Nonlinear Control Approaches for SI Engine Model with Uncertainties

Rui Yan, Haizhou Li, Zhao Yang Dong and Huajin Tang

Abstract—Air-fuel ratio control is a challenging control problem for port fuel injected and throttle body fuel injected spark ignition (SI) engines, since the dynamics of air manifold and fuel injection of the SI engines are highly nonlinear and often with unmodeled uncertainties and disturbance. This paper presents nonlinear control approaches for multi-input multi-output engine models, by developing adaptive control and learning control design methods. Theoretical proofs are established which ensure proposed controllers are able to give asymptotical tracking performance. Adaptive control and learning control approaches are capable of dealing with both constant uncertainty and time-varying periodic uncertainty. Simulation results illustrate the efficacy of the proposed controllers.

I. INTRODUCTION

Automotive engine control is one of the most complex control problems for control engineers and researchers. A lot of research work has been done for the design and analysis of engine controls. The objective of a typical control system for an internal combustion (IC) engine is not only to maintain the desired engine speed, but also to satisfy regulations relating to exhaust emissions. Moreover, the feedback control strategy should be robust so that the designed performance could be maintained when the engine is operating at different operating points, with different aging history of the components, or under vastly different environmental conditions. A key development was the introduction of closed loop control in the 70s [1], [2]. As more capable microprocessors become available through 90s, it is practical to entertain the use of various nonlinear control and estimation algorithms for automotive engine control practice [3], [4], [5], [6], where linear quadratic control, Kalman filtering, observer based and model predictive control methods were developed. Most of the control approaches are based on a single variable output system, i.e., only air-fuel ratio is controlled. Observer-based fuel injection control algorithm [7] was developed to achieve better AFR control performance by fast response and small amplitude chattering of the AFR as compared to using sliding mode control method. However, the success of the suggested observation control methods depends largely on the accuracy of the plant model.

In [8], a sliding mode control method is applied to the engine model derived from the fuel wall wetting dynamics of engine intake manifold [9] and Hendricks’ model [10]. The engine model is a nonlinear, MIMO, coupled system having three states \( x = (\dot{m}_f, n, p_i) \) and three inputs \( u = (\dot{m}_f, \theta, \alpha) \). By using the sliding mode techniques to a multi-input multi-output IC engine model, [8] demonstrated that undesirable chattering effects at the actuators. The particular choice of sliding surface is input dependent which produces a dynamic controller. Such dynamic policies are desirable in sliding mode control as they effectively filter out the chattering of the control signal. The designed controller is valid over the entire operating range. The air-fuel ratio is stabilized to its ideal value and the speed and manifold pressure track satisfactorily given trajectories. The researchers in Ford Motor Company [11], [12] studied a diesel engine model and considered a MIMO control system. They proposed a control design method by employing a recently developed control Lyapunov function based nonlinear control because it possesses a guaranteed robustness property equivalent to gain and phase margins. By applying the input-output linearization method and the control Lyapunov function, the controller can be designed. However, when the system contains uncertainties, the proposed controller is not able to keep tracking performance. In recent years a number of theoretical advances in nonlinear control systems have been achieved to a level where they can be applied to solve practical control design problems. Although the nonlinear control methods applying Lyapunov theory and adaptive control have been studied in various mechatronics areas, from actuator control to power steering and large scale system [13], [14], [15], [16], there is a lack of complete and systematic treatments of applying adaptive control [17] and learning control [18], [19] methodologies to engine control.

In this paper, we present the controller design methods by using adaptive control and learning control strategies. Uncertainties and disturbance are important characteristics of the engine, therefore the controller should have a good robust property to uncertainties and disturbance. We design the controllers for the engine based on this consideration and present the approach to deal with the uncertainties by devising suitable control algorithm. Rigorous proofs are established and simulation results are also presented to validate our theoretical results.

II. THE ENGINE MODEL DESCRIPTION

There are a number of IC engine dynamic models in the literature. Among them the mean value engine model (MVEM) developed by Hendricks et al. [20], [21], [10], [22] is mathematically compact and can be parameterized for different engines easily. Thus, the MVEM is adopted widely for engine control systems [23].
A. The Mean Value Engine Model
The engine model comprises three subsystems: fueling, air intake and engine speed dynamic systems. These subsystems are described briefly in the following.

(1) Fueling System
The fluid film flow model describes the dynamics of the fluid flow through the manifold. The fluid flow has two components: fuel vapor flow and fuel film flow, denoted by \( \dot{m}_{fi}, \dot{m}_{ff} \), respectively. The total flow \( \dot{m}_f \) is not measurable. The dynamics of the submodel is described as:

\[
\begin{align*}
\dot{m}_{ff} &= \frac{1}{\tau_f}(-\dot{m}_{ff} + X\dot{m}_{fi}) \quad (1a) \\
\dot{m}_{fi} &= (1 - X)\dot{m}_{fi} \quad (1b) \\
\dot{m}_f &= \dot{m}_{ff} + \dot{m}_{fi} \quad (1c) \\
X &= k_{x1} + k_{x2}\frac{\dot{m}_{ap}}{\dot{m}_{ap}(\text{max})} \quad (1d) \\
\lambda &= \frac{\dot{m}_{ap}}{14.67\dot{m}_f} = h(\cdot) \quad (1e)
\end{align*}
\]

where \( X\dot{m}_{fi} \) is the rate at which injected fuel is deposited on manifold as fuel film.

(2) Engine Speed Dynamics
The submodel is derived from energy conservation laws, is described as:

\[
\begin{align*}
\dot{n} &= \frac{1}{\eta_i}(H_u\eta_i\dot{m}_f(t - \tau_d) - P_l - P_b) \quad (2) \\
P_l &= n(k_1 + k_2n + k_3n^2) + n(-k_4 + k_5n)P_i \\
P_b &= k_6n^3
\end{align*}
\]

where \( \eta_i = \eta_i\eta_p\eta_p\eta_\lambda\eta_\theta, \eta_i(n) = k_0(1 - k_7n^{-0.36}) \), \( \eta_p(n) = k_8 + k_9n + k_{10}n^2 \), \( \eta_\lambda = -k_{11} + k_{12}\lambda - k_{13}\lambda^2 \), \( \eta_\theta = e^{-\theta^2/\theta_{mbt}^2} \), \( \eta_{mbt} = \min(\min(\theta_1, \theta_2), 45) \), \( \theta_1 = k_{14}\theta_4 + k_{15} + n_{47} \), \( \theta_2 = k_{16}\theta_4 + k_{17} + n_{47} \), and \( n_{47} = 4.7n \), for \( n < 4.8 \) \( n_{47} = 4.7 \times 4.8 \) for otherwise.

(3) Air Flow System
The air mass flowing through the manifold is depicted by the following equations:

\[
\begin{align*}
\dot{p}_i &= \frac{R}{V_{i}}(-\dot{m}_{ap} + \dot{m}_{at}) \quad (3) \\
\dot{m}_{ap} &= \frac{V_{d}}{120RT_{i}}(k_{18}\dot{p}_i + k_{19})n \\
\dot{m}_{at} &= m_{at1}\frac{p_0}{(T_0 + \beta_2(p_r)\beta_1(\alpha) = k_{20}\beta_2(p_r))} \quad (4)
\end{align*}
\]

Remark 1: The physical meanings of the above symbols are given in Appendix.

B. MIMO Engine Plants
Regulation AFR has received overwhelming attention in engine control since maintaining AFR at the stoichiometric value can obtain the best balance between power output and fuel consumption, and also effectively reduce the pollution emission, because a stoichiometric value produces relatively low CO and NOx emissions while enabling good aftertreatment conversion efficiency.

Define \( x = (x_1, x_2, x_3) = (\dot{m}_f, n, p_i) \). The SI engine model is given as follows:

\[
\begin{align*}
\dot{x}_1 &= \dot{\dot{m}}_f = \frac{1}{\tau_f}((-\dot{m}_f + \dot{m}_{fi}) + (1 - X)\dot{m}_{fi}) \\
&= \frac{1}{\tau_f}(-x_1 + \dot{m}_{fi}) + (1 - X)\dot{m}_{fi} \\
\dot{x}_2 &= \dot{n} = \frac{1}{\eta_i}(H_u\eta_i\dot{m}_f - P_l + P_b) \\
&= -\frac{1}{Ix_2}(P_l + P_b) + \frac{1}{Ix_2}H_u\eta_i\eta_p\eta_\lambda x_1 e^{-\theta^2/\theta_{mbt}^2} \\
\dot{x}_3 &= \dot{p}_i = \frac{RT_{i}}{V_i}(-\dot{m}_{ap} + \dot{m}_{at}) \\
&= -\frac{RT_{i}}{V_i}\dot{m}_{ap} + \frac{RT_{i}}{V_i}k_{20}\beta_2(p_i)(1 - \cos(\alpha - \alpha_0)) \\
&= -\frac{RT_{i}V_d}{V_i120RT_{i}}(k_{18}\dot{p}_i + k_{19})x_2 \\
&\quad + \frac{RT_{i}}{V_i}k_{20}\beta_2(x_3)(1 - \cos(\alpha - \alpha_0)) \quad (4)
\end{align*}
\]

In the above equation of \( \dot{x}_3 \), the first term represents the impact of air flow into the cylinder and the second term represents that of the air flow past the throttle plate (namely the orifice equation). The time delay \( \tau_d \) for \( \dot{m}_f \) is assumed to be zero.

In this paper, we discuss that the system has three inputs \( u = (\dot{m}_{fi}, \theta, \alpha) \), and three outputs \( y = (\lambda, n, p_i) \). Thus the SI engine model can be rewritten as:

\[
\begin{align*}
\dot{x} &= f(x) + Gu \\
y &= h(x)
\end{align*}
\]

where

\[
f = \begin{pmatrix}
-x_1/\tau_f \\
-x_1^2/\tau_f RT_{i}V_{i} \frac{k_{18}\dot{p}_i + k_{19}}{V_{i120RT_{i}}} x_2
\end{pmatrix}
\]

\[
G = \text{diag}(|g_1, g_2, g_3|)
\]

\[
h = \left( \begin{array}{c}
1 \\
0 \\
0
\end{array} \right)\frac{RT_{i}}{V_i}k_{20}\beta_2(x_3)
\]

\[
\dot{u} = \begin{pmatrix}
\frac{1}{\tau_f}u + (1 - X)\dot{u} \\
\frac{RT_{i}}{V_i}k_{20}\beta_2(x_3)
\end{pmatrix}, \quad \dot{h}(x) = \begin{pmatrix}
h(x) \\
\frac{RT_{i}}{V_i}k_{20}\beta_2(x_3)
\end{pmatrix}
\]

and

\[
h(x) = \lambda = \frac{\dot{m}_{ap}}{14.67\dot{m}_f} = \frac{1}{14.67} \frac{V_d}{120RT_{i}}(k_{18}\dot{p}_i + k_{19})n \\
= \frac{V_d}{14.7 \times 120RT_{i}}(k_{18}x_3 + k_{19}) \quad (7)
\]

III. Adaptive Control for the Engine System with Constant Uncertainties
In this section, we design the controller based on adaptive control method to deal with uncertainties in the engine dynamic system. We assume there exists a constant uncertainty...
(disturbance) in the air-fuel dynamics:

\[
\begin{align*}
\dot{x}_1 &= f_1 + g_1 \hat{u}_1 + d_1, \\
\dot{x}_2 &= f_2 + g_2 \hat{u}_2 + d_2, \\
\dot{x}_3 &= f_3 + g_3 \hat{u}_3 + d_3.
\end{align*}
\]

Outputs of the system are given by \( y = (\lambda, x_2, x_3) = \hat{h}(x) \), where \( \lambda = h(x) \).

Define \( e_1 = \lambda - r, e_2 = x_2 - x_{2d}, e_3 = x_3 - x_{3d} \), then the error dynamics is derived as

\[
\begin{align*}
\dot{e}_1 &= \frac{\partial h}{\partial x_1} \dot{x}_1 + \frac{\partial h}{\partial x_2} \dot{x}_2 + \frac{\partial h}{\partial x_3} \dot{x}_3 \\
&= \left( \frac{\partial h}{\partial x_1} f_1 + \frac{\partial h}{\partial x_2} f_2 + \frac{\partial h}{\partial x_3} f_3 \right) \\
&\quad + \left( \frac{\partial h}{\partial x_1} g_1 \hat{u}_1 + \frac{\partial h}{\partial x_2} g_2 \hat{u}_2 + \frac{\partial h}{\partial x_3} g_3 \hat{u}_3 \right) \\
&\quad + \left( \frac{\partial h}{\partial x_1} d_1 + \frac{\partial h}{\partial x_2} d_2 + \frac{\partial h}{\partial x_3} d_3 \right) \\
&= L_h f + \sum_{i=2}^{3} L_{hi} g_i \hat{u}_i + L_{h1} g_1 \hat{u}_1 + L_h \theta,
\end{align*}
\]

\[
\begin{align*}
\dot{e}_2 &= \dot{x}_2 - x_{2d} = f_2 + g_2 \hat{u}_2 - x_{2d} + d_2, \\
\dot{e}_3 &= \dot{x}_3 - x_{3d} = f_3 + g_3 \hat{u}_3 - x_{3d} + d_3,
\end{align*}
\]

where \( L_h = [L_{h1} L_{h2} L_{h3}] = [\frac{\partial h}{\partial x_1} \frac{\partial h}{\partial x_2} \frac{\partial h}{\partial x_3}], f = [f_1 f_2 f_3]^T \) and \( \theta = [\theta_1 \theta_2 \theta_3]^T = [d_1 d_2 d_3]^T \).

It is noted that the value of each \( L_{hi} g_i \) is dependent on the state variables \( x \). It is reasonably assumed that \( L_{hi} g_i \) and \( g_i \) are nonzero for some range of state variables, therefore, the inverse functions for \( L_{hi} g_i \) and \( g_i \) exist in such range of state variables. The above equations can be rewritten as

\[
\begin{align*}
\dot{e}_1 &= L_h g_1 ((L_h g_1)^{-1} L_h f + (L_h g_1)^{-1} \sum_{i=2}^{3} L_{hi} g_i \hat{u}_i + \hat{u}_1 + (L_h g_1)^{-1} L_h \theta), \\
\dot{e}_2 &= g_2 (g_2^{-1} f_2 + \hat{u}_2 - g_2^{-1} x_{2d} + g_2^{-1} d_2), \\
\dot{e}_3 &= g_3 (g_3^{-1} f_3 + \hat{u}_3 - g_3^{-1} x_{3d} + g_3^{-1} d_3). \\
\end{align*}
\]

The control objective is to look for a suitable controller \( \hat{u}_i \) such that the tracking error \( e_i(t) \) converges to zero as \( t \rightarrow \infty \) even if the system has some constant uncertainties. In order to realize the aims, the adaptive controller \( \hat{u}_i \) is designed as follows:

\[
\begin{align*}
\dot{u}_1 &= -C_1 (L_h g_1)^{-1} e_1 - (L_h g_1)^{-1} L_h f \\
&\quad - (L_h g_1)^{-1} \sum_{i=2}^{3} L_{hi} g_i \hat{u}_i, \\
\hat{u}_i &= -C_1 g_i^{-1} e_i - g_i^{-1} f_i + g_i^{-1} x_{id} - g_i^{-1} d_i, \quad i = 2, 3
\end{align*}
\]

where \( \theta = [\theta_1 \theta_2 \theta_3] \) and the parametric updating law is

\[
\dot{\theta} = q L_h^T e_1,
\]

\[
\dot{d}_1 = q_1 e_1, \quad i = 2, 3
\]

with the feedback gains \( q > 0 \) and \( q_1 > 0, j = 2, 3 \).

The globally asymptotical tracking property is ensured by the following theorem.

**Theorem 1:** For the air-fuel dynamics with constant uncertainties, the adaptive controller (10) and parametric updating law (11) guarantee that \( \lim_{t \rightarrow \infty} e_i(t) = 0, \ i = 1, 2, 3 \).

**Proof:** To prove the convergence of the tracking error, we define the following Lyapunov function

\[
V = V_1 + V_2 + V_3,
\]

where \( V_1 = \frac{1}{2} e_1^2 + \frac{1}{q_1} (\theta - \hat{\theta})^T (\theta - \hat{\theta}) \) and \( V_2 = \frac{1}{2} e_i^2 + \frac{1}{q_i} (d_i - \hat{d}_i)^2, \ i = 2, 3 \). Substituting the adaptive controller (10) into (9), we obtain

\[
\begin{align*}
\dot{e}_1 &= -C_1 e_1 + (\theta - \hat{\theta})^T L_h^T, \\
\dot{e}_i &= -C_1 e_i + (d_i - \hat{d}_i), \quad i = 2, 3
\end{align*}
\]

Differentiating \( V \) with respect to time \( t \) gives \( \dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \). Furthermore, according to (13), we obtain

\[
\begin{align*}
\dot{V}_1 &= e_1 \dot{e}_1 + \frac{1}{q_1} (\theta - \hat{\theta})^T (-\dot{\theta}), \\
\dot{V}_i &= e_i \dot{e}_i + \frac{1}{q_i} (d_i - \hat{d}_i)(\dot{d}_i - \dot{\hat{d}}_i), \quad i = 2, 3
\end{align*}
\]

Therefore, we have

\[
\begin{align*}
\dot{V} &= -C_1 e_1^2 + C_2 e_2^2 + C_3 e_3^2 + \frac{1}{q_1} (\theta - \hat{\theta})^T (q L_h^T e_1 - \dot{\theta}) \\
&\quad + \sum_{i=2}^{3} \frac{1}{q_i} (d_i - \hat{d}_i)(q_i e_i - \dot{\hat{d}}_i).
\end{align*}
\]

Considering the parametric updating law in (11), we further have \( V = -C_1 e_1^2 + C_2 e_2^2 + C_3 e_3^2 \leq 0 \). Since \( V \) is a continuous function of \( e_i, \hat{\theta}, \hat{d}_i, i = 2, 3 \), this guarantees that \( V(0) \) is bounded for any initial values \( e(0), \hat{\theta}(0) \) and \( \hat{d}(0) \), i.e., \( V(0) < \infty \). It is also noted that \( 0 \leq V(t) \leq V(0) \). Integrating the above inequality yields \( \int_0^\infty C_1 e_i^2(t) dt \leq V(0) < \infty \). This implies that \( \int_0^\infty e_i^2(t) dt \) is bounded and \( e_i \) is bounded, by noting that \( C_1 > 0 \). Secondly, \( \hat{e}_i \) is bounded from equation (13). Therefore, we conclude that \( \lim_{t \rightarrow \infty} e_i(t) = 0 \) according to Barbalat’s Lemma [27].

**IV. LEARNING CONTROL FOR THE ENGINE SYSTEM WITH TIME-VARYING PERIODIC UNCERTAINTIES**

In this section, we design the controller based on learning control method to deal with time-varying periodic uncertainties in the engine dynamic system.

As described in the previous section, we consider that the state dynamics with uncertainties are written as

\[
\begin{align*}
\dot{x}_1 &= f_1 + g_1 \hat{u}_1 + d_1(t), \\
\dot{x}_2 &= f_2 + g_2 \hat{u}_2 + d_2(t), \\
\dot{x}_3 &= f_3 + g_3 \hat{u}_3 + d_3(t)
\end{align*}
\]

where the uncertainty \( d_i(t), i = 1, 2, 3 \) is time varying and assumed to be periodic with period \( T_i \), i.e., \( d_i(t) = d_i(t - T_i) \). Without loss of generalization, assume \( T_1 \leq T_2 \leq T_3 \).
Denote the same state errors as Section III. Then we can get the same error dynamics as the equation in (9). The learning controller is given as follows:

\[ \tilde{u}_i = -C_1 (L_{h1}g_1)^{-1} e_1 - (L_{h1}g_1)^{-1} \cdot L_h f \]

\[ \tilde{u}_i = -C_i g_i^{-1} e_i - g_i^{-1} \tilde{x}_i - g_i^{-1} \tilde{d}_i(t), \quad i = 2, 3 \]

where \( \hat{\theta}(t) = [\hat{\theta}_1(t), \hat{\theta}_2(t), \hat{\theta}_3(t)]^T \) and the parametric updating law is

\[ \hat{\theta}_i(t) = \left\{ \begin{array}{ll} q L_{h_i} e_i, & t \in [0, T_1) \\ \hat{\theta}_i(t - T_1) + q L_{h_i} e_i, & t \in [T_1, \infty), \quad i = 1, 2, 3, \end{array} \right. \]

\[ \hat{d}_i(t) = \left\{ \begin{array}{ll} q_i e_i, & t \in [0, T_1) \\ \hat{d}_i(t - T_1) + q_i e_i, & t \in [T_1, \infty), \quad i = 2, 3 \end{array} \right. \]

where \( q > 0 \) and \( q_j > 0, \quad j = 2, 3 \) are constants.

To facilitate the convergence analysis, the positive-definite function will be adopted: \( V = V_1 + V_2 + V_3 \), where

\[ V_1 = \left\{ \begin{array}{ll} \frac{1}{2} q_1 e_1^2 + \frac{1}{2 q_1} \int_{0}^{t} \hat{\theta}_1^2(\tau) d\tau, & t \in [0, T_1) \\ \frac{1}{2} q_1 e_1^2 + \frac{1}{2 q_1} \int_{t-T_1}^{t} \hat{\theta}_1^2(\tau) d\tau, & t \in [T_1, \infty), \end{array} \right. \]

\[ V_2 = \left\{ \begin{array}{ll} \frac{1}{2} q_i e_i^2 + \frac{1}{2 q_i} \int_{0}^{t} \hat{d}_i^2(\tau) d\tau, & t \in [0, T_1) \\ \frac{1}{2} q_i e_i^2 + \frac{1}{2 q_i} \int_{t-T_1}^{t} \hat{d}_i^2(\tau) d\tau, & t \in [T_1, \infty), \quad i = 2, 3 \end{array} \right. \]

with \( \hat{\theta}_i(t) = \theta_i(t) - \tilde{\theta}_i(t) \) and \( \hat{d}_i(t) = d_i(t) - \tilde{d}_i(t), \quad i = 2, 3 \).

In the following we summarize two important properties associated with functionals, which will be used in subsequent derivations with the Lyapunov Krassovskii functional.

**Property 1:** Let \( \phi \in R \) and \( T > 0 \) be a finite constant. The upper right-hand derivative of \( \int_{t-T_1}^{t} \phi^2(\tau) d\tau \) is \( \phi^2(t - T_1) \).

**Property 2:** Let \( d(t), \tilde{d}(t), \hat{d}(t), f(t) \in R \), and assume that the following relation hold \( d(t) = (t - T_1) \), \( \tilde{d}(t) = d(t) - \tilde{d}(t), \hat{d}(t) = \hat{d}(t) + f(t) \). Then the upper right-hand derivative of \( \int_{t-T_1}^{t} d^2(\tau) d\tau \) is \(-2d(t)f(t) - \tilde{d}^2(t)\).

The proof of the above properties can be seen from [26].

**Theorem 2:** The control law (16) with the parametric updating law parameter (17) warrants the asymptotical convergence

\[ \lim_{t \to \infty} \int_{0}^{t} e_i^2(\tau) d\tau = 0. \]

**Proof:** Let us first derive the upper right-hand derivative of \( V \) for \( t \in [0, T_1] \), which is

\[ \dot{V}(t) = \sum_{i=1}^{3} e_i \dot{e}_i + \frac{1}{2 q_i} \sum_{i=1}^{3} \hat{\theta}_i^2 + \frac{1}{2 q_i} \hat{d}_i^2. \]  

Substituting the learning controller in (16) into (9) yields

\[ e_i \dot{e}_i = -C_i e_i + \sum_{i=1}^{3} e_i L_{h_i} \hat{\theta}_i \]

\[ e_i \dot{e}_i = -C_i e_i^2 + e_i \dot{d}_i, \quad i = 2, 3. \]

Thus according to the parameter updating law in (17), for \( t \in [0, T_1] \),

\[ \dot{V} = -\sum_{i=1}^{3} C_i e_i^2 - \sum_{i=1}^{3} e_i L_{h_i} \dot{\theta}_i + \sum_{i=1}^{3} e_i \dot{d}_i \]

\[ + \frac{1}{2 q_i} \sum_{i=1}^{3} \hat{\theta}_i^2 + \frac{1}{2 q_i} \hat{d}_i^2. \]

\[ = -\sum_{i=1}^{3} C_i e_i^2 - \sum_{i=1}^{3} e_i L_{h_i} \dot{\theta}_i + \sum_{i=1}^{3} e_i \dot{d}_i \]

\[ + \frac{1}{2 q_i} \sum_{i=1}^{3} \hat{\theta}_i^2 + \frac{1}{2 q_i} \hat{d}_i^2. \]

\[ = \sum_{i=2}^{3} \frac{1}{2 q_i} \hat{d}_i^2 + \frac{1}{2 q_i} \hat{d}_i^2. \]

Since for any \( c_i > 0 \) and \( \tilde{c}_i > 0 \), \( \frac{1}{q_i} \hat{\theta}_i \leq \frac{1}{q_i} \hat{\theta}_i^2 + \frac{1}{4 q_i} \hat{d}_i^2 \) and \( \frac{1}{q_i} \hat{d}_i \leq \frac{2}{q_i} \hat{d}_i^2 + \frac{1}{4 q_i} \hat{d}_i^2 \). Let \( 0 < \frac{1}{q_i} < \frac{1}{2 q_i} \) and \( 0 < \frac{1}{q_i} < \frac{1}{2 q_i} \). We have

\[ \dot{V} \leq \sum_{i=1}^{3} \frac{1}{4 q_i} \hat{d}_i^2 + \frac{1}{4 q_i} \hat{d}_i^2. \]

\[ \leq \sum_{i=2}^{3} \frac{1}{2 q_i} \hat{d}_i^2 + \frac{1}{4 q_i} \hat{d}_i^2. \]

Since \( \dot{\theta}_i(t) \) and \( \dot{d}_i(t) \) are periodic, there exists a finite bound \( M_1 \geq \dot{\theta}_i(t), \forall t \in [0, T_1] \) and \( M_2 \geq \dot{d}_i(t), \forall t \in [0, T_1] \). Thus, \( \dot{V} < 0 \) outside the region

\[ \left\{ (e_i, \dot{\theta}_i, \dot{d}_i) | \dot{V} \leq \sum_{i=1}^{3} \frac{1}{4 q_i} \hat{d}_i^2 + \frac{1}{4 q_i} \hat{d}_i^2 \right\}. \]

where

\[ \dot{V} = \sum_{i=1}^{3} \frac{1}{4 q_i} \hat{d}_i^2 + \frac{1}{4 q_i} \hat{d}_i^2 + \sum_{i=2}^{3} \frac{1}{2 q_i} \hat{d}_i^2. \]

Therefore, \( \dot{V} \) is finite in the interval \( [0, T_1] \).

The upper right-hand derivative of \( V \) according to Property 1 for \( t \in (T_1, T_2) \) should be

\[ \dot{V} = \sum_{i=1}^{3} e_i \dot{e}_i + \frac{1}{2 q_i} (\hat{\theta}_i^2(t) - \hat{\theta}_i^2(t - T_1)) \]

\[ + \frac{1}{2 q_i} \sum_{i=2}^{3} \hat{\theta}_i^2 + \frac{1}{2 q_i} \hat{d}_i^2. \]

Comparing with (20), only one term \( \frac{1}{2 q_i} \hat{\theta}_i^2 \) is changed into \( \frac{1}{2 q_i} (\hat{\theta}_i^2(t) - \hat{\theta}_i^2(t - T_1)) \) in the above equation. According to Property 2 and the parameter updating law (17), we can further derive the following relationship

\[ (\hat{\theta}_i^2(t) - \hat{\theta}_i^2(t - T_1)) \]

\[ = 2(\hat{\theta}_1(t - T_1) - \hat{\theta}_1(t))\hat{d}_1(t) - \frac{1}{2}(\hat{\theta}_1(t - T_1) - \hat{\theta}_1(t))^2 \]

\[ = -2q e_1 L_{h1} \hat{\theta}_1 - (\hat{\theta}_1(t - T_1) - \hat{\theta}_1(t))^2. \]
Thus we will obtain $\dot{V}$ for $t \in [T_1, T_2)$
\[
\dot{V} \leq -\sum_{i=1}^{3} C_i e_i^2 - \frac{1}{2q} \sum_{i=1}^{3} \hat{\theta}_i^2 + \frac{1}{q} \sum_{i=2}^{3} \theta_i^2 + \frac{1}{2q} \sum_{i=2}^{3} d_i^2 + \frac{1}{q} \sum_{i=2}^{3} d_i d_\bar{d}_i,
\] (25)
which is similar as (22). Therefore the finiteness of $V$ for $t \in [T_1, T_2)$ can be easily obtained.

Similarly, the upper right-hand derivative of $V$ for $t \in [T_2, T_3)$ is
\[
\dot{V} \leq -\sum_{i=1}^{3} C_i e_i^2 - \frac{1}{2q} \sum_{i=1}^{3} \hat{\theta}_i^2 + \frac{1}{q} \sum_{i=2}^{3} \theta_i^2 + \frac{1}{2q} \sum_{i=2}^{3} d_i^2 + \frac{1}{q} d_i d_\bar{d}_i.
\] (26)
Then $V$ is also finite in the time interval $[T_2, T_3)$.

The upper right-hand derivative of $V$, according to the above deduction for $t \in [T_3, \infty)$, should be
\[
\dot{V} = \sum_{i=1}^{3} e_i \dot{e}_i + \frac{1}{2q} \sum_{i=1}^{3} (\theta_i^2(t) - \dot{\theta}_i^2(t - T_1)) + \sum_{i=2}^{3} \frac{1}{2q} (d_i^2(t) - d_i^2(t - T_1)) \leq -\sum_{i=1}^{3} C_i e_i^2.
\] (27)

Now let us derive the convergence property $\lim_{t \to \infty} \int_{t-T_3}^{t} e_i^2(\tau) d\tau = 0$ using the fact (27) that $V$ for $t \in [T_3, \infty)$ is negative semi-definiteness. Suppose that $\lim_{t \to \infty} \int_{t-T_3}^{t} e_i^2(\tau) d\tau \neq 0$. Then there exist an $\epsilon > 0$, a $t_0 \geq T_3$ and a sequence $t_i \to \infty$ with $i = 1, 2, \cdots$ and $t_{i+1} \geq t_i + T_3$ such that $\int_{t_i-T_3}^{t_i} e_i^2(\tau) d\tau > \epsilon$ when $t_i > t_0$. Hence from (27), we obtain for $t > T_3$
\[
\lim_{t \to \infty} V \leq V(T_3) - \lim_{t \to \infty} \sum_{j=1}^{3} \int_{t_j-T_3}^{t_j} e_i^2(\tau) d\tau.
\]
Since $V(T_3)$ is finite, the above relation implies $\lim_{t \to \infty} V \to -\infty$, a contradiction to the positiveness property of $V$.

This completes the proof.

V. Simulation Results

The simulations are carried out for a 1275cc British Leyland engine for which data has been reported by Hendrics et al [10] and by [8]. The values of the parameters used in the simulations are adopted from their work, as given below: $I = (\frac{2 \pi}{60})^2 \times 0.5 \text{ kg-m}^2$, $\tau_f = 0.25$, $X = 0.22$, $V_r = 646 \times 10^{-6}$, $H_o = 4.3 \times 10^4$, $V_d = 1.275$, $R = 287 \times 10^{-5}$, $T_i = 308$, $T_o = 297$, $P_o = 1.013$, $k_1 = 1.673$, $k_2 = 0.272$, $k_3 = 0.0135$, $k_4 = 0.969$, $k_5 = 0.206$, $k_6 = 0.558$, $k_7 = 0.392$, $k_8 = 0.9301$, $k_9 = 0.2154$, $k_{10} = 0.1657$, $k_{11} = 47.31$, $k_{12} = 2.6214$, $k_{13} = -56.55$, $k_{14} = 57.34$, $k_{15} = 0.952$, $k_{16} = 0.075$, $k_{20} = 7.32 \sqrt{\frac{\pi}{2}}$.

In this example, we will apply the designed learning controller to give the desired air-fuel ratio and track the speed and air pressure correctly when facing time-varying periodic disturbances. In the following, the simulations are performed with the disturbance $d = (0.1 \sin(2\pi t), 0, 0)^T$. According to Theorem 2 established, the learning controller can guarantee asymptotic tracking when there is time periodic disturbance in the control plant. It can be seen from Fig. 1a, the air-fuel ratio is maintained at a good level, only with a miniature deviation less than 0.1% of the magnitude. On the other hand, the crank shaft speed and intake manifold air pressure are exactly tracked to the desired signals (Fig. 1b, c). Furthermore, the corresponding learning control signals are given in Fig. 2.

VI. Conclusion

This paper studied the spark ignition engine control problems. To deal with uncertainties in the engine modeling, two controller design methods based on adaptive control and learning control were proposed by considering two types of uncertainties: constant and time-varying periodic uncertainty. With rigorous analysis, we proved the convergence properties of the adaptive and learning control laws. The simulation results show that the controller using adaptive control and learning control is able to achieve the control objectives in the event of uncertainty and disturbance.

REFERENCES


APPENDIX

\( \dot{m}_{i,p} \) Air mass flow rate into cylinder (kg/s).

\( \dot{m}_{i,t} \) Air mass flow rate past throttle plate (kg/s).

\( p_a \) Atmosphere pressure (1.013 bar).

\( T_a \) Atmosphere temperature (K).

\( k_i \) Constants. \( n \) Crank shaft speed (krpm).

\( \dot{m}_{f} \) Cyclinder port fuel flow (kg/s).

\( X \) Fraction of \( \dot{m}_{f,i} \) which is deposited on manifold as fuel film.

\( \tau_f \) Fuel evaporation time constant (0.25s).

\( \dot{m}_{f,f} \) Fuel film mass flow (kg/s).

\( H_u \) Fuel heating value (4.3 \times 10^4 \text{kJ/kg}).

\( \dot{m}_{f,v} \) Fuel vapor mass flow (kg/s).

\( \eta_i \) Indicated efficiency.

\( \dot{m}_{f,i} \) Injected fuel mass flow (kg/s).

\( P_b \) Load power (kW).

\( p_i \) Manifold air pressure (bar).

\( \theta_{mbt} \) Max break torque spark timing.

\( \lambda \) Normalized air-fuel ratio.

\( P_l \) Pumping & friction power (kW).

\( \theta \) Spark advance angle (degrees).

\( \alpha \) Throttle opening angle (degrees).

\( I \) Total moment of inertia (0.5/(2\pi/60)^2 \text{kgm}^2).