Impact of Selfish Power Control on the Stability in Wireless Networks*

Pei Li, Yinlong Xu, Chunpeng Zhang, and Yuchong Hu
Department of Computer Science and Technology, University of Science and Technology of China
Anhui Province High Performance Computing Key Laboratory
Hefei, Anhui, P.R. China
lipei@mail.ustc.edu.cn, ylxu@ustc.edu.cn, cpzhang@mail.ustc.edu.cn, churhu@mail.ustc.edu.cn

Abstract—Power control can help to reduce energy consumption in wireless networks, while the selfish power control of a link may affect the performance of the neighbors. This paper studies the impact of selfish power control on the stability in wireless networks where each link is under a throughput requirement. Employing the modulation scheme Non-coherent Frequency Shift Keying (NFSK), we prove that there is an optimal reception rate and a corresponding optimal transmission strategy to minimize the energy consumption for a link with fixed channel condition, and further present a transmission strategy selection algorithm for links with varying channel condition. A round-based repeated non-cooperative game is proposed, assuming that there is only one link which can adjust its transmission strategy in each unit time and the link always adopts the self-optimal strategy myopically. It is observed that this game cannot converge in some cases, since links are motivated to move transmission power from high interference durations to low interference ones. To bring the game into a steady state, a penalty function is introduced and the performance of various penalty functions is investigated. Simulation results show that heavier penalty leads to faster convergence, but has no effect on the energy consumption of the steady state.

Keywords—power control; throughput requirement; stability; wireless network

I. INTRODUCTION

Power control in wireless networks has been extensively studied recently, since energy efficiency is one of the key issues in wireless networks and power control can help to reduce energy consumption. There has been much work considering power control in the presence of selfish users. Most work [2-5] studies a power control game, where each sender adjusts its transmission power to maximize a given utility function, typically some function related to signal-to-interference-plus-noise ratio (SINR) or throughput. Given QoS constraints on a link, the sender can select its transmission strategy to minimize the energy consumption. Some work [6, 7] aims to achieve energy efficiency with joint power control and rate adaptation subject to some QoS constraints. Repeated game has been applied to model the interaction among the selfish users [8-11]. In [8], C. Long et al. develop a non-cooperative power control algorithm with repeated game for ad hoc networks, and show that intelligent learning behavior with self-incentive dynamics eventually converges to steady state.

Many modulation schemes are more energy-efficient at lower transmission rates, so significant power savings can be achieved by transmitting data as slow as possible, often termed lazy packet scheduling [12]. However, we observe that it’s not the case in NFSK [1], which is one of the most popular modulation schemes in wireless networks (e.g. CDMA networks [2, 3]). In NFSK, the transmission rate is fixed, and the reception rate is uniquely determined by the bit error rate (BER). Therefore, a sender can determine the reception rate at a receiver by adjusting its transmission power, which determines the reception power and the BER consequently, given the interference level and the distance between the sender and the receiver. Suppose that each link is subject to a throughput requirement. We find that the lowest reception rate doesn’t mean the minimal energy consumption, and there exists an optimal reception rate which determines the optimal transmission strategy. Then a round-based repeated non-cooperative game is proposed, assuming that there is only one link which can adjust its transmission strategy in each unit time and the link always adopts the self-optimal strategy myopically. It is observed that this game cannot converge in some cases, since links are motivated to move transmission power from high interference durations to low interference ones. To bring the game into a steady state, a penalty function is introduced and the performance of various penalty functions is investigated through simulation.

The rest of this paper is organized as follows. The system model is introduced in Section II. The existence of the optimal transmission strategy is proved and a transmission strategy selection algorithm is proposed in Section III. A game-theoretic description for the selfish power control problem is presented in Section IV. The stability of the proposed game is studied in Section V. Finally, the paper is concluded in Section VI.

II. SYSTEM MODEL

A wireless network is modeled as a graph $G=(V,E)$, where $V$ is the node set, and $E$ is the directed-link set. There are $N$ links in the network. The sender or the receiver of one link can be the sender or the receiver of another one. Since we only have to consider the impact of senders on receivers, the geographical distance between the sender of Link $i$ and the receiver of Link $j$ is denoted by $d_{ij}$. The throughput requirement (i.e., required average reception rate) on Link $i$ is represented by $\lambda_i$.

The signal attenuation model defines the mapping from the transmission power $P_s$ of the sender to the reception power $P_r$ of the receiver. In this paper, we adopt the path loss model [13]

$$P_r = c \cdot \frac{P_s}{d^\alpha} \quad (1)$$

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as the signal attenuation model, where $d$ is the geographical distance between the sender and the receiver, and $c$ and $k$ are constants determined by the network environments. We call $c/d$ the link gain.

Modulation is a mapping from the SINR to the BER, which determines the reception rate under a given transmission rate. In order to facilitate the analysis and bring insight on power control, we take NFSK into account, which is also investigated in [2, 3]. The reception rate of Link $i$ is formulated as

$$R_i = \frac{LR}{M} \left(1 - \frac{1}{2} e^{\frac{\gamma_i}{2}} \right)^M,$$

(2)

where $R$ is the transmission rate, $L$ is the length of payload in each packet, $M$ is the packet length, and $\gamma_i$ is the SINR at the receiver of Link $i$. $\gamma_i$ is calculated as

$$\gamma_i = \frac{\sum_{j \in \mathcal{N}, j \neq i} p_j g_{ij} + \sigma_i}{\sum_{j \in \mathcal{N}, j \neq i} g_{ij}},$$

(3)

where $p_i$ is the transmission power of Link $i$, $g_{ij}$ is the link gain from the sender of Link $j$ to the receiver of Link $i$, and $\sigma_i^2$ is the additive White Gaussian noise (AWGN) at the receiver of Link $i$. Assume that AWGN is identical for all nodes at any time in the network, then we only have to consider the interference. The configuration in [2, 3] is applied here, and the values for the parameters are listed in Table I.

### III. TRANSMISSION STRATEGY SELECTION

In this section, we study the transmission strategy selection for some link with given channel condition (i.e., the distribution of interference at the receiver is known). To simplify the analysis, we first assume that the channel condition at the receiver is fixed. This assumption holds if other links are so far away that the interference can be ignored, or their transmission powers are fixed. We prove that there is an optimal reception rate which determines the optimal transmission strategy for the link. Finally, we present a transmission strategy selection algorithm for links with varying channel condition in general scenarios.

Assume that there is only one link which can adjust its strategy in each unit time. Then a selected link can predetermine the interference in the current unit time by that in the previous one. We further assume that the receiver computes the transmission strategy, and send it to the sender at the beginning of some unit time through an additional channel. The reason for making these two assumptions is that the sender cannot be aware of the channel condition at the receiver in wireless networks.

#### A. Fixed Channel Condition

First, some definitions are introduced which will be used in this paper. A strategy adopted by a link characterizes the link transmission behavior, which describes the distribution of the transmission power in a unit time. The optimal strategy is the one which consumes the least energy. If the transmission power of a link is constant (various) in a unit time, we say the link adopts a *simple (complex)* strategy. We present Lemma 1 and Lemma 2 to prove Theorem 1.

| **TABLE I. VALUES FOR PARAMETERS IN NFSK** |
|-----------------|---------|
| $M$             | 80 b    |
| $L$             | 64 b    |
| $R$             | 10$^6$ b/s |
| $\sigma^2$      | 5x10^{-12} V^2 |
| $c$             | 7.75x10^{-3} |
| $k$             | 3.6     |

**Lemma 1:** For a link with fixed channel condition, there is a simple strategy which consumes not more energy than any other simple strategies.

**Proof:** Suppose the link is Link $i$, and the throughput requirement is $\lambda_i$. From (2) and (3), we get the reception rate $R_i$ and the channel time occupation $\alpha_i$.

$$\alpha_i = \frac{\lambda_i}{R_i}, \quad R_i \in [\lambda_i, R_i^{\text{max}}].$$

(4)

$R_i$ should not be less than $\lambda_i$, otherwise the expected average reception rate cannot be achieved, and $R_i$ should not be more than $R_i^{\text{max}}$, which is the maximal reception rate determined by the maximal transmission power $p_i^{\text{max}}$. From (2) and (3), we also have

$$p_i = \sum_{j \in \mathcal{N}, j \neq i} \left( \frac{g_{ij}}{p_j} + \sigma_i^2 \right) \ln \left( 2 \left( \frac{R_i M}{LR} \right)^{\frac{1}{\alpha_i}} \right).$$

(5)

Let $E_i$ be the energy consumption of Link $i$ in a unit time, then

$$E_i = p_i \alpha_i.$$  

(6)

From (4), (5) and (6), $E_i$ can be written as a function over $R_i$. Note that the interference and the AWGN at the receiver are fixed. We prove that (the detail analysis refers to Appendix)

$$\frac{d^2 E_i}{dR_i^2} > 0.$$  

(7)

In other words, the function $E_i$ is strictly convex over $R_i$. Since the feasible range of $R_i$ is compact, there must be a minimal value for $E_i$. To get the minimal energy consumption $E_i^{\text{min}}$ and the optimal reception rate $R_i^{\text{opt}}$, we solve the following transcendental equation

$$\frac{dE_i}{dR_i} = 0,$$

(8)

which is

$$\frac{1}{M} \frac{R_i M}{LR} \left( \frac{R_i M}{LR} \right)^{\frac{1}{\alpha_i}} \ln \left( 2 \left( \frac{R_i M}{LR} \right)^{\frac{1}{\alpha_i}} \right) = 0.$$  

(9)

Let the solution of (9) be $R_i^\star$. Note that $R_i^\star$ is only determined by $M$, $L$, and $R$, and it can be calculated by numerical methods. Since $R_i^\star$ may not be in the feasible range of $R_i$, it concludes

$$R_i^{\text{opt}} = \begin{cases} R_i^\star, & \text{if } R_i^\star > R_i^{\text{max}}, \\ R_i^{\text{max}}, & \text{if } \lambda_i \leq R_i^\star \leq R_i^{\text{max}}, \\ \lambda_i, & \text{if } R_i^\star < \lambda_i. \end{cases}$$  

(10)

From (4), (5) and (10), the optimal channel time occupation $\alpha_i^{\text{opt}}$ and the optimal transmission power $p_i^{\text{opt}}$ can be got. Therefore, the simple strategy which minimizes the energy consumption is found and Lemma 1 holds.

$$\blacksquare$$
Lemma 2: For a link with fixed channel condition, the optimal strategy must be a simple strategy.

Proof: The lemma is proved by contradiction. Suppose the optimal strategy is a complex strategy. It consists of \( n (n \geq 2) \) different powers \( p_1, p_2, \ldots, p_n \) and the corresponding reception rates and transmission durations are \( R_1, R_2, \ldots, R_n \) and \( t_1, t_2, \ldots, t_n \) respectively. Take two different reception rates \( R_i, R_j \) \((R_i < R_j)\) into account. The total throughput is \( R_i t_i + R_j t_j \), and the total energy consumption is \( P_i t_i + P_j t_j \). Note that there is a simple strategy, whose reception rate is \((R_i t_i + R_j t_j)/(t_i + t_j)\) and transmission duration is \( t_i + t_j \). From the convexity property of the energy consumption function (see Fig. 1), it is known that the energy consumption of this simple strategy is less than \( P_i t_i + P_j t_j \). So the complex strategy can be modified to reduce energy consumption. Hence the optimal strategy cannot be a complex strategy and Lemma 2 holds.

From Lemma 1 and 2, we conclude with Theorem 1.

Theorem 1: For a link with fixed channel condition, there is a simple strategy which consumes not more energy than any other strategies.

B. Varying Channel Condition

Let power efficiency be the change amount of the reception rate obtained by increasing or decreasing the transmission power by one unit. Note that when the power unit is infinitesimal, the power efficiency for increasing one unit power equals to that for deceasing one. So we use the derivative of the reception rate \( R_i \) at the transmission power \( p_i \) (i.e., \( dR_i/dp \)) to denote the power efficiency. Then the optimal strategy should meet the requirement that the power efficiencies are the same at all occupied durations. Otherwise, decreasing the transmission power in the lowest power efficiency duration and increasing the power by the same amount in the highest power efficiency duration can help to increase throughput.

Algorithm 1 is proposed to compute a transmission strategy, which is similar to the water-filling algorithm. To increase the throughput, Link \( i \) needs to fill the power to some time durations. In every step, it can either occupy a new duration with the lowest interference, or increase the power in an occupied duration with the highest power efficiency. Which action is more efficient depends on the cost, which is defined as the energy consumption while increasing the throughput by one unit. Note that when the power unit and the time duration approach infinitesimal, the computed strategy can be considered as an optimal one. Therefore, when a link is chosen to adjust its transmission strategy, it should employ Algorithm 1 to minimize its energy consumption.

IV. GAME-THEORETIC DESCRIPTION

Suppose the time is slotted, and each unit time is divided into \( T \) slots. The strategy of Link \( i \) in a unit time can be defined as a transmission power allocation vector

\[
s_i = (p_{i1}, p_{i2}, \ldots, p_{iT}),
\]

where \( p_{ij} \) is the power for Link \( i \) in Slot \( j \). The strategy vectors of all links define a strategy matrix \( S \) and the strategy matrix except for the strategy of Link \( i \) is denoted by \( S_i \).

Algorithm 1 Transmission strategy selection for Link \( i \)

1: while true do
2: if Link \( i \) is chosen to adjust its strategy do
3: clear up the previous transmission strategy
4: while the throughput requirement has not been achieved do
5: if occupying the new duration with the lowest interference is more efficient do
6: occupy this duration using the optimal reception rate
7: else
8: increase the power in the occupied duration with the highest power efficiency by one unit
9: end if
10: modify the power in other occupied durations to ensure that the power efficiencies are the same
11: end while
12: adopt the computed transmission strategy
13: else
14: adopt the previous transmission strategy
15: end if
16: end while

\[
S = (s_1, s_2, \ldots, s_N)^T
\]

\[
S_i = (s_{i1}, \ldots, s_{i1}, s_{i1}', \ldots, s_{iN})^T
\]

Assume that links are rational and the objective of each link is to minimize its energy consumption, which is

\[
C_i = \sum_{t=1}^{T} p_{ij} \frac{1}{T}.
\]

Then the strategy matrix \( S'=(s_1', \ldots, s_N')^T \) defines a Nash equilibrium (NE), if for every Link \( i \), we have

\[
C_i(s_i, s_i') \leq C_i(s_i', s_i')
\]

for every strategy \( s_i' \). Note that in a NE, none of the links can unilaterally change its strategy to decrease its energy consumption. So each NE corresponds to a steady state.

The selfish power control problem is modeled as a repeated game, and in each unit time there is only one link which can adjust its transmission strategy to guarantee an average reception rate no less than \( \lambda_i \). These links are scheduled in a round-robin way. From Section III-B, it’s known that links should employ Algorithm 1 to minimize energy consumption.

V. STABILITY

The stability of the proposed game is studied in this section. To simplify the analysis, a scenario with only two links is first considered to show the existence of instability. Then a penalty function is introduced to ensure convergence, which is validated by simulation. In the simulations, the configuration in Table I is applied, and a unit time is considered to be a second, which is divided into 100 slots (i.e., \( T = 100 \)).

Assume that there are two links in the scenario, which is depicted in Fig. 2, and \( S'=(s_1', s_2')^T \) defines a NE, where there are not only slots for these two links to transmit exclusively but also slots to transmit simultaneously. It’s known that neither link can unilaterally modify its strategy to decrease energy consumption. Take three sample slots into account as in Fig. 3. Note that for Link 1, since the interference in Slot 2 is higher than in Slot 1 and the power efficiencies should be the same in these two slots, the transmission power of Link 1 in Slot 2 must be larger. Then if Link 2 is chosen to adjust its strategy, it must move the transmission power from Slot 2 to Slot 1, since the interference in Slot 1 is lower. Hence this state is not stable and...
\( S^* \) cannot be a NE. Actually, when the total required average reception rate (i.e., \( \lambda_1^* + \lambda_2^* \)) is set at an unlucky value, there must be slots for exclusive transmissions as well as for simultaneous ones. Then both links are motivated to move transmission power from high interference durations to low interference ones, which leads to instability. The analysis is validated by simulation. Let \( d_{11}, d_{22}, d_{12}, d_{21}, \lambda_1, \lambda_2 \) be 1000m, 1000m, 2500m, 2000m, 5000b/s, 5000b/s respectively, and the simulation is run for 200 seconds (i.e., 100 rounds). The average reception rates for the links in the first and last 20 seconds are presented in Fig. 4, where it’s shown that this game cannot converge even after a sufficiently long time.

In order to bring the game into a steady state, a penalty is added to the energy consumption function of each link as

\[
C_i = \sum_{(j \neq i)} p_{ij} T + f(r) \Delta P_i^*,
\]

where \( f(r) \) is an ascending function over round \( r \), and \( \Delta P_i^* \) is the amount of increased power when Link \( i \) changes its strategy. \( \Delta P_i^* \) can be formulated as

\[
\Delta P_i^* = \sum_{j \neq i} \frac{p_{ij} - p_{ij}'}{T} \left[ p_{ij} > p_{ij}' \right],
\]

where \( p_{ij} (p_{ij}') \) is the power in the current (previous) strategy, and the function \( [p_{ij} > p_{ij}'] \) is defined as

\[
[p_{ij} > p_{ij}'] = \begin{cases} 1, & \text{if } p_{ij} > p_{ij}', \\ 0, & \text{if } p_{ij} \leq p_{ij}'. \end{cases}
\]

When some link increases its transmission power in some slots, it has to pay for the actions, and \( f(r) \) is the price for increasing a unit power. As time goes by, \( f(r) \) increases, but the income (i.e. the decrement in the energy consumption) for a link to move a unit power from the lowest power efficiency slot to the highest power efficiency one is finite. So finally the penalty will exceed the income, and links will stop this selfish power adaptation behavior, which results in convergence. This approach is validated through simulation. The two-link scenario is taken into account, and the function \( f(r) = 0.001 r^2 \) is applied. The simulation is run for 20 seconds (i.e. 10 rounds), and the reception rates over time are given in Fig. 5, where it’s known that this game converges only after a few rounds.

A general scenario with 10 random links is also studied, which is depicted in Fig. 6. The required average reception rate for each link is 1600b/s. We focus on two links, Link 5 and Link 7, which stand for the links in high and low contention areas respectively. The simulation without penalty is run for 1000 seconds (i.e., 100 rounds). The average reception rates for these two links in the first and last 100 seconds are presented in Fig. 7, where it’s known that the game cannot converge too. To investigate the influence of the penalty function on the steady
state, a series of penalty functions are considered, whose \( f(r) \) is
generalized as (Note that other ascending functions can also be
applied here)
\[
f(r) = a \cdot r^b, \quad a > 0, \quad b \geq 1. \tag{19}
\]
In the simulations, \( b \) is fixed at 2 and \( a \) varies with the values
0.1, 0.01 and 0.001. The simulations are run for 50, 100 and
400 seconds respectively, and the average reception rates for
Link 5 and Link 7 are presented in Fig. 8, where it’s known that
these links can always reach steady states, and heavier penalty
leads to faster convergence. The total energy consumption for
all the links is depicted in Fig. 9, where it’s shown that heavier
penalty doesn’t mean more total energy consumption, since
different penalty functions may bring the game into different
steady states, and which steady state to converge has no
relation to the convergence speed. The energy consumptions
for Link 5 and Link 7 are presented in Fig. 10, where it’s known
that the energy consumption for an individual link also has no
relation to the convergence speed.

VI. CONCLUSION
This paper studies the impact of selfish power control on the
stability in wireless networks employing NFSK, where each
link is under a throughput requirement. It’s proved that there is
an optimal transmission strategy, and a transmission strategy
selection algorithm is presented. A round-based repeated non-
cooperative game is proposed, and it’s observed that links will
keep on changing their strategies in some cases, which results
in instability. A penalty function is introduced, and it’s shown
that ascending penalty functions can always result in
convergence, and heavier penalty leads to faster convergence
but has no effect on the energy consumption of the steady state.

However, the proposed game requires that links are
scheduled in a synchronous and round-robin way, which means
many information exchanges and seems impractical in large-
scale wireless networks. We aim to relax this constraint in our
future work. Moreover, how to find a penalty function, which
leads to fast convergence as well as low energy consumption, is
also of interest.

APPENDIX
Proof of (7): From (4), (5) and (6), we have
\[
\frac{d^2E_i}{dr_i^2} = \frac{\gamma_i}{R_i^3M} \sum_{k=1}^{n} \frac{g_i}{g_k} + \sigma_i \tag{20}
\]
To prove (7) is equivalent to prove
\[
f(R_i) = \frac{1}{1-\left(1-\frac{R_i}{R_{\text{max}}}\right)^{\frac{1}{\tau}} - 2M^2 \ln \left(1-\left(\frac{R_i}{R_{\text{max}}}\right)^{\frac{1}{\tau}}\right)} > 0 \tag{21}
\]
\( f(R_i) \) can be rewritten as
\[
f(R_i) = \frac{1}{1-\left(1-\frac{R_i}{R_{\text{max}}}\right)^{\frac{1}{\tau}} - 2M^2 \ln \left(1-\left(\frac{R_i}{R_{\text{max}}}\right)^{\frac{1}{\tau}}\right)} \tag{22}
\]
(21) will always hold, if
\[
g(R_i) = -\frac{(3M-1)^2}{4} - 2M^2 \ln \left(1-\left(\frac{R_i}{R_{\text{max}}}\right)^{\frac{1}{\tau}}\right) > 0 \tag{23}
\]
Note that \( g(R_i) \) is an ascending function over \( R_i \). So if
\[
R_i > LR_i \left(1-\frac{1}{2} \left(\frac{R_i}{R_{\text{max}}}\right)^{\frac{1}{\tau}}\right)^{\frac{1}{\tau}} \tag{24}
\]
(23) will always hold.

Applying the configuration in Table I, it is observed that if
\( R_i \geq 4.86 \times 10^{-3}/b_s, \) (24) will hold. Note that \( R_i \in [\lambda_i, R_{\text{max}}] \) and
the minimal throughput requirement \( \lambda_{\text{min}} \geq 4.86 \times 10^{-3}/b_s \) is a
practical assumption, so \( R_i \geq 4.86 \times 10^{-3}/b_s \) holds. Then (24),
(23) and (21) hold sequentially, and (7) is proved.

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