Dynamic Spectrum Access with Statistical QoS Provisioning: A Distributed Learning Approach Beyond Expectation Optimization

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Abstract—This article investigates the problem of dynamic spectrum access with statistical quality of service (QoS) provisioning for dynamic canonical networks, in which the channel states are time-varying from slot to slot. In the existing work with time-varying environment, the commonly used optimization objective is to maximize the expectation of a certain metric (e.g., throughput or achievable rate). However, it is realized that expectation alone is not enough since some applications are sensitive to the channel fluctuations. Effective capacity is a promising metric for time-varying service process since it characterizes the packet delay violating probability (regarded as an important statistical QoS index), by taking into account not only the expectation but also other high-order statistic. We formulate the interactions among the users in the time-varying environment as a non-cooperative game, in which the utility function is defined as the achieved effective capacity. We prove that it is an ordinal potential game which has at least one pure strategy Nash equilibrium. In addition, we propose a multi-agent learning algorithm which is proved to achieve stable solutions with uncertain, dynamic and incomplete information constraints. The convergence of the proposed learning algorithm is verified by simulation results. Also, it is shown that the proposed multi-agent learning algorithm achieves satisfactory performance.

Index Terms—dynamic spectrum access, statistical QoS, effective capacity, multi-agent learning, distributed channel selection, potential game.

I. INTRODUCTION

DYNAMIC spectrum access (DSA) has been regarded as one of the most important technology for future wireless networks since it provides flexible and efficient spectrum usage. With the significant advances in cognitive radios in the last decade [1]–[3], DSA can be implemented in more intelligent and smart manners [4]–[6]. Generally, there are two main application scenarios [7]: open-access, in which all users are equal to access the spectrum, and primary-secondary access, in which the spectrum is owned by the primary users and can be used by the secondary users when it is idle. For decision-making, it has been shown that the methodologies for the two scenarios are mostly overlapped [4].

A number of existing studies, e.g., [8]–[13], have considered intelligent spectrum access for static wireless networks in which the channel states remain unchanged during the selection procedure. However, it has been realized that although the assumption of static channel leads to mathematical tractability, it is not generally true since the spectrum are always time-varying in wireless environment [14]–[16]. To track the channel dynamics, an instinctive approach is to reiterate the selection algorithms in each quasi-static period. This method, however, is off-line, costly and inefficient, and is even not feasible for fast-varying channels. Thus, it is timely important to develop on-line intelligent channel selection algorithms for dynamic wireless networks.

In this article, we consider a dynamic wireless canonical networks, in which the channel states are time-varying and there is no information exchange among the users. In a few existing researches for dynamic networks with time-varying channels, e.g., [14]–[17], the commonly used optimization objective is to maximize the expectation of a certain metric, e.g., the expected throughput. However, only considering the expectation is not enough for practical applications. For example, in real-time multimedia applications, higher expected transmission rate as well as lower fluctuation are desirable, which implies that not only the expectation but also other statistic, e.g., the variance, should be taken into account for dynamic wireless networks. A promising metric is the effective capacity, which is defined as the maximum packet arrival rate that a time-varying service process can support while a statistical quality-of-service (QoS) constraint on delay violating probability can be met [18], [19]. Mathematically, effective capacity takes into account the expectation and all other statistics; further, it degrades the expectation if the statistical QoS index is sufficiently small. Therefore, we use effective capacity as the optimization metric in this article.

The considered DSA network encounters uncertain, dynamic and incomplete information constraints for the decision procedure. Specifically, the channel states are not deterministic at each slot and change from slot to slot, and a user can only monitor its chosen channel and know nothing about other users. Furthermore, the introduction of effective capacity into dynamic cognitive radio networks leads to additional challenges. In comparison, the expectation admits additive property in the time domain while the effective capacity does not. It should be pointed out that the main concern of this paper is to consider both expectation and other statistic in dynamic wireless networks. Thus, except for the used effective capacity, other forms of optimization metric can also be used. We will explain this more specific later.
not. In particular, an expected value can be obtained by cumulatively averaging the random payoffs in a long period. However, effective capacity does not admit the additive property due to its nonlinearity. Thus, the task of designing effective-capacity oriented intelligent channel selection approaches for multiple users with the uncertain, dynamic and incomplete information constraints remains unsolved and is challenging.

Since the decisions of the uses are interactive, we formulate the problem of dynamic spectrum access with statistical QoS constraints as a non-cooperative game. We prove that the proposed game is an ordinal potential game which has at least one pure strategy Nash equilibrium (NE). Due to the uncertain, dynamic and incomplete information constraints, existing game-theoretic algorithms, e.g., the best response [20], fictitious play [21], spatial adaptive play [8] and regret learning [9], can not be applied to the considered dynamic networks. The reason is that they are originally designed for static systems with complete information. It is known that users in cognitive radios are able to observe the environment, learn from history experiences, and make intelligent decisions [3].

Following this methodology, we propose a multi-agent learning algorithm to achieve the Nash equilibria of the formulated dynamic spectrum access game with QoS provisioning. To summarize, the main contributions of this article are:

1) We formulate the problem of dynamic spectrum access with QoS provisioning as a non-cooperative game, in which the utility function of each user is defined as the effective capacity characterized by a QoS index. In particular, the utility function takes into account not only the expectation of the achievable transmission rate but also other statistic. We prove that the game is an ordinal potential game and hence has at least one pure strategy NE point.

2) We propose a multi-agent learning algorithm to achieve the pure strategy NE points of the game with unknown, dynamic and incomplete information constraints. The proposed algorithm is fully distributed and autonomous, since it only relies on the individual information of a user and does not need information about other players. Simulation results show that the proposed learning algorithm achieves satisfactory performance.

Note that there are some previous work which also considered effective capacity in dynamic spectrum access/cognitive radio networks, e.g., [19], [22]–[25]. The main differences in methodology are: i) most existing studies considered optimization of effective capacity in a centralized manner, while we consider this problem in a distributed manner, ii) we consider the interactions among multiple users and propose a multi-agent learning algorithm to achieve stable solutions, and iii) the effective capacity can not be obtained by cumulatively averaging the random payoffs in a long period due to its nonlinearity, which brings new challenges for the learning solutions.

The rest of the article is organized as follows. In Section II, we give a brief review of related work. In Section III, we present the system model and formulate the problem. In Section IV, we present the dynamic spectrum access game and investigate the properties of its NE, and propose a multi-agent learning algorithm for achieving stable solutions. In Section V, simulation results are presented. Finally, we present discussion and draw conclusion in Section VI.

II. RELATED WORK

The problem of dynamic spectrum access in both open-access and primary-secondary access scenarios has been extensively investigated in the context of cognitive radio, e.g., [8]–[13], [26]–[29]. These work mainly focused on static networks, in which the channel states remain unchanged during the learning and decision procedure. However, it has been realized that the assumption of static channel is not always true in practice.

Recently, the problem dynamic spectrum access with varying channel states began to draw attention, using e.g., Markovian decision process (MDP) [15], online learning algorithms for multi-armed bandit (MAB) problems [17], and game-theoretic learning [14], [16]. The commonly used optimization metric in these work is to maximize the expected achievable transmission rate, which does not consider the QoS requirement in the packet delay. In addition, the algorithms in MDP and MAB models are mainly for scenarios with single user. Compared with those existing studies, this work is differentiated in that a statistical QoS requirement in packet delay is considered for a multi-user DSA network with time-varying channels.

It is noted that multi-agent learning algorithms for game-theoretic solutions in wireless networks have been an active topic. Specifically, stochastic learning automata [30] based algorithms for wireless communications can be found in the literature, e.g., distributed channel selection for opportunistic spectrum access [14], [16], [31], distributed power control [52], precoding selection for MIMO systems [33], spectrum management [34] and cooperative coordination design [35] for cognitive radio networks. Furthermore, Q-learning based dynamic spectrum access was reported in [36]–[38], various combined fully distributed payoff and strategy-reinforcement learning algorithms for 4G heterogeneous networks were studied in [39], a trial-and-error learning approach for self-organization in decentralized networks was studied in [40], and several variations of logit-learning algorithms were studied in [12], [13], [41]–[43]. In methodology, all of the above mentioned algorithms are originally designed for maximizing the expectation and hence can not be applied in this work.

We consider a new optimization metric that takes into account not only the expectation but also other high-order moments. It leads to new challenges in analyzing the game properties as well as designing the learning procedure. However, it should be pointed out that risk-sensitive game [44] admits the same utility function with that in this paper, which implies it also considers both expectation and other high-order statistic. The key differences in this paper are: (i) the effective capacity has physical meaning for wireless communications, (ii) we show that the dynamic spectrum access game with effective capacity optimization is an ordinal potential game, and prove the convergence of the proposed multi-agent learning algorithm.
The most related work is [45], in which a game-theoretic optimization approach for effective capacity in cognitive femtocells was studied. The key difference in this work is that we focus on formulating the game model as well as designing multi-agent learning with uncertain, dynamic and incomplete information constraints. Nevertheless, the authors of [45] only focused on game formulation and analysis. Another related work is [46], in which a satisfaction equilibrium approach is proposed for QoS provisioning in decentralized networks.

Note that NE may be inefficient due to its inherent non-cooperative nature. There are some other solutions beyond NE to improve the efficiency, e.g., pricing [47], auction [48], Nash bargaining [49], and coalitional games [50], [51]. The key difference in this paper is that the proposed solution does not need information exchange while these solutions need information exchange among users, which may cause heavy communication overhead. Also, there are some variants of NE for OSA optimization in the literature, e.g., correlated equilibrium (CE) [11], [52] and evolutionary stable state (ESS) [53], [54]. Compared with existing CE and ESS solutions, the main difference in this paper is that the optimization objective considers both expectation and other high-order statistic.

III. SYSTEM MODELS AND PROBLEM FORMULATION

A. System model

We consider a distributed canonical network consisting of $N$ users and $M$ channels. A user in canonical networks is a collection of multiple entities with intra-communications and there is a head managing the whole community [55]. Examples of users in canonical networks given by, e.g., a WLAN access point with the serving client [56] and a cluster head with its belonged members. For presentation, denote the user set as $\mathcal{N}$, i.e., $\mathcal{N} = \{1, \ldots, N\}$, and the channel set as $\mathcal{M}$, i.e., $\mathcal{M} = \{1, \ldots, M\}$. Due to fading in wireless environment, the transmission rate of each channel is always time-varying. To capture the rate fluctuations, the finite rate channel model is applied [57]. In particular, the rate set of channel $m$ is denoted as $\mathcal{S}_m = \{s_{m1}, s_{m2}, \ldots, s_{MK}\}$, where $s_{mk}$ indicates that the channel can support certain transmission rate (packets/slot). The corresponding rate-state probabilities are given by $\Pi_m = \{\pi_{m1}, \ldots, \pi_{MK}\}$ and the expected transmission rate of channel $m$ is given by $\bar{s}_m = \sum_k \pi_{mk}s_{mk}$.

We assume that time is divided into slots with equal length and the transmission rate of each channel is block-fixed in a slot and changes randomly in the next slot. Specifically, the achievable transmission rate of channel $m$ for user $n$ in slot $i$ is denoted as $r_{nm}(i)$, which is randomly chosen from the rate set $\mathcal{S}_m$. We consider heterogeneous spectrum in this article. Specifically, the transmission rate set and the corresponding probability set vary from channel to channel [1].

The task of each user is to choose an appropriate channel to access. Without loss of generality, we assume that the number of users is larger than that of the channels, i.e., $N > M$. When more than one user chooses the same channel, they share the channel using some multiple access mechanisms, e.g., CSMA. There is no central controller and no information exchange among the users, which means that the users should choose appropriate channels through learning and adjusting.

Denote $a_n$ as the chosen channel of user $n$, i.e., $a_n \in \mathcal{M}$. For presentation, we assume that perfect CSMA is applied to resolve contention among the users. Thus, the instantaneous achievable transmission rate of user $n$ is given by:

$$r_n(i) = \begin{cases} s_{an}(i), & \text{w.p. } 1 + \frac{1}{1+\sum_{n'\in\mathcal{N},n\neq n'}I(a_n,a_{n'})} \frac{1}{1+\sum_{n'\in\mathcal{N},n\neq n'}I(a_n,a_{n'})} \\ 0, & \text{w.p. } 1 - \frac{1}{1+\sum_{n'\in\mathcal{N},n\neq n'}I(a_n,a_{n'})} \end{cases}$$

(1)

where $s_{an}(i)$ is the instantaneous transmission rate of channel $a_n$ in time $t$, and $I(a_n,a_{n'})$ is the following indicator function:

$$I(a_n,a_{n'}) = \begin{cases} 1, & a_n = a_{n'} \\ 0, & a_n \neq a_{n'} \end{cases}$$

(2)

B. Preliminary of effective capacity

Since the channel transmission rate are time-varying, one candidate optimization metric is to maximize the expected transmission rate of user $n$, i.e., $\max_{a_n} \mathbb{E}[r_n(t)]$. It is noted that such an objective is not enough since the rate fluctuation may cause severe delay-bound violating probability whereas the expected rate cannot reflect this event.

To study the effect of time-varying transmission rate, one would take into account not only the expectation but also the variance and other higher-order moments. Among all possible solutions, the theory of effective capacity of time-varying service process is a promising approach. Therefore, we use effective capacity to study the problem of opportunistic channel access in heterogeneous spectrum.

Using large deviations theory [58], it was shown in [59] that for a dynamic queuing system with stationary arrival and service processes, the probability that the stationary queue length $Q(t)$ is large than a threshold $x$ is given by:

$$\lim_{x \to \infty} \frac{\log \Pr\{Q(t) > x\}}{x} = -\theta,$$

(3)

where $\theta$ serves as the exponential decay rate tail distribution of the stationary queue length. Therefore, for sufficiently large $x$, the queue length violating probability can be approximated by $\Pr\{Q(t) > x\} \approx e^{-\theta x}$. It is shown that larger $\theta$ corresponds to strict QoS requirement while small $\theta$ implies loose QoS requirement. Furthermore, it is shown that for a stationary traffic with fixed arrival rate $\lambda$, the delay-bound violating probability and the length-bound violating probability is related by:

$$\Pr\{D(t) > d\} \leq c \sqrt{\Pr\{Q(t) > q\}},$$

(4)

where $c$ is some positive constant and $q = \lambda d$. From the above analysis, it is seen that both the queue length violating

$^2$The feature of heterogeneous spectrum is caused by the flexible spectrum usage pattern in current wireless communication systems. Examples are given by: (i) in radio network, the channels are occupied by the primary users with different probabilities and (ii) in heterogeneous networks, the channels belong to different networks have different rate sets.

$^3$It is emphasized that the analysis and results presented in this article can be easily extended to practical CSMA systems by multiplying a modified factor in the formulation of the individual achievable rate.
probability and delay-bound violating probability are determined by the exponential decay rate $\theta$, which specifies the QoS requirement. Thus, we will pay attention to $\theta$ in this article.

For a time-varying service process with independent and identical distribution (i.i.d), the effective capacity is defined as follows [59]:

$$C(\theta) = -\frac{1}{\theta} \log \left( E[e^{-\theta x(t)}] \right), \quad (5)$$

where $x(t)$ is the time-varying service process, and $\theta$ is the statistical QoS index as specified by [3].

The properties of effective capacity can be analyzed as follows [44]:

- For a given time-varying service, it is a decreasing function with respect to $\theta$, i.e.,
  $$\theta_1 > \theta_2 > 0 \Rightarrow C(\theta_1) > C(\theta_2). \quad (6)$$

- For each $\theta > 0$, the effective capacity is always less than the expected capacity, i.e.,
  $$C(\theta) < E[x(t)], \forall \theta > 0, \quad (7)$$

  which can be proved by Jensen’s inequality [60].

- As $\theta$ approaches zero, the effective capacity degrades to the expected capacity, i.e.,
  $$\lim_{\theta \to 0} C(\theta) = E[x(t)]. \quad (8)$$

- If $\theta$ is sufficiently small, by performing Taylor expansion, we have:
  $$C(\theta) = E[x(t)] - \frac{\theta}{2} \text{var}[x(t)] + o(\theta), \quad (9)$$

  where $\text{var}[x(t)]$ is the variance of $x(t)$, and $o(\theta)$ is the infinitely small quantity of higher order.

From (6) to (11), it is seen that the effective capacity takes into account not only the expectation but also other moments (including the variance and other high-order moments) to capture the fluctuation in the time-varying service rate.

C. Problem formulation

For the considered dynamic spectrum access system, we use the effective capacity as the optimization metric. Specifically, denote $\theta_n$ as the statistical QoS index of user $n$, then the achievable effective capacity of user $n$ is given by

$$C_n(a_n, a_{-n}, \theta_n) = -\frac{1}{\theta_n} \log \left( E[e^{-\theta_n x_n(t)}] \right), \quad (10)$$

where $r_n(t)$ is the instantaneous transmission rate as specified by [1]; and $a_{-n}$ is the channel selection profile of all the users except user $n$.

For each user, the optimization objective is to choose a channel to maximize the effective capacity. It has been pointed out that information is key to decision-making problems [4]. For the considered dynamic spectrum access with statistical QoS provisioning, the information constraints can be summarized as follows:

- **Uncertain**: the instantaneous channel transmission rates are not deterministic, and the event of successfully accessing a channel in a slot is random.
- **Dynamic**: the instantaneous channel transmission rate is time-varying.
- **Incomplete**: the rate-state probabilities of each channel are unknown to the users, and a user does not know the QoS index of other users. Moreover, there is no information exchange among the users.

Due to the above uncertain, dynamic and incomplete information constraints, it is challenging to achieve desirable solutions even in a centralized manner, not to mention in an autonomous and distributed manner. Learning, which is core of cognitive radios [11], would achieve satisfactory performance in complex and dynamic environment. In the following, we propose a multi-agent learning approach to solve this problem.

IV. Multi-agent Learning Approach

Since there is no central controller and no information exchange, the users make their decisions autonomously and distributively. Furthermore, the decisions are interactive. This motivates us to formulate a non-cooperative game to capture the interactions among users. The properties of the formulated game are investigated. However, due to the uncertain, dynamic and incomplete information constraints, most existing game-theoretic algorithms cannot be applied. Therefore, we propose a multi-agent learning approach for the users to achieve desirable solutions autonomously and distributively.

A. Dynamic spectrum access game with QoS provisioning

The dynamic channel access game with QoS provisioning is denoted as $G = \{N, \theta_n, A_n, u_n\}$, where $N$ is the player (user) set, $A_n$ is the action space of player $n$, $\theta_n$ is the QoS index of player $n$ and $u_n$ is the utility function of player $n$. The action space of each player is exactly the available channel set, i.e., $A_n \subseteq \mathcal{M}$, $\forall n \in N$. In this game, the utility function is exactly the achievable effective capacity, i.e.,

$$u_n(a_n, a_{-n}) = -\frac{1}{\theta_n} \log \left( E[e^{-\theta_n x_n(t)}] \right), \quad (12)$$

In non-cooperative games, each player maximizes its individual utility. Therefore, the proposed dynamic spectrum access game with QoS provisioning can be expressed as:

$$G : \max u_n(a_n, a_{-n}), \forall n \in N \quad (13)$$

For a channel selection profile $(a_n, a_{-n})$, denote the set of users choosing channel $m$ as $C_m$, i.e., $C_m = \{n \in N : a_n = m\}$, then the number of users choosing channel $m$ can be expressed as $c_m(a_n, a_{-n}) = |C_m|$.

where $\alpha_1$ and $\alpha_2$ are the weighted coefficients determined by the specific practical applications. The reasons for using effective capacity as the optimization goal in this paper are twofold: (i) effective capacity takes into both expectation and other statistic into account, and (ii) it has physical meanings related to QoS provisioning for time-varying OSA networks.
B. Analysis of Nash equilibrium (NE)

In this subsection, we present the concept of Nash equilibrium (NE), which is the most well-known stable solution in non-cooperative game models, and analyze its properties. A channel selection profile $\alpha^* = (a^*_1, \ldots, a^*_n)$ is a pure strategy NE if and only if no player can improve its utility function by deviating unilaterally \cite{11}, i.e.,

$$u_n(a^*_n, a_{-n}^*) \geq u_n(a_n, a_{-n}^*), \forall n \in N', \forall a_n \in A_n$$

To investigate the properties of the formulated game, we first present the following definitions.

Definition 1. A game is an exact potential function (EPG) if there exists an exact potential function $\phi_e : A_1 \times \cdots \times A_N \rightarrow R$ such that for all $n \in N$, all $a_n \in A_n$, and $a_n' \in A_n$,

$$u_n(a_n, a_{-n}) - u_n(a_n', a_{-n}) = \phi_e(a_n, a_{-n}) - \phi_e(a_n', a_{-n})$$

(15)

In other words, the change in the utility function caused by an arbitrary unilateral action change of a user is the same with that in the exact potential function.

Definition 2. A game is an ordinal potential function (OPG) if there exists an ordinal potential function $\phi_o : A_1 \times \cdots \times A_N \rightarrow R$ such that for all $n \in N$, all $a_n \in A_n$, and $a_n' \in A_n$, the following holds:

$$u_n(a_n, a_{-n}) - u_n(a_n', a_{-n}) > 0 \iff \phi_o(a_n, a_{-n}) - \phi_o(a_n', a_{-n}) > 0$$

(16)

In other words, if the change in the utility function caused by an arbitrary unilateral action change is increasing, the change in the ordinal potential function keeps the same trend.

According to the finite improvement property \cite{20}, both EPG and OPG admits the following two promising features: (i) every EPG (OPG) has at least one pure strategy Nash equilibrium, and (ii) an action profile that maximizes the exact (ordinal) potential function is also a Nash equilibrium.

To investigate the properties of the formulated game, we first study an auxiliary dynamic spectrum access game with expected transmission rate serving as the utility function (denoted as $G_{\alpha}$), i.e., $u_n(a_n, a_{-n}) = E[r_n(i)]$.

Lemma 1. The auxiliary dynamic spectrum access game with expected transmission rate serving as the utility function is an EPG.

Proof: The following proof follows similar lines of proof given in \cite{14}. In this game, the achievable expected transmission rate of an arbitrary user $n \in C_m$ is given by:

$$\tilde{u}_n(a_n, a_{-n}) = E[r_n(i)] = \frac{E[r_{nm}(i)]}{\epsilon_m} = \frac{\tilde{s}_m}{\epsilon_m},$$

(17)

where $\tilde{s}_m$ is the expected transmission rate of channel $m$.

We define the following exact potential function $\phi_o : A_1 \times \cdots \times A_N \rightarrow R$ for the auxiliary channel access game $G_{\alpha}$:

$$\phi_o(a_n, a_{-n}) = \sum_{m=1}^{M} \sum_{l=1}^{c_m} \varphi_m(l),$$

(18)

where $\varphi_m(l) \triangleq \frac{\tilde{s}_m}{\epsilon_m}$. The above function is also known as Rosenthal's potential function \cite{61}.

If an arbitrary player $n$ unilaterally changes its channel selection from $a_n$ to $a_n'$, then the change in its utility function caused by this unilateral change is expressed as:

$$\tilde{u}_n(a_n', a_{-n}) - \tilde{u}_n(a_n, a_{-n}) = \varphi_m(c_{a_n'} + 1) - \varphi_m(c_{a_n}).$$

(19)

From a high-level perspective, the unilateral channel selection change of player $n$ can be equivalently regarded as if it is moved from channel $a_n$ to $a_n'$. Therefore, it only has impact on players that chose channels $a_n$ and $a_n'$, which implies that the change in the exact potential function is given by:

$$\phi_o(a_n', a_{-n}) - \phi_o(a_n, a_{-n}) = \left( \sum_{l=1}^{c_{a_n}+1} \varphi_m(l) + \sum_{l=1}^{c_{a_n}-1} \varphi_m(l) \right) - \left( \sum_{l=1}^{c_{a_n'}+1} \varphi_m(l) + \sum_{l=1}^{c_{a_n}} \varphi_m(l) \right)$$

$$= \varphi_m(c_{a_n'} + 1) - \varphi_m(c_{a_n}).$$

(20)

Combining (19) and (20) yields the following equation:

$$\tilde{u}_n(a_n', a_{-n}) - \tilde{u}_n(a_n, a_{-n}) = \phi_o(a_n', a_{-n}) - \phi_o(a_n, a_{-n}).$$

(21)

According to Definition 1, Lemma 1 is proved.

Based on the above auxiliary game, we are ready to investigate the properties of the formulated opportunistic channel access game with QoS provisioning.

Theorem 1. The dynamic spectrum access game with QoS provisioning is an OPG, which has at least one pure strategy Nash equilibrium.

Proof: Refer to Appendix A.

C. Multi-agent learning for achieving Nash equilibria

Since the formulated dynamic spectrum access game is an OPG as characterized by Theorem 1, it has at least one pure strategy Nash equilibrium. In the literature, there are large number of learning algorithms for an OPG to achieve its Nash equilibria, e.g., best (better) response \cite{20}, fictitious play \cite{21}, and no-regret learning \cite{9}. However, these algorithms require the environment to be static and need to know information of other users in the learning process, which means that these algorithms can not be applied to the considered dynamic system. In the following subsection, we propose a multi-agent learning algorithm to achieve the Nash equilibria of the formulated opportunistic channel access game in the presence of unknown, dynamic and incomplete information constraints.

For the formulated dynamic spectrum access game with QoS provisioning, the utility function of player $n$ can be rewritten as:

$$u_n(a_n, a_{-n}) = \lim_{T \rightarrow \infty} - \frac{1}{T \theta_n} \log \left( \sum_{i=1}^{T} e^{-\theta_n r_n(i)} \right).$$

(24)

It is seen that the utility function does not enjoy the additive property with respect to the random payoff part $r_n(i)$. On the contrary, it leads to multiplicative dynamic programming in essence \cite{44}. To cope with this problem, we estimate the
Algorithm 1: Multi-agent learning algorithm for dynamic spectrum access with QoS provisioning

Initialization: set the iteration index \( i = 0 \), the initial channel selection probability vector as \( p_n(0) = \left( \frac{1}{M}, \ldots, \frac{1}{M} \right) \), and the initial estimation \( Q_{nm}(0) = 0, \forall n, m \). Let each player \( n \) randomly select a channel \( a_n(0) \in A_n \) with equal probabilities.

Loop for \( i = 0, \ldots. \)

Channel access and get random payoff: with the channel selection profile \( (a_n(i), a_{-n}(i)) \), the players contend for the channels and get random payoffs \( r_n(i) \), which are determined by (1).

Update estimation: each player updates the estimations according to the following rules:

\[
Q_{nm}(i + 1) = Q_{nm}(i) + \lambda_i I(a_n(i), m) \left(\frac{1 - e^{-\theta_n r_n(i)}}{\theta_n} - Q_{nm}(i)\right),
\]

where \( \lambda_i \) is the step factor, \( I(a_n(i), m) = 1 \) if \( a_n(i) = m \) and \( I(a_n(i), m) = 0 \) otherwise.

Update channel selection probabilities: each player updates its channel selection probabilities using the following rule:

\[
p_{nm}(i+1) = \frac{p_{nm}(i)(1 + \eta_i)Q_{nm}(i)}{\sum_{m'}=1 p_{nm'}(i)(1 + \eta_i)Q_{nm'}(i)}, \forall n, m
\]

where \( \eta_i \) is the learning parameter. Based on the updated mixed strategy, the players choose the channel selection \( a_n(i + 1) \) in the next iteration.

End loop

following approximated part by performing Taylor expansion of the logarithmic function, specifically,

\[
u_n(a_n, a_{-n}) = \frac{1 - E[e^{-\theta_n r_n(i)}]}{\theta_n} + o(r_n(i)),
\]

where \( o(r_n(i)) \) is the infinitely small quantity of higher order.

By omitting the logarithmic term, we define \( u'_n(a_n, a_{-n}) = \frac{1 - E[e^{-\theta_n r_n(i)}]}{\theta_n} \), which is an approximation of \( u_n(a_n, a_{-n}) \). It is can be proved that \( u'_n(a_n, a_{-n}) \) has some important properties with \( u_n(a_n, a_{-n}) \). In particular, \( \lim_{\theta_n \to 0} u'_n(a_n, a_{-n}) = E[r(i)] \).

For the expected part of \( u'_n(a_n, a_{-n}) \), it can be written as:

\[
y_n(T) = \frac{1}{T + 1} \sum_{i=0}^{T} e^{-\theta_n r_n(i)},
\]

which can be further re-written in the following recursive form:

\[
y_n(T) = (1 - \frac{1}{T+1})y_n(T-1) + \frac{1}{T+1}e^{-\theta_n r_n(T)} = y_n(T-1) + \frac{1}{T+1}(e^{-\theta_n r_n(T)} - y_n(T-1))
\]

Based on the above recursive analysis, we propose a multi-agent learning algorithm for the channel access game with QoS provisioning. To begin with, we extend the formulated dynamic spectrum access game into a mixed strategy form. Formally, let \( P(i) = (p_1(i), \ldots, p_n(i)) \) denotes the mixed strategy profile in slot \( i \), where \( p_n(i) = (p_{n1}(i), \ldots, p_{nM}(i)) \) is the probability vector of player \( n \) choosing the channels. The underlying idea of the proposed multi-agent learning algorithm is that each player chooses a channel, receives a random payoff, and then updates its channel selection in the next slot. Specifically, it can be summarized as follows: i) in the first slot, each player chooses the channels with equal probabilities, i.e., \( p_n(0) = (\frac{1}{M}, \ldots, \frac{1}{M}) \), \( \forall n \in N \), ii) at the end of slot \( t \), player \( n \) receives random payoff \( r_n(t) \) and constructs estimation \( Q_{nm} \) for the aggregate reward of choosing each channel, and iii) it updates its mixed strategy based on the estimations. Formally, the illustrative paradigm of the multi-agent learning algorithm for dynamic spectrum access with QoS provisioning is shown in Fig. 1 and the procedure is formally described in Algorithm 1.

The properties of the proposed multi-agent learning algorithm are characterized by the following theorems. First, using the method of ordinal differential equalization (ODE) approximation, the long-term behaviors of the probability matrix sequence \( P(i) \) and the estimation sequence \( Q(i) \) are characterized. Secondly, the stable solutions of the approximated ODE are analyzed.

To begin with, we define \( \omega_n(m, p_{-n}) \) as the expected value of \( u'_n(a_n, a_{-n}) \) when player \( n \) chooses channel \( m \) while all other players choose their channels according to the mixed strategies, i.e.,

\[
\omega_n(m, p_{-n}) = E_{a_{-n}}[u'_n(a_n, a_{-n})]\big|a_n=m
\]

\[
= \sum_{a_k, k \neq n} u'_n(a_1, \ldots, a_{n-1}, m, a_{n+1}, \ldots, a_N) \prod_{k \neq n} p_{kk} \tag{28}
\]

**Theorem 2.** With sufficiently small \( \lambda_i \) and \( \eta_i \), the channel selection probability matrix sequence \( p_{nm}(i) \) can be approximately characterized by the following ODE:

\[
\frac{dp_{nm}(t)}{dt} = p_{nm}(t) \left[ \omega_n(m, p_{-n}) - \sum_{m'=1}^{M} p_{nm'}(t)\omega_m(m', p_{-n}) \right] \tag{29}
\]

**Proof:** Refer to Appendix B.

For the proposed multi-agent algorithm, the stable solutions of (29) and the Nash equilibria of the formulated channel access game with approximated utility function \( u'_n(a_n, a_{-n}) \) are related by the following proposition.

**Proposition 1.** The following statements are true for the proposed multi-agent algorithm:

1. All the stable stationary points of the ODE are Nash equilibria.
2. All Nash equilibria are the stationary points of the ODE.

**Theorem 3.** With sufficiently small \( \lambda_i \) and \( \eta_i \), the proposed multi-agent algorithm asymptotically converges to Nash equilibria of the formulated dynamic spectrum access game with approximated utility function \( u'_n(a_n, a_{-n}) \).

**Proof:** Refer to Appendix C.

**Remark 1.** It is noted that the estimation update rule is based on the recursive equalization specified as (27). Also, it
is noted the proposed algorithm is distributed and uncoupled, i.e., each player makes the decisions autonomously and does not to know information about other players.

Although the above convergence analysis is for the game with the approximated utility function $u'_i$, the convergence for the original game can be expected. The reason is that the approximated utility function is close to the original utility function. In particular, its convergence will be verified by simulation results in the next section.

V. Simulation Results and Discussion

We use the finite state channel model to characterize the time-varying transmission rates of the channels. Specifically, with the help of adaptive modulation and coding (AMC), the channel transmission rate is classified into several states according to the received instantaneous signal-to-noise-ratio (SNR). The state classification is jointly determined by the average received SNR $\gamma$ and the target packet error rate $p_e$. The HIPERLAN/2 standard [62] is applied in the simulation study, in which the channel rate set is given by $\{0, 1, 2, 3, 6\}$. Here, the rate is defined as the transmitted packets in a slot. To make it more general, we consider Rayleigh fading and set different average SNR for the channels. Using the method proposed in [63], the state probabilities can be obtained for a given average SNR and a certain packet error rate. Taking $\gamma = 5$ dB and $p_e = 10^{-3}$ as an example, the state probabilities are given by $\pi = \{0.3376, 0.2348, 0.2517, 0.1757, 0.002\}$. Furthermore, the learning parameters are set to $\lambda_i = \frac{1}{i}$ and $\eta_i = 0.1$ unless otherwise specified. In the simulation study, we first present the convergence behaviors of the proposed multi-agent learning algorithm, and then investigate the effective capacity performance.

A. Convergence Behavior

In this subsection, we study the convergence behavior of the proposed multi-agent learning approach. Specifically, there are eight users and five channels with average received SNR being 5dB, 6dB, 7dB, 8dB and 9dB respectively. For convenience of presentation, the QoS indices of all the users are set to $\theta = 10^{-2}$.

For an arbitrarily chosen user, the evolution of channel selection probabilities are shown in Fig. 2. It is noted that the selection properties converge to a pure strategy ($\{0,0,1,0,0\}$) in about 400 iterations. In addition, the evolution of the estimation $Q$ is shown in Fig. 3. It is noted from the figure that the estimation values also converge. These results validate the convergence of the proposed multi-agent learning algorithm with uncertain, dynamic and incomplete information. The evolution aggregate effective capacity of the users are shown in Fig. 4. It is noted that the aggregate effective capacity finally converges to about 8.6 packets/slot, which implies the convergence of all the users.

We study the convergence behavior versus the learning parameter $\eta$ for different QoS indices and the comparison results for different parameters are shown in Fig. 5. These results are obtained by performing 200 independent trials and then taking the expectation. It is noted from the figure that he convergence behaviors are different for different QoS indices. In particular, for relatively small QoS indices, e.g., $\theta = 10^{-1}$, the trend is opposite. Also, it is noted although it takes about 2000 iterations for the proposed multi-agent learning algorithm to converge, it achieves satisfactory performance rapidly (e.g., it achieves 90% performance in about 500 iterations). Thus, the choice of the algorithm iteration is application-dependent.
Fig. 2. The evolution of channel selection probabilities of an arbitrarily chosen user (the number of uses is $N = 8$ and the QoS indices of the users are set to $\theta = 10^{-2}$).

Fig. 3. The evolution of estimation for the multi-agent learning algorithm (the number of uses is $N = 8$ and the QoS indices of the users are set to $\theta = 10^{-2}$).

Fig. 4. The evolution of aggregate effective capacity of all the users (the number of uses is $N = 8$ and the QoS indices of the users are set to $\theta = 10^{-2}$).

Fig. 5. The convergence behaviors versus different learning parameter $\eta$ for different QoS indices (the number of users is $N = 8$).

B. Throughput performance

In this subsection, we evaluate the throughput performance of the proposed multi-agent learning algorithm. There are also five channels with the average received SNR being 5dB, 6dB, 7dB, 8dB and 9dB respectively. The number of users is increasing from 5 to 25. We study the achievable effective capacities of the users with different QoS indices. Furthermore, we compare the proposed multi-agent learning algorithm with the random selection approach. Under the uncertain, dynamic and incomplete information, random selection is an instinctive approach. For convenience of simulating, the QoS indices of all the users are set to the same.

1) Impact of QoS indices: To begin with, the achievable effective capacities of the users with different QoS indices are shown in Fig. 6. The results are obtained by taking 5000 independent trials and then taking expectation. It is noted that for a given QoS index, e.g., $\theta = 10^{-2}$, increasing the number of the users leads to significant increases in the aggregate effective capacity when the number of users is small, e.g., $N \leq 11$. However, it is also shown that the increase in the aggregate effective capacity becomes trivial when the number of users is large, e.g., $N > 11$. The reason is that the access opportunities are abundant when the number of the users is small while they are saturated when the number of users is large. Also, for a given number of users, e.g., $N = 7$, the achievable aggregate increases as the QoS indices decrease. In particular, as the QoS indices become sufficiently small, e.g., $\theta < 10^{-3}$, the achievable effective capacity moderates. The reasons are as follows: 1) smaller value of QoS index implies loose QoS requirements in the packet violating probability and hence leads to larger effective capacity, and 2) when the QoS index approaches zero, say, when it becomes sufficiently small, the effective capacity degrades to the expected capacity. It is noted that the presented results in this figure comply with the properties of the effective capacity, which were analyzed in Section III.B.

2) Performance for scenarios with small QoS indices: In the first comparison scenario, the QoS indices of the users are set to $\theta = 10^{-2}$. The comparison results are shown in
The QoS of indices of the users (N)
The aggregate effective capacity of all the users (packets/slot).

Fig. 6. The achievable aggregate effective capacity of the users for different statistical QoS indices.

The proposed multi-agent learning approach
Random selection approach

Fig. 7. The comparison results between the proposed multi-agent learning approach and random approach for relatively loose QoS requirements (the QoS indices are set as $\theta = 10^{-2}$).

The results are obtained by taking 5000 independent trials and then taking expectation. It is noted from the figure that the proposed multi-agent learning algorithm significantly outperforms the random selection approach while the performance gap decreases as the number of users increases. In addition, it is noted that the achievable performance of both approaches increase rapidly as $N$ increases when the number of users is small, e.g., $N < 15$, while it becomes moderate when the number of users is large, e.g., $N > 20$. The reasons are: 1) when the multi-agent learning approach finally converges to a pure strategy, the users are spread over the channels. On the contrary, the users are in disorder with the random selection approach, which means that some channels may be crowded while some others may be not occupied by any user. 2) the access opportunities are abundant when the number of users is small, which means that adding a user to the system leads to relatively significant performance improvement. On the contrary, the access opportunities are saturated when the number of users is large, which means that the performance improvement becomes small. 3) when the number of users becomes sufficiently larger, the users are asymptotically uniformly spread over the channels. Thus, the performance gap between the two approaches is trivial.

3) Performance for scenarios with large QoS indices: In the second comparison scenario, the QoS indices of the users are set to $\theta = 10^{-1}$, which corresponds to more strict QoS requirement. The simulation results are shown in Fig. 8. It is noted from the figure that there are some similar trend with those for the first scenario, e.g., 1) the proposed multi-agent learning algorithm also significantly outperforms the random selection approach while the performance gap decreases as the number of users increases, and 2) the achievable performance of both approaches becomes moderate when the number of users is large.

It is noted that the performance of the multi-agent learning approach increase when the number of users is small while it decreases as the number of users is large. The reason is that the QoS requirements for this scenario is more strict. Therefore, adding users to the system results in system performance drop.

4) Performance comparison with an existing learning approach for expected throughput optimization: In order to
validate the proposed learning approach for effective capacity optimization, we compare it with an existing stochastic learning automata algorithm (SLA), which is an efficient solution for expected throughput optimization in dynamic and unknown environment \(^{[14]}\). Specifically, the SLA algorithm is implemented for maximizing the expected throughput explicitly rather than maximizing the effective capacity, and then the achievable effective capacity is calculated over the converging channel selection profile. The QoS indices of the users are randomly chosen from the following set \( A = [0.2 \times 10^{-1}, 0.5 \times 10^{-1}, 10^{-1}, 2 \times 10^{-1}, 5 \times 10^{-1}, 0.2 \times 10^{-2}, 0.5 \times 10^{-2}, 10^{-2}, 10^{-3}] \) and the learning step size of SLA is set to \( b = 0.08 \).

The comparison results are shown in Fig. 9. It is noted from the figure that the performance of the proposed learning algorithm is better than the SLA algorithm when \( N \) is large, which follows the fact that the SLA algorithm is for expected throughput optimization and is not for effective capacity optimization. However, when the number of users is small, i.e., \( N \) is small, the SLA approach performs better. The reasons can be analyzed as follows: (i) the competition among users is slight in this scenario, and (ii) the SLA algorithm converges to more efficient channel selection profiles in this scenario. The presented results again validate the effectiveness of the proposed multi-agent learning approach for effective capacity optimization.

VI. CONCLUSION

In this article, we investigated the problem dynamic spectrum access with statistical QoS provisioning. In particular, the channel states are time-varying from slot to slot. In most existing work with time-varying environment, the commonly used optimization objective is to maximize the expectation of a certain metric. However, expectation alone is not enough since some applications are sensitive to the fluctuations. We considered effective capacity, which takes into account not only the expectation but also other-order moments, to characterize the statistical QoS constraints in packet delay. We formulated the interactions among the users in the time-varying environment as a non-cooperative game and proved it is an ordinal potential game which has at least one pure strategy Nash equilibrium. In addition, we proposed a multi-agent learning algorithm which is proved to achieve stable solutions with uncertain, dynamic and incomplete information constraints. The convergence of the proposed learning algorithm is verified by simulation. Also, it is shown that the proposed algorithm achieves satisfactory performance. In future work, we plan to establish a general distributed optimization framework which considers the expectation and other higher-order moments. In particular, a multi-agent learning approach for optimizing the outage capacity in dynamic spectrum access networks with time-varying channels is ongoing.

Due to the fact that the considered dynamic spectrum access network is fully distributed and autonomous, NE solution is desirable in this work. However, when information exchange is available, some more efficient solutions beyond NE, e.g., the before-mentioned Nash bargaining and coalitional games, should be developed. In future work, we also plan to develop solutions beyond NE for spectrum management in 5G heterogeneous networks, in which there is a controller in charge for the small cells and information exchange is feasible.

APPENDIX A

PROOF OF THEOREM 1

For easy analysis, we first omit the logarithmic term in the utility function in (12) and denote

\[
v_n(a_n, a_{-n}) = E[e^{-\theta_n r_n(i)}].
\]

(A.1)

For an arbitrary player \( n \in C_m \), we have:

\[
v_n(a_n, a_{-n}) = E[e^{-\theta_n \frac{s_{mk}}{c_m}}] = \sum_{k=1}^{K} \pi_{mk} e^{-\theta_n \frac{s_{mk}}{c_m}},
\]

(A.2)

where \( s_{mk} \) is the random transmission rate of channel \( m \) and \( \pi_{mk} \) is the corresponding probability. For presentation, denote \( v_n^{(k)}(a_n, a_{-n}) = \pi_{mk} e^{-\theta_n \frac{s_{mk}}{c_m}}, \) for \( k = 1, \ldots, K \), which are a family of functions. Define \( \phi_v^{(k)}(a_n, a_{-n}) : A_1 \times \cdots \times A_N \to R \) as

\[
\phi_v^{(k)}(a_n, a_{-n}) = \sum_{m=1}^{M} \sum_{l=1}^{L} \pi_{mk} e^{-\theta_n \frac{s_{mk}}{c_m}},
\]

(A.3)

and

\[
\phi_v(a_n, a_{-n}) = \sum_{k=1}^{K} \phi_v^{(k)}(a_n, a_{-n}).
\]

(A.4)

Now, suppose that player \( n \) unilaterally changes its channel selection from \( a_n \) to \( a_n' \) (denote \( a_n' = m' \) for presentation), the change in \( v_n^{(k)}(a_n, a_{-n}) \) caused by this unilateral change can be expressed as:

\[
v_n^{(k)}(a_n', a_{-n}) - v_n^{(k)}(a_n, a_{-n}) = \pi_{mk} e^{-\theta_n \frac{s_{mk}}{c_m(a_n', a_{-n})}} - \pi_{mk} e^{-\theta_n \frac{s_{mk}}{c_m(a_n, a_{-n})}}.
\]

(A.5)

Accordingly, the change in \( \phi_v^{(k)}(a_n, a_{-n}) \) is given by (A.7), which is shown in the top of next page. The change in the channel selection of player \( n \) only affects the users in channel \( m \) and \( m' \). Furthermore, we have \( c_{m'}(a_n', a_{-n}) = c_{m'}(a_n', a_{-n}) + 1 \) and \( c_m(a_n', a_{-n}) = c_m(a_n, a_{-n}) - 1 \). Therefore, (A.7) can be further expressed as (A.8). Combining (A.5) and (A.8), the changes in \( v_n^{(k)}(a_n', a_{-n}) \) and \( \phi_v^{(k)}(a_n, a_{-n}) \) are related by (A.9). Therefore, for all \( n \in N \), all \( a_n \in A_n \), and \( a_n' \in A_n' \), it always holds that

\[
v_n(a_n', a_{-n}) - v_n(a_n, a_{-n}) = \phi_v(a_n', a_{-n}) - \phi_v(a_n, a_{-n}).
\]

(A.6)

Letting

\[
\phi_u(a_n, a_{-n}) = -\frac{1}{\theta_n} \frac{d}{d x} \log(\phi_v(a_n, a_{-n})),
\]

(A.12)

Due to the monotony of the logarithmic function, i.e., \( \frac{d}{d x} \log(x) \) is monotonously decreasing with respect to \( x \), the inequalities as specified by (A.10) and (A.11) always hold. Combining (A.10), (A.11) and (A.6) yields the following inequality:

\[
(u_n(a_n', a_{-n}) - u_n(a_n, a_{-n})) (\phi_u(a_n', a_{-n}) - \phi_u(a_n, a_{-n})) > 0,
\]

(A.13)
which always holds for all \( n \in \mathcal{N}, \alpha_n \in \mathcal{A}_n \) and \( \alpha_n'\in \mathcal{A}_n \).

According to Definition 2, it is proved that the formulated opportunistic channel access game with QoS provisioning is an OPG with \( \phi_v \) serving as an ordinal potential function. Therefore, Theorem 1 is proved.

**APPENDIX B**

**Proof of Theorem 2**

The following proof follows the lines for the proof in [44], which are mainly based the theory of stochastic approximation.

First, the expected changes of the estimation \( Q_{nm}(i) \) in one slot is as follows:

\[
E \left( \frac{Q_{nm}(i) + Q_{nm}(i+1) - 2Q_{nm}(i)}{\lambda_i} p_n(i) \right) = p_n(i) \left( 1 - \frac{1 - E[e^{-\delta Q_{nm}(i)}]}{\theta_n} \right) - Q_{nm}(i). \tag{B.1}
\]

If the step factor \( \lambda_i \) is sufficiently small, the discrete time process (B.3) can be approximated by the following differential equation:

\[
\frac{dQ_{nm}(t)}{dt} = p_n(i) \left( 1 - \frac{E[e^{-\delta_n r_{nm}(i)}]}{\theta_n} \right) - Q_{nm}(t). \tag{B.2}
\]

Second, the changes of the channel selection probability in one slot is as follows:

\[
= \frac{1}{n_i} \sum_{m'=1}^{M} p_{nm'}(i) \left( 1 + \eta_n \right) \frac{Q_{nm'}(i)}{\eta_n} - p_n(i).
\tag{B.3}
\]

Using the fact that \( \frac{(1+\eta_n)^{m-1}}{m} \rightarrow x \) as \( \eta_n \rightarrow 0 \), and taking the conditional expectation, the discrete time process (B.3) can be approximated by the following differential ordinal equation:

\[
\frac{dp_{nm}(t)}{dt} = p_{nm}(t) \left( E[Q_{nm}(t)] - \sum_{m'=1}^{M} p_{nm'}(t) E[Q_{nm'}(t)] \right). \tag{B.4}
\]

Furthermore, according to the asymptotic convergence of the estimation update process [44], we have \( E[Q_{nm}(t)] \rightarrow \omega_n(m, p_n) \) for (B.2). Therefore, Theorem 2 is proved.

**APPENDIX C**

**Proof for Theorem 3**

It is seen that \( u_n'(a_n, a_n - \cdot) = 1 - \frac{v_n(a_n, a_n - \cdot)}{\theta_n} \), where \( v_n(a_n, a_n - \cdot) \) is defined in (A.30). Therefore, there also exists an ordinal potential function for \( u_n'(a_n, a_n - \cdot) \). Specially, the potential function for \( a_n' \) is expressed as:

\[
\phi_v(a_n, a_n - \cdot) = \frac{1 - \phi_v(a_n, a_n - \cdot)}{\theta_n}, \tag{C.1}
\]

where \( \phi_v(a_n, a_n - \cdot) \) is characterized by (A.4).

We define the expected value of the potential function over mixed strategy profile \( \mathbf{P} \) as \( \Phi_v(\mathbf{P}) \) and the expected value of the potential function when player \( n \) chooses a pure strategy \( m \) while all other active players employ mixed strategies \( \mathbf{p}_n \) as \( \Phi_v(m, \mathbf{p}_n) \). Since \( \Phi_v(\mathbf{P}) = \sum_p p_{nm} \Phi_v(m, \mathbf{p}_n) \), the variation of \( \Phi_v(\mathbf{P}) \) can be expressed as follows:

\[
\frac{\partial \Phi_v(\mathbf{P})}{\partial p_{nm}} = \Phi_v(m, \mathbf{p}_n) \tag{C.2}
\]

We can re-write the ODE specified by (29) as follows:

\[
\frac{dp_{nm}(t)}{dt} = p_{nm}(t) \left[ \sum_{m'=1}^{M} p_{nm'}(t) \omega_n(m', \mathbf{p}_n) - \sum_{m'=1}^{M} p_{nm'}(t) \omega_n(m', \mathbf{p}_n) \right] \tag{C.3}
\]

The derivation of \( \Phi_v(\mathbf{P}) \) is given by (C.4), which is shown in the top of next page. According to the properties of EPG and OPG, we have:

\[
\Phi_v(m, \mathbf{p}_n) - \Phi_v(m', \mathbf{p}_n) \\
\times \left[ \omega_n(m, \mathbf{p}_n) - \omega_n(m', \mathbf{p}_n) \right] > 0 \tag{C.5}
\]

Therefore, we have \( \frac{d\Phi_v(\mathbf{P})}{dt} \geq 0 \), which implies that \( \Phi_v(\mathbf{P}) \) increases as the algorithm iterates. Furthermore, since \( \Phi_v(\mathbf{P}) \)
\[
\frac{d\Phi_n(P)}{dt} = \sum_{m,n} \frac{\partial \Phi_n(P)}{\partial p_{nm}} \frac{dp_{nm}}{dt}
\]

\[
= \sum_{n,m} \Phi'(m, p_{n-m}) p_{nm}(t) \left[ \sum_{m'=1}^{M} p_{nm'}(t) \omega(m_{n-m'}, p_{n-m}) - \sum_{m'=1}^{M} p_{nm'} \omega(m', p_{n-m}) \right]
\]

\[
= \sum_{n,m} \Phi'(m, p_{n-m}) p_{nm}(t) \left[ \sum_{m'=1}^{M} p_{nm'}(t) \omega(m_{n-m'}, p_{n-m}) - \omega(m', p_{n-m}) \right]
\]

\[
= \frac{1}{2} \sum_{n,m,n',m'} p_{nm}(t) p_{nm'}(t) \left[ \Phi'(m, p_{n-m}) - \Phi'(m', p_{n-m}) \right] \omega(m_{n-m'}, p_{n-m}) - \omega(m', p_{n-m}) \right]
\]

Finally, we have the following relationships:

\[
\frac{d\Phi_n(P)}{dt} = 0 \Rightarrow \omega_n(m, p_{n-m}) - \omega_n(m', p_{n-m}) = 0, \forall n, m, m'
\]

The last equation shows that \( P \) eventually converges to the stationary point of (29). Therefore, according to Proposition 1, it is proved that the proposed multi-agent learning algorithm converges to Nash equilibria of the formulated opportunistic channel access game with approximated utility function \( u_n(a_n, a_{-n}) \), which proves Theorem 5.

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