a_j^w], [\prod_{j=1}^n b_j^w], [1 - \prod_{j=1}^n (1 - c_j)^w], [1 - \prod_{j=1}^n (1 - d_j)^w])$$  (9)

where $w = (w_1, w_2, ..., w_n)^T$ is the weight vector of $\tilde{\alpha}_j$ (j = 1, 2, ..., n), with $w_j > 0$ and $\sum_{j=1}^n w_j = 1$. Especially, if $a_j = b_j$ and $c_j = d_j$ for all $j$, i.e., all $\tilde{\alpha}_j$ (j = 1, 2, ..., n) are reduced to the intuitionistic fuzzy numbers, then the IIFWG operator is reduced to the intuitionistic fuzzy weighted geometric (IFWG) operator [3], which has the following form:

$$IFWG_n(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = ([\prod_{j=1}^n a_j^w], [1 - \prod_{j=1}^n (1 - c_j)^w])$$  (10)

If $a_j + c_j = 1$, for all $j$, i.e., all $\tilde{\alpha}_j$ (j = 1, 2, ..., n) are reduced to the ordinary fuzzy numbers, then the IFWG operator is reduced to the FWG operator, which has the following form:

$$FWG_n(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = ([\prod_{j=1}^n a_j^w], [1 - \prod_{j=1}^n a_j^w])$$  (11)

In the following we develop some new geometric operators for aggregating interval-valued intuitionistic fuzzy information.

Definition 3.2. Let $\tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j])$ (j = 1, 2, ..., n) be a collection of IVIFNs. An interval-valued intuitionistic fuzzy ordered weighted geometric (IFOWG) operator of dimension $n$ is a mapping $IFOWG: \Theta^n \rightarrow \Theta$ that has an associated vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$, such that $\omega_j > 0$ and $\sum_{j=1}^n \omega_j = 1$. Furthermore,

$$IFOWG_n(\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = (\tilde{\alpha}_{\sigma(1)})^{\omega_1} \odot (\tilde{\alpha}_{\sigma(2)})^{\omega_2} \odot \cdots \odot (\tilde{\alpha}_{\sigma(n)})^{\omega_n}$$  (12)

where $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of (1, 2, ..., n) such that $\tilde{\alpha}_{\sigma(j-1)} \geq \tilde{\alpha}_{\sigma(j)}$ for all $j$. Especially, if $\omega = (1/n, 1/n, ..., 1/n)^T$ then the IFOWG operator is reduced to the IIFG operator.

Theorem 3.2. Let $\tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j])$ (j = 1, 2, ..., n) be a collection of IVIFNs, then their aggregated value by using the IIFOWG operator is also an IVIFN, and satisfies:
\[ IIFOWG_\omega (\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left( \prod_{j=1}^{n} a_{\sigma(j)}^{\alpha_j}, \prod_{j=1}^{n} b_{\sigma(j)}^{\alpha_j} \right) \] (13)

where \( \omega = (\alpha_1, \alpha_2, ..., \alpha_n) \) is the weighting vector of the IIFOWG operator, with \( \alpha_j > 0 \) and \( \sum_{j=1}^{n} \alpha_j = 1 \), which can be determined by using the normal distribution based method [17]. Especially, if all \( \tilde{\alpha}_j \) (\( j = 1, 2, ..., n \)) are reduced to the intuitionistic fuzzy numbers, then the IIFOWG operator is reduced to the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator [3], which has the following form:

\[ IFOWG_\omega (\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left( \prod_{j=1}^{n} a_{\sigma(j)}^{\alpha_j}, 1 - \prod_{j=1}^{n} (1 - c_{\sigma(j)})^{\alpha_j} \right) \] (14)

If all \( \tilde{\alpha}_j \) (\( j = 1, 2, ..., n \)) are reduced to the ordinary fuzzy numbers, then the IIFOWG operator is reduced to the FOWG operator, which has the following form:

\[ FOWG_\omega (\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = \left( \prod_{j=1}^{n} a_{\sigma(j)}^{\alpha_j}, 1 - \prod_{j=1}^{n} a_{\sigma(j)}^{\alpha_j} \right) \] (15)

Consider that the IIFWG operator weights only the IVIFNs and the IIFOWG operator weights only the ordered positions of the IVIFNs. In what follows, we develop an interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator, which weights both the given IVIFN and its ordered position.

**Definition 3.3.** An interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator is a mapping \( IIFHG : \Theta^* \rightarrow \Theta \), which has an associated vector \( \omega = (\omega_1, \omega_2, ..., \omega_n) \) with \( \omega_j > 0 \) and \( \sum_{j=1}^{n} \omega_j = 1 \), such that

\[ IIFHG_{\omega,\alpha} (\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = (\tilde{\alpha}(\tilde{\omega_j})^\alpha, \tilde{\sigma}(\tilde{\alpha})^\alpha \otimes \cdots \otimes (\tilde{\alpha}(\tilde{\omega_n})^\alpha) \] (16)

where \( \tilde{\alpha}(\tilde{\omega_j}) \) is the \( j \)th largest of the weighted IVIFNs \( \tilde{\alpha}(\tilde{\omega_j}, \tilde{\alpha}_j, ..., \tilde{\alpha}_n) \) (\( \tilde{\omega} = (\omega_1, \omega_2, ..., \omega_n) \) is the vector of \( \tilde{\alpha}(\tilde{\omega_j}, \tilde{\alpha}_j, ..., \tilde{\alpha}_n) \) and \( \tilde{\alpha}(\tilde{\omega_j}) \) is the balancing coefficient, which plays a role of balance, (in this case, if the vector \( (\omega_1, \omega_2, ..., \omega_n) \) approaches \( (1/n, 1/n, ..., 1/n) \), then the vector \( (\tilde{\alpha}(\tilde{\omega_j}, \tilde{\alpha}_j, ..., \tilde{\alpha}_n) \) approaches \( (\tilde{\alpha}(\tilde{\omega_j}, \tilde{\alpha}_j, ..., \tilde{\alpha}_n) \).

**Theorem 3.3.** Let \( \tilde{\alpha}_j = [(\tilde{a}_j, \tilde{b}_j), [c_j, d_j]] \) \( (j = 1, 2, ..., n) \) be a collection of IVIFNs, then their aggregated value by using the IIFHG operator is also an IVIFN, and satisfies:

\[ IIFHG_{\omega,\alpha} (\tilde{\alpha}_1, \tilde{\alpha}_2, ..., \tilde{\alpha}_n) = (\prod_{j=1}^{n} \tilde{\alpha}_j^{\alpha_j}, \prod_{j=1}^{n} \tilde{\alpha}_j^{\alpha_j}) \] (17)

where \( \omega = (\omega_1, \omega_2, ..., \omega_n) \) is the weighting vector of the IIFHG operator, with \( \omega_j > 0 \) and \( \sum_{j=1}^{n} \omega_j = 1 \), \( \tilde{\beta}(\tilde{\alpha}_j, \tilde{\beta}(\tilde{\alpha}_j, \tilde{\alpha}_j, ..., \tilde{\alpha}_n)) \) is the \( j \)th largest of the weighted IVIFNs \( \tilde{\beta}(\tilde{\alpha}_j, \tilde{\alpha}_j, ..., \tilde{\alpha}_n) \), \( \tilde{\alpha}(\tilde{\alpha}_j, \tilde{\alpha}_j, ..., \tilde{\alpha}_n) \) is the weighted vector of the IVIFNs \( \tilde{\alpha}_j \), \( \tilde{\alpha}_j \), \( \tilde{\alpha}_j \), \( \tilde{\alpha}_j \), \( \tilde{\alpha}_j \) with \( \tilde{\alpha}_j > 0 \) and \( \sum_{j=1}^{n} \tilde{\alpha}_j = 1 \). Especially, if \( w = (1/n, 1/n, ..., 1/n) \), then the IIFHG operator is reduced to the IIFVG operator; if \( \omega = (1/n, 1/n, ..., 1/n) \), then the IIFHG operator is reduced to the IIFVG operator.

**4. Illustrative example**

In this section, a multiple attribute decision-making problem involves the prioritization of a set of information technology improvement projects (adapted from [18]) is used to illustrate the developed procedures. The information management steering committee of Midwest American Manufacturing Corp. must prioritize for development and implementation a set of six information technology improvement projects \( x_j \) \( (j = 1, 2, ..., 10) \), which have been proposed by area managers. The committee is concerned that the projects are prioritized from highest to lowest potential contribution to the firm’s strategic goal of gaining competitive advantage in the industry. In assessing the potential contribution of each project, three factors are considered, \( B_1 \) – productivity, \( B_2 \) – differentiation, and \( B_3 \) – management. The productivity factor assesses the potential of a proposed project to increase effectiveness and efficiency of the firm’s manufacturing and service operations. The differentiation factor assesses the potential of a proposed project to fundamentally differentiate the firm’s products and services from its competitors, and to make them more desirable to its customers. The management factor assesses the potential of a proposed project to assist management in improving their planning, controlling and decision-making activities. The following is the list of proposed information systems projects: 1) \( A_1 \) – Quality Management Information, 2) \( A_2 \) – Inventory Control, 3) \( A_3 \) – Customer Order Tracking, 4) \( A_4 \) – Materials Purchasing Management, 5) \( A_5 \) – Fleet Management, 6) \( A_6 \) – Design Change.
Management, 7) A4 – Electronic Mail, 8) A8 – Customer Returns and Complaints, 9) A9 – Employee Skills Tracking, 10) A10 – Budget Analysis. Suppose that the weight vector of B1, B2 and B3 is \( w = (0.5,0.3,0.2)^T \), and the committee represents the characteristics of the projects \( A_i \) (\( i=1,2,...,\ldots,10 \)) by the IVIFNs \( \tilde{\alpha}_{ij} \) (\( i = 1, 2, \ldots, 10; \ j = 1,2,3 \)) with respect to the factors \( B_j \ (j = 1,2,3) \), listed in Table 1.

### Table 1. The characteristics of the projects

<table>
<thead>
<tr>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>([0.5, 0.6], [0.2, 0.3])</td>
<td>([0.3, 0.4], [0.4, 0.6])</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>([0.5, 0.6], [0.4, 0.5])</td>
<td>([0.1, 0.3])</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>([0.6, 0.7], [0.2, 0.4])</td>
<td>([0.3, 0.4])</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>([0.5, 0.7], [0.4, 0.5])</td>
<td>([0.1, 0.2])</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>([0.7, 0.8], [0.1, 0.2])</td>
<td>([0.4, 0.5])</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>([0.5, 0.6], [0.4, 0.5])</td>
<td>([0.1, 0.3])</td>
</tr>
<tr>
<td>( A_7 )</td>
<td>([0.4, 0.5])</td>
<td>([0.1, 0.2])</td>
</tr>
<tr>
<td>( A_8 )</td>
<td>([0.2, 0.4])</td>
<td>([0.3, 0.4])</td>
</tr>
<tr>
<td>( A_9 )</td>
<td>([0.5, 0.7])</td>
<td>([0.4, 0.5])</td>
</tr>
<tr>
<td>( A_{10} )</td>
<td>([0.4, 0.5])</td>
<td>([0.4, 0.5])</td>
</tr>
</tbody>
</table>

To rank the given ten projects, we first weight all the IVIFNs \( \tilde{\alpha}_{ij} \) (\( i=1, 2, \ldots, 10; \ j=1,2,3 \)) by the vector \( w = (0.5,0.3,0.2)^T \) of attribute weights \( B_j \) (\( j=1,2,3 \)) and multiplies these values by the balancing coefficient \( n=3 \), and gets the weighted IVIFNs \( \tilde{\beta}_{ij} = \alpha_{ij}^{w_n} \), \( i=1, 2, \ldots, 10; \ j=1,2,3 \), and utilize the formula (17) (let \( w = (0.2429,0.5142, 0.2429)^T \) be its weighting vector derived by the normal distribution based method [16]) to get the overall values \( \tilde{\alpha}_i \) (\( i=1,2,\ldots,10 \)) of the projects \( A_i \) (\( i=1,2,\ldots,10 \)):

\[
\tilde{\alpha}_1 = ([0.3941,0.4990], [0.2852,0.4380])
\]

\[
\tilde{\alpha}_2 = ([0.02109,0.4307], [0.2626,0.4063])
\]

\[
\tilde{\alpha}_3 = ([0.4685,0.6018], [0.2584,0.3679])
\]

\[
\tilde{\alpha}_4 = ([0.3608,0.5777], [0.2328,0.3458])
\]

\[
\tilde{\alpha}_5 = ([0.3237,0.5826], [0.1986,0.3679])
\]

\[
\tilde{\alpha}_6 = ([0.4544,0.5854], [0.1490,0.2935])
\]

\[
\tilde{\alpha}_7 = ([0.4266,0.6362], [0.2200,0.3187])
\]

\[
\tilde{\alpha}_8 = ([0.4247,0.6321], [0.2732,0.3679])
\]

\[
\tilde{\alpha}_9 = ([0.3964,0.5635], [0.2751,0.3748])
\]

\[
\tilde{\alpha}_{10} = ([0.5237,0.6560], [0.1218,0.3440])
\]

By (6), we calculate the scores of the overall values \( \tilde{\alpha}_i \) (\( i = 1,2,\ldots, 10 \)):

\[
s(\tilde{\alpha}_1) = 0.0850, \ s(\tilde{\alpha}_2) = -0.0137, \ s(\tilde{\alpha}_3) = 0.2220, \ s(\tilde{\alpha}_4) = 0.1797, \ s(\tilde{\alpha}_5) = 0.1699, \ s(\tilde{\alpha}_6) = 0.2987, \ s(\tilde{\alpha}_7) = 0.2621, \ s(\tilde{\alpha}_8) = 0.2079, \ s(\tilde{\alpha}_9) = 0.1550, \ s(\tilde{\alpha}_{10}) = 0.3570
\]

Since

\[
s(\tilde{\alpha}_{10}) > s(\tilde{\alpha}_6) > s(\tilde{\alpha}_7) > s(\tilde{\alpha}_3) > s(\tilde{\alpha}_4) > s(\tilde{\alpha}_8) > s(\tilde{\alpha}_5) > s(\tilde{\alpha}_1) > s(\tilde{\alpha}_2),
\]

then \( A_{10} \rightarrow A_6 \rightarrow A_7 \rightarrow A_3 \rightarrow A_4 \rightarrow A_8 \rightarrow A_9 \rightarrow A_1 \rightarrow A_2 \), hence, the project \( A_{10} \) has the highest potential contribution to the firm’s strategic goal of gaining competitive advantage in the industry.

### 4. Conclusions

We have developed some interval geometric aggregation operators, including the interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator, and interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator, for dealing with interval-valued intuitionistic fuzzy information. The interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator and the IIFOWG operator are the special cases of the IIFHG operator. We have also illustrated the developed operators by a multiple attribute decision making problem involving the prioritization of a set of information technology improvement projects. The applications of the developed operators in other fields such as decision making, pattern recognition, supply chain management and medical diagnosis, etc., will be the direction for future research.

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### References