Distributed Channel Selection in CRAHNs with Heterogeneous Spectrum Opportunities: A Local Congestion Game Approach

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SUMMARY This letter investigates the problem of distributed channel selection in cognitive radio ad hoc networks (CRAHNs) with heterogeneous spectrum opportunities. Firstly, we formulate this problem as a local congestion game, which is proved to be an exact potential game. Then, we propose a spatial best response dynamic (SBRD) to rapidly achieve Nash equilibrium via local information exchange. Moreover, the potential function of the game reflects the network collision level and can be used to achieve higher throughput.

key words: cognitive radio ad hoc networks, distributed channel selection, potential game, local congestion game, best response dynamic

1. Introduction

In order to address interactions among multiple cognitive radio (CR) users, game-theoretic distributed channel selection for CR networks is a contemporary research topic. Currently, most studies, for example [1]–[4], were based on an assumption that the action of a CR user substantially affects all other users. However, this assumption does not hold in cognitive radio ad hoc networks (CRAHNs). In CRAHNs, CR users are located spatially in a region, which leads to the following two distinct features: (i) the transmission of a CR user only interferes with its neighbors rather than with all other users, and (ii) the spectrum opportunities vary from user to user. The above features were not well addressed in previous literature, and hence the problem of distributed channel selection in CRAHNs needs to be re-investigated.

To address local interactions in CRAHNs, we resort to the graphical game [5], in which the payoff of each player is only dependent on its own action and the action profile of its neighboring players. Moreover, motivated by congestion game [6], we formulate the problem of distributed channel selection in CRAHNs as a local congestion game, in which the utility of a user is defined as a function of the number of its neighbors who select the same channel. It is proved that the proposed local congestion game is an exact potential game, and exhibits good properties. Most importantly, the potential function of the proposed game can reflect the network collision level. Specifically, higher potential function value implies lower network collision level. Thus, the Nash equilibrium (NE) solutions, in which the potential function is globally or locally maximized, represent an increase in network throughput. Also, a spatial best response dynamic (SBRD), which rapidly converges to NE via local information exchange, is proposed.

It is noted that the application of the graphical game models in CR networks is just beginning to draw attention [5]. Our work is differentiated from this reference in the following two aspects: (i) our proposed game is an exact potential game which enjoys good properties, and its potential function reflects the network collision level, and (ii) the spectrum opportunities in our work are heterogeneous.

2. System Model and Problem Formulation

2.1 System Model

We consider a CRAHN involving \( N \) cognitive transmitter-receiver pairs and \( M \) licensed channels, \( N > M \). For presentation, we refer to a cognitive transmitter-receiver pair as a CR user interchangeably. The licensed channels are owned by the primary users and can be opportunistically used by the CR users when not occupied by the primary users. Denote the set of the CR users as \( \mathcal{N} \), i.e., \( \mathcal{N} = \{1, 2, \ldots, N\} \), and the set of the licensed channels as \( \mathcal{M} \), i.e., \( \mathcal{M} = \{1, \ldots, M\} \). All the channels are independently occupied by the primary users and assumed to have the same transmission rate. An example of the considered CRAHN is shown in Fig. 1.

First, we characterize the heterogeneous spectrum opportunities by the channel availability vector \( \mathbf{C}_i \), \( i \in \mathcal{N} \). Specifically, \( \mathbf{C}_i = (C_{i1}, C_{i2}, \ldots, C_{im}) \), where \( C_{im} = 1 \) means that channel \( m \) is available for user \( i \) while \( C_{im} = 0 \) means

![Fig. 1](image)

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that it is not available. Moreover, it is assumed that the spectrum opportunities are static or vary slowly in time.

Secondly, we characterize the limited range of mutual interference in the CRAHN by an undirectional graph $G = (N, E)$. The interference graph $G$ consists of a set of nodes which is exactly the CR user set $N$, and a set of edges $E \subset N \times N$. Denote each edge as an un-ordered pair $(i, j)$; then, if two CR users are connected by an edge, it means that they can hear each other. As a result, they interfere with each other when simultaneously transmitting on the same channel. Moreover, the neighboring CR users can exchange information with each other on graph $G$.

2.2 Problem Formulation

It is assumed that all the CR users can sense all channels, but can transmit on only one channel [7]. For presentation, we consider slotted Aloha transmission mechanism for the CR users\(^\dagger\). Let $s_n$ denote the number of neighbors who select the same channel with CR user $n$. Then, the system-centric objective is to find the optimal joint channel selection profile such that the following network throughput is maximized:

$$
(P1) : \quad \max_n P(1 - p)^{s_n},
$$

where $p$ denotes the transmission probability of each CR user in a slot. It can be seen that problem P1 is a combinatorial optimization problem, and its optimal solution can be obtained by exhaustive search in a centralized manner, which has exponential complexity. However, since there is no centralized controller in CRAHNs, a distributed approach with low complexity is desirable.

3. Local Congestion Game Based Channel Selection

3.1 Local Congestion Game Model

We formulate the problem of distributed channel selection in CRAHNs as a local congestion game. Formally, the game is denoted as $G_c = (N, \{A_n \mid n \in N\}, \{J_n \mid n \in N\}, \{u_n \mid n \in N\})$, where $N = \{1, \ldots, N\}$ is the set of players (CR users), $A_n = \{m \in M : C_{nm} = 1\}$ is the set of actions (available channels) for player $n$, $J_n$ is the set of neighbors of player $n$, i.e., $J_n = \{i \in N : (n, i) \in E\}$, and $u_n$ is the utility function of player $n$. Since player $n$ is only affected by its neighbors, then the utility function can be defined as follows:

$$
\phi(a_n, a_{n'} : a_{n''}) = -|I_n(a_n, a_{n'})| - |I_n(a_n, a_{n''})| - |I_n(a_{n'}, a_{n''})|.
$$

where $-|I_0(a_n, a_{n'})| = 0$ denotes the utility function of player $k$ after unilaterally changing the selection of player $n$ and $K = N \setminus \{I_n(a_n, a_{n'}) \cup I_n(a_{n'}, a_n)\}$, where $A|B$ means that $B$ is excluded from $A$. Since player $n$’s action only affects the payoffs of its neighboring players, the following equations are easily known:

$$
|I_k(a_n, a_{n'})| = 1, \forall k \in K(a_n, a_{n'}).
$$

The motivation of the above utility function is twofold. First, the individual throughput of CR user $n$ is given by $R_n = p(1 - p)^{s_n}$, where $s_n = |I_n(a_n, a_{n'})|$. Then, minimizing the number of interfering neighbors is equivalent to maximizing the throughput of CR user $n$. Secondly, it is motivated by the congestion game [6], in which the utility function is dependent on the number of players selecting the same action. The difference is that the utility function of local congestion game is dependent on its neighbors whereas that of congestion game is dependent on all other players.

3.2 Analysis of Nash Equilibrium (NE)

In this subsection, we define Nash equilibrium (NE) of $G_c$ and investigate its properties.

Definition 1: A channel selection profile $a^* = (a_1^*, \ldots, a_N^*)$ is a pure strategy NE of $G_c$ if and only if no CR user can improve its utility function by deviating unilaterally, i.e.,

$$
-|I_n^*(a_n, a_{n'})| \geq -|I_n(a_n, a_{n'})|, \forall n \in N, \forall a_n \in A_n, a_n \neq a_n^*.
$$

Theorem 1: $G_c$ is an exact potential game which has at least one pure strategy NE, and its potential function reflects the network collision level.

Proof: We construct the potential function as follows:

$$
\phi(a_n, a_{n'}) = -\frac{1}{2} \sum_{m \in N} |I_n(a_m, a_{n'})|.
$$

Suppose that an arbitrary player $n$ unilaterally changes its action from $a_n$ to $a'$. Then the change in its individual utility function caused by this unilateral change is given by:

$$
u_n(a_n, a_{n'}) - \nu_n(a_n, a_{n''}) = |I_n(a_n, a_{n'})| - |I_n(a_n, a_{n''})|.
$$

On the other hand, the change in the potential function caused by this unilateral change is given by:

$$
\phi(a_n, a_{n'}) - \phi(a_n, a_{n''}) = \frac{1}{2} \sum_{m \in N} |I_m(a_n, a_{n'})| - |I_m(a_n, a_{n''})| - \sum_{k \in E_n(a_n, a_m)} |I_n(a_n, a_{n'})| - |I_n(a_k, a_{n'})| - |I_n(a_k, a_{n''})| - |I_n(a_k, a_{n''})|.
$$

\dagger The used Aloha is just for the purpose of illustration, and the discussion in this letter can easily be extended to other transmission models, e.g., carrier sensing multiple access.
Based on (7)–(10), we have:
\[
\phi(a_n, a_{-n}) - \phi(a_n, a_{-n}) = |I_n(a_n, a_{Jn})| - |I_n(\bar{a}_n, a_{Jn})|.
\] (11)

From (6) and (11), it is seen that the change in individual utility function caused by any player’s unilateral deviation is the same as the change in the potential function. Then, by the definition of the exact potential game (EPG) [8], it is known that \( G_c \) is an EPG which has at least one pure strategy NE.

It is noted that the potential function specified by (5) is exactly equal to the negative number of the total neighboring CR users who select the same channel in the CRAHN, and can hence reflect the network collision level. Instinctively, larger potential function value implies lower network collision level, and higher network throughput can hence be obtained. Thus, Theorem 1 is proved. \( \square \)

However, normally multiple NEs exist in game \( G_c \), and the potential function is globally or locally maximized at any NE point [8]; moreover, the number of the NE points is difficult to obtain.

3.3 Achieving NE via Local Information Exchange

It is known that in the EPG if exactly one player is scheduled to change its strategy using best response dynamic (BRD) (this is referred to as traditional BRD) in each iteration, then it always makes the potential function increase and finally converges to a NE point in finite iterations [8]. However, the convergence speed of traditional BRD is lower since there is only one player changing its action in each iteration. Thus, we resort to the method in which multiple players concurrently change strategies using BRD. We then propose the spatial best response dynamic (SBRD), which is described in Algorithm 1.

By assuming that there is a common control channel, Step 2 of SBRD can naturally be achieved through the implementation of the 802.11 DCF mechanism in an autonomous fashion [4]. The difference in our work is that there are multiple non-neighboring users selected, whereas there is only one user selected in [4].

**Remark 1:** The procedure of traditional BRD is similar to the proposed SBRD, except that there is only one CR user chosen to change its selection according to (12).

**Theorem 2:** SBRD converges to a pure strategy NE of \( G_c \), and the number of iterations needed for convergence is less than \( |\phi(a(0))| \) for any initial channel selection profile \( a(0) \).

**Proof:** First, we have the following inequality:
\[
0 \leq |I_n(a_n, a_{Jn})| \leq |J_n|, \forall n \in N,
\] (13)
where we use the fact that \( I_n(a_n, a_{Jn}) \subseteq J_n, \forall n \in N \).

Then, from (5) and (13), we have the following value region for the potential function \( \phi \):
\[
\begin{align*}
\frac{1}{2} \sum_{n \in N} |J_n| & \leq \phi(a) \leq 0, \forall a \in A_1 \otimes \cdots \otimes A_N.
\end{align*}
\] (14)

It is known from SBRD that before arriving at a NE, there are always \( k(m) \) players concurrently changing channel selection in iteration \( m, k(m) \geq 1 \). Let \( \Omega(m) \) denote the set of players changing channel selection in iteration \( m \). Since these players are not neighbors, then their strategy changing does not affect each other. Now, combining the properties of EPG and BRD, the change of the potential function from iteration \( m \) to \( m + 1 \) is given by:
\[
\begin{align*}
\phi(a(m + 1)) - \phi(a(m)) &= \sum_{m \in \Omega(m)} \left( |I_n(a_n(m), a_{J_n}(m))| - |I_n(a_n(m + 1), a_{J_n}(m))| \right) \geq k(m) \geq 1
\end{align*}
\] (15)

It is known from (15) that each iteration will make the potential function \( \phi \) increase at least by one. Moreover, it is known that any maxima of the potential function are NE points [8]. Thus, according to (14), Theorem 2 follows. \( \square \)

**Remark 2:** It is seen that the increase in potential function using SBRD is larger than that of using traditional BRD in each iteration. Thus, we claim that the proposed SBRD converges to NE faster than traditional BRD. In addition, it is noted that \( |\phi(a(0))| \) also serves as the upper bound of converging iterations of traditional BRD.

4. Simulation Results and Discussion

In the simulation study, we consider a CRAHN consisting of four primary users and 20 CR users, as shown in Fig. 2. We assume that there are four licensed channels. For simplicity, it is assumed that the licensed channels are independently occupied by the primary users with the same probability \( \theta \), \( 0 < \theta < 1 \). However, note that the spectrum opportunities vary slowly in time, or are at least static during the convergence towards NE.

First, we compare the convergence speed of SBRD and the traditional BRD. The cumulative distribution function (CDF) of the convergence iterations for different spectrum opportunities and different initial channel selection profiles are shown in Fig. 3.
is shown in Fig. 3. It is noted that the convergence speed of SB RD is faster than that of traditional BRD. The reason is that there are multiple CR users updating their selections simultaneously in each iteration, as stated before.

Secondly, we evaluate the throughput performance of the proposed game based channel selection scheme in Fig. 4.

For a given channel idle probability, the simulation results are obtained by simulating 10^6 trials for different spectrum opportunities and initial channel selection profiles. It is noted that the game based approach always outperforms random approach and the throughput gap increases significantly when the channel idle probability increases.

To summarize, the promising characteristic of the proposed local congestion game is that its potential function reflects the network collision level, and is globally or locally maximized at any NE point. Thus, a higher network throughput can be achieved. However, since there may be multiple NEs, and some of them may lead to suboptimal network throughput; then, the problem of how to achieve the optimal NE remains an interesting but challenging task.

5. Conclusion

We formulated the problem of distributed channel selection in cognitive radio ad hoc networks (CRAHNs) as a local congestion game. The proposed game is proved to be an exact potential game and hence exhibits good properties. Most importantly, the potential function reflects the network collision level and hence can lead to a higher network throughput. Moreover, a spatial best response dynamic (SB RD) that rapidly converges to NE via local information exchange was proposed. Future work focused on designing an algorithm to achieve the optimal NE is ongoing.

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