LASSO based stimulus frequency recognition model for SSVEP BCIs

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A B S T R A C T

Steady-state visual evoked potential (SSVEP) has been increasingly used for the study of brain–computer interface (BCI). How to recognize SSVEP with shorter time and lower error rate is one of the key points to develop a more efficient SSVEP-based BCI. To achieve this goal, we make use of the sparsity constraint of the least absolute shrinkage and selection operator (LASSO) for the extraction of more discriminative features of SSVEP, and then we propose a LASSO model using the linear regression between electroencephalogram (EEG) recordings and the standard square-wave signals of different frequencies to recognize SSVEP without the training stage. In this study, we verified the proposed LASSO model offline with the EEG data of nine healthy subjects in contrast to canonical correlation analysis (CCA). In the experiment, when a shorter time window was used, we found that the LASSO model yielded better performance in extracting robust and detectable features of SSVEP, and the information transfer rate obtained by the LASSO model was significantly higher than that of the CCA. Our proposed method can assist to reduce the recording time without sacrificing the classification accuracy and is promising for a high-speed SSVEP-based BCI.

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1. Introduction

Steady-state visual evoked potential (SSVEP) may be elicited over occipital scalp areas with the same frequency as the stimulus and may often include its harmonics when a subject pays attention to a repetitively flickering stimulus [1,2]. In recent years, SSVEP-based brain–computer interfaces (BCIs) have been increasingly studied and have demonstrated strength including a higher information transfer rate (ITR) and less training to both the system and the user than other BCIs (e.g. P300-based BCIs, event-related desynchronization/synchronization (ERD/ERS)-based BCIs) [3–6]. SSVEP-based BCIs typically depend on external visual stimuli in the form of an array of light sources, where each light source flickers with a distinct frequency [1,2,4,7]. The outstanding property of SSVEP is that the frequency components of the target stimulus focused by a subject significantly present in the EEG spectrum, and hence the target stimulus can be inferred by classifying the frequency components of recorded EEG [2,8,10].

In order to obtain faster speed of a SSVEP-based BCI, one option is to recognize the target stimulus frequency accurately within shorter time, and then in this case, a better algorithm to recognize frequencies of EEG signal should be provided. The traditional frequency recognition method in SSVEP-based BCIs is power spectral density analysis (PSDA) and the power spectral density (PSD) is estimated typically by the fast Fourier transform (FFT) from the EEG signal within a time window (TW) [9–12]. The frequency corresponding to the peak of PSD is considered as the target stimulus frequency. When using the PSDA, the length of TW required in SSVEP-based BCIs is typically more than 3 s [13–15], and such duration may limit the real-time speed of SSVEP-based BCIs. Wu et al. [16] proposed the stability coefficient (SC) method to recognize SSVEP, and the SC was shown to be better than the PSDA in using a shorter TW with the cost of training data to form the SC model. Lin et al. [5] used canonical correlation analysis (CCA) to recognize SSVEP. Bin et al. [3] developed an online SSVEP-based BCI system by the CCA with 2 s TW. The CCA can improve the speed of a SSVEP-based BCI in contrast to the PSDA. Indeed, a high-speed SSVEP-based BCI does require a shorter TW, and hence this study is devoted to achieving this goal through developing an algorithm to extract more discriminative features from EEG recordings within a shorter TW (i.e. using fewer samples).

When fewer samples are available, the sparse signal processing plays an important role [17]. Sparsity analysis has been proved to be very useful in blind signal reconstruction [18–20], image and signal detection [21,22]. Recently, the robustness of sparsity constraint has been exhibited to occlusion and corruption in face recognition [23,24]. As a popular model selection and shrinkage estimation method, least absolute shrinkage and selection operator (LASSO) proposed by Tibshirani [25] can provide an analytical solution and a low-variance estimate with high interpretability for a linear regression due to its sparsity constraint [26,27]. In this study, we attempt to make use of the sparsity constraint of the LASSO to extract more

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discriminative features of SSVEP from a shorter TW. We propose a
LASSO model using the linear regression between EEG recordings
and the standard square-wave signals of different frequencies to
recognize SSVEP without the training stage. To validate the pro-
posed LASSO model, the CCA is used for comparison with the EEG
data collected from nine healthy subjects, and information trans-
fer rate (ITR) [28–30] is calculated to evaluate the communication
performance.

2. Experiments

2.1. Subjects

Nine healthy right-handed volunteers (S1–S9, aged from 21 to
28 years, seven males and two females) participated in our ex-
periment. All of them had normal or corrected to normal vision and
no experience with a SSVEP-based BCI. The subjects were seated
in a comfortable chair 50 cm from a standard 17 inch CRT moni-
tor (85 Hz refresh rate, 1024 × 768 screen resolution) in a shielded
room.

2.2. Stimulation unit and experimental layout

The command icons, stimuli and the layout used in this study
are presented in Fig. 1. The first row in Fig. 1 shows four command
icons (‘Volume ON’, ‘Volume +’, ‘Volume −’, ‘Volume OFF’). The fre-
cuencies of the four flickering squares in the second row were f1:
6.1 Hz, f2: 7.1 Hz, f3: 8.5 Hz, f4: 10.6 Hz’ corresponding to the four
commands, respectively. These four commands could be used to
adjust the volume of a device.

2.3. Experimental schedule

Each subject completed six runs with 10 min rest after each run.
Each run consisted of four parts, and one part was for one of the
four command icons. In each part, a cued target icon was presented
in the middle of the blank screen for 1 s, and then the four squares
appeared and flickered simultaneously. The subject was required
to focus on the flickering square corresponding to the cued target
icon for 4 s. Hence, 24 (i.e. 6 × 4) segments of 4 s EEG data were
acquired from each subject. Fig. 2 shows the timing of each part.

2.4. EEG recordings

EEG signals were recorded at 250 Hz sampling rate from three
channels O1, Oz, O2 which were placed on the standard posi-
tions of the 10–20 international system and each channel was
amplified by A Nuamps (NuAmp, Neuroscan, Inc.) amplifier with
a sensitivity of 100 μV. The average of two mastoid electrodes (A1,
A2) was used as reference and the ground electrode was placed
on the forehead. The EEG signals were bandpass filtered between
5 and 35 Hz.

3. Methods

3.1. SSVEP frequency recognition model based on LASSO estimate

3.1.1. LASSO estimate

Consider a standard linear regression model for the observations
of the response \( \mathbf{y} \in \mathbb{R}^n \) [26,31]:

\[
\mathbf{y} = \mathbf{X} \hat{\beta} + \mathbf{e},
\]

where \( \mathbf{y} \) is a \( n \times 1 \) vector, \( \mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_p) \) denotes a \( n \times p \) design
matrix, and \( \mathbf{e} \) represents a noise vector with the zero mean and
constant variance. As introduced in [25], the LASSO estimate is then
given by:

\[
\hat{\beta} = \arg \min_{\beta} \left( \| \mathbf{y} - \mathbf{X} \beta \|_2^2 + \lambda \| \beta \|_1 \right),
\]  

where \( \| \cdot \|_2 \) and \( \| \cdot \|_1 \) denote the \( l_2 \)-norm and \( l_1 \)-norm respectively.

\( \lambda \) is a penalty parameter which encourages a sparse solution \( \hat{\beta} \)
(i.e. it has many entries equal to zero). The optimization problem
depicted by Eq. (2) can be solved by quadratic programming [32].

3.2. Experiments

The results of LASSO optimization are shown in Fig. 3. The per-
formance of the proposed model is compared with the traditional
CCA model. The CCA model was run using the same EEG data and
SSVEP frequency information as the proposed model. The compari-
on showed that the LASSO model significantly outperformed the
CCA model in terms of communication performance. The results
are summarized in Table 1. The achieved ITR for the proposed
model was significantly higher than that of the CCA model (p < 0.05).

4. Discussion

4.1. Conclusion

In conclusion, we have proposed a LASSO model for the linear
regression between EEG recordings and SSVEP frequencies. The
model successfully recognized SSVEP frequencies without the
training stage and outperformed the traditional CCA model. The
results suggest that the LASSO model is a promising approach for
the control of BCI systems.

4.2. Future work

Further studies are needed to investigate the performance of the
proposed model under different experimental conditions. The
model could be extended to include additional features such as
higher-order spectral information and movement-related poten-
tials. Additionally, the model could be applied to real-world
BCI applications to evaluate its practical utility.

Fig. 1. Experimental images and layout. The colors of the four flickering squares in the second row are red in the experiment.

Fig. 2. The timing of each part.
The LASSO estimate usually offers an analytical solution and a low-variance estimate with high interpretability for a linear regression [26,27].

### 3.1.2. Frequency components recognition model

To construct the SSVEP recognition model, we assume that $X$ is a standard signal set consisting of four symmetric square-wave signals which correspond to the four stimulus frequencies respectively. The symmetric square-wave signal can be decomposed into the Fourier series of its harmonics [3,5]:

$$S_i = \begin{pmatrix} \sin(2\pi f_i t) \\ \cos(2\pi f_i t) \\ \vdots \\ \sin(2\pi K f_i t) \\ \cos(2\pi K f_i t) \end{pmatrix}^T, \quad i = 1, 2, 3, 4, \ldots, n$$

where $f_i$ are the four symmetric square-wave signals, $f_i$ denotes the $i$th stimulus frequency (fundamental frequency), $K$ is the number of harmonics, $F$ is the sampling rate, and $n$ represents the number of sampling points and the superscript $T$ denotes the transpose transform. In this study, the fundamental frequency and second harmonic components are used to analyze the SSVEP, hence $N$ equals to 2. Then, $X = [S_1, S_2, S_3, S_4]_{i=1,16}$. Suppose the vector $y$ derived from the LASSO estimate contains the EEG data within a TW at one channel. In this study, the length of a TW was set to be from 0.5 to 4 s with an interval of 0.5 s. The LASSO estimator $\hat{Y}$ between the EEG signal $y$ and the standard signal set $X$ can be computed from Eq. (2) by the quadratic programming [32]. Here, $\hat{Y} = [\hat{h}_1, \hat{h}_2, \hat{h}_3, \hat{h}_4, \ldots, \hat{h}_{14}, \hat{h}_{15}, \hat{h}_{16}]^T$ in which the entries $\hat{h}_1, \hat{h}_2, \hat{h}_3, \hat{h}_4, \hat{h}_{14}, \hat{h}_{15}, \hat{h}_{16}$ (i.e. $i = 1, 2, 3, 4$) implicate the contribution degree (CD) of the $i$th stimulus frequency $f_i$ and its harmonic to the EEG signal. Since $\hat{Y}$ is sparse to some extent (i.e. most entries in $\hat{Y}$ equal to zero), the CD of different stimulus frequencies in the standard signal set $X$ to the EEG signal $y$ can be classified.

The synthetic measurement for the CDs of different stimulus frequencies at used channels is defined as:

$$CD_i = \frac{\sum_{k=1}^{M} \sum_{j=1}^{2K} |\hat{h}_{ij}|}{M}$$

in which $M$ is the number of used channels which are O1, O2 and O2 here, $K$ represents the number of harmonics (K = 2) and $CD_i$ denotes the contribution degree of $i$th square-wave signal $S_i$ (i.e. $i$th stimulus frequency $f_i$) to the EEG signals at the three channels. The maximal $CD_i$ implies the dominant frequency component in the EEG signals. Thus, the target stimulus frequency $f_{\text{target}}$ can be recognized by Eq. (5). The detailed explanation of LASSO for recognizing SSVEP frequency components is illustrated in Fig. 3.

$$f_{\text{target}} = \max_i (CD_1, CD_2, CD_3, CD_4)$$

### 3.2. Comparison method

In our study, we also implemented canonical correlation analysis (CCA) and compared the results with that of our proposed LASSO model. CCA is a well-known multivariable method developed by Hotelling [33] in statistics. CCA extends ordinary correlation to two sets of variables [34] and is widely used in several fields, such as functional magnetic resonance imaging (fMRI) [35], and climate research [36]. Consider two multidimensional random variables $A$ and $B$ and their linear combinations $a = A^T W_a$ and $b = B^T W_b$, respectively. A canonical correlation coefficient $\rho$ is then computed through finding the weight vectors $W_a$ and $W_b$ which maximize the correlation between $a$ and $b$, by solving the following optimization problem:

$$\max_{W_a, W_b} \rho(a, b) = \frac{E[a^T b]}{\sqrt{E[a^T a] E[b^T b]}} = \sqrt{\frac{E[W_a^T A^T W_b]}{E[W_a^T A^T W_a] E[W_b^T B^T W_b]}}$$

(6)

The maximum of $\rho$ is the maximum canonical correlation. Lin et al. [5] used the CCA to recognize stimulus frequency in SSVEP BCI for the first time. They reported that the CCA was better than the PSDA using multiple channels. Also, an online SSVEP BCI based on the CCA was developed by Bin et al. [3]. As introduced in [3,5], $A$ contains multi-channel (O1, O2, O2) EEG signals, and $B = S_i$. Then, the canonical correlation coefficient $\rho_i$ reflecting the correlation between the $i$th stimulus frequency $f_i$ and the EEG signals was computed through Eq. (6). The fundamental frequency and second harmonic were still used for the CCA. In order to be consistent with the work of Lin et al. [5], only the maximum correlation coefficient was used as the classification basis. Then, the target stimulus frequency $f_{\text{target}}$ could be recognized by Eq. (7).

$$f_{\text{target}} = \max_i (\rho_1, \rho_2, \rho_3, \rho_4)$$

(7)

### 3.3. Information transfer evaluation

In this study, information transfer rate (ITR) [28–30] was adopted to evaluate the communication speed of our BCI. If $N$ possible choices exist in one trial, if each choice is of the identical probability to be selected by the user, if the probability ($P$) that the desired selection will indeed be chosen always keeps invariant, and if each of the other (i.e. undesired) choices has the same probability of selection, the bit rate or bits/trial ($Br$) can be computed as:

$$Br = \log_2 N + P \times \log_2 P + (1 - P) \times \log_2 \left(1 - \frac{1}{N-1}\right)$$

(8)

Then, the ITR (bits/min) is equal to $Br$ multiplied by the selection speed (i.e. trials per minute).

### 4. Results

#### 4.1. Classification accuracy

Fig. 4 describes the classification accuracy of the LASSO model and the CCA for all subjects. From the averaged accuracy, there was no evident difference between accuracies of the two methods when the length of TW was more than 2.5 s. However, the classification accuracy of the LASSO model was obviously higher than that of the CCA when the length of TW was less than 2.5 s. That is, the LASSO model is better than the CCA for SSVEP frequency recognition in using a short TW less than 2.5 s.

The Bonferroni corrected $t$-test was used to analyze the performance of LASSO model and CCA. Table 1 exhibits the $p$ value of the classification accuracy differences between each two TW lengths for each method. For the LASSO model, the classification accuracies were not significantly different (see Table 1). For the CCA, when the TW lengths were less than 2 s, the classification accuracies were significantly different from each other. This result means that the LASSO model was more robust than the CCA when using a shorter TW less than 2 s.

#### 4.2. Information transfer rate

The highest information transfer rate (ITR) with the corresponding classification accuracy and TW duration was also used...
to evaluate the performance of the LASSO model and the CCA (see Table 2). The corresponding classification accuracy difference was not significant between the LASSO model and the CCA ($p = 0.64$) while the highest ITR of the LASSO model was significantly higher than that of the CCA ($p < 0.05$). The corresponding TW length of the LASSO model was significantly shorter than that of the CCA ($p < 0.05$). In other words, the LASSO model could improve the speed of BCI system without sacrificing classification accuracy in contrast to the CCA.

### 4.3. Classification features

Fig. 5 displays the features of different target frequencies obtained by the LASSO model (contribution degree, CD) and the CCA (maximum correlation coefficient, CC). Here, the features extracted from 1 s TW were used to analyze the performance of the two methods. For most subjects, CD was higher than CC for the target frequency and CD was lower than CC for the non-target frequency. This implies that the LASSO model produces more detectable fea-

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>0.5 s vs. 1 s</th>
<th>1 s vs. 1.5 s</th>
<th>1.5 s vs. 2 s</th>
<th>2 s vs. 2.5 s</th>
<th>2.5 s vs. 3 s</th>
<th>3 s vs. 3.5 s</th>
<th>3.5 s vs. 4 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>LASSO</td>
<td>$p = 0.086$</td>
<td>$p = 0.15$</td>
<td>$p = 0.81$</td>
<td>$p = 1.0$</td>
<td>$p = 0.28$</td>
<td>$p = 0.51$</td>
<td>$p = 0.59$</td>
</tr>
<tr>
<td>CCA</td>
<td>$p &lt; 0.01$</td>
<td>$p &lt; 0.01$</td>
<td>$p &lt; 0.05$</td>
<td>$p &lt; 0.13$</td>
<td>$p &lt; 0.05$</td>
<td>$p &lt; 0.16$</td>
<td>$p &lt; 0.20$</td>
</tr>
</tbody>
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*Fig. 3. Explanation of the LASSO model for recognizing SSVEP frequency components. $y_i$ denotes the EEG signals with $t$ seconds length of time window from $i$th channel. $\hat{\beta}_i$ is the LASSO estimator corresponding to $y_i$, $CD_i$ denotes the contribution degree of $i$th square-wave signal (i.e. $i$th stimulus frequency $f_i$) to the EEG signals from three channels (O1, Oz, O2).*

*Fig. 4. Classification accuracy obtained by the LASSO model and the CCA, across different time window (TW) lengths, for all nine subjects and the averaged accuracy.*
Fig. 5. Classification features of different four target frequencies \(f_1-f_4\). The black bar is the contribution degree (CD) of different stimulus frequencies in the LASSO model, and the white bar is the maximum correlation coefficient (CC) of different stimulus frequencies in the CCA. The abscissa 1, 2, 3, 4 correspond to the stimulus frequencies \(f_1, f_2, f_3, f_4\) respectively. The features are extracted from 1 s TW, averaged over six runs and then normalized to [0,1].
Fig. 6. Time–frequency representation of SSVEP ($f_1$–$f_4$) through complex Morlet wavelet transform from the bandpass filtered (5–35 Hz) EEG signal on channel Oz and averaged over six runs.
tures than that obtained from the CCA. The target frequency \( f_2 \) of S5, the target frequencies \( f_2 \) and \( f_4 \) of S6, and the target frequencies \( f_2 \) and \( f_6 \) of S8 were classified incorrectly by the CCA due to the indiscriminate features. However, the LASSO model extracted more robust and distinct features making the target frequencies classified accurately. For S3, the target frequency could not be classified correctly neither by the LASSO model nor by the CCA. This is because that this subject was not able to focus on the flicker stimulus, hence the evoked SSVEP might be too weak (see Fig. 6). It has been reported that SSVEP-based BCI was inapplicable for some subjects due to their low attention to the stimulus or weak SSVEP response [1,4,12]. Fig. 6 depicts the time–frequency representation of SSVEP at channel Oz by complex Morlet wavelet transform. While the SSVEP frequency spectrums can be observed for most subjects, they are still disturbed by some non-related frequency spectrums resulting from spontaneous EEG or other noises to varying extent.

5. Discussion

The frequency components of SSVEP usually cannot be detected accurately with a short period (e.g. 2 s) of EEG recordings due to the interference from non-related EEG and other noises [16]. In this study, a frequency recognition model based on LASSO was proposed to detect SSVEP. The results show that LASSO model improved the speed of SSVEP BCIs without sacrificing classification accuracy compared to CCA. LASSO model was more robust than CCA in using different time window lengths (see Table 1). In our experiment, the highest information transfer rate of the LASSO model was significantly higher than that of the CCA (\( p < 0.05 \)), while the corresponding classification accuracies of the two methods had no statistical difference (\( p = 0.64 \)) (see Table 2). The average TW length required by the LASSO model in our study was only 1.1 s (with an average ITR of 39.2 bit/min) which was significantly shorter than the average TW length of 1.8 s required by the CCA (with an average ITR of 25.9 bit/min) (\( p < 0.05 \)). It is also shorter than that required by the PSDA reported in [13–15].

Fig. 6 shows the EEG time–frequency representation of SSVEP. For most subjects (except for subject 3), SSVEP frequency components (containing the fundamental frequency and second harmonic) appeared after the stimulus was onset and could be observed. However, the SSVEP frequency components were still affected by other irrelevant frequency components. In this case, the proposed LASSO model yielded higher classification accuracy compared to the CCA within a short TW less than 2.5 s (see Fig. 4). We further show the features extracted by the two methods respectively from a TW of 1 s. The LASSO model offered more discriminative features in contrast to the CCA (see Fig. 5). The reasons are: 1. For the CCA, the correlation information may be scattered in several coefficients if EEG signals are contaminated by noises [5]. SSVEP frequency components are vulnerable to the interference from non-related EEG or noises within a short TW. Thus, the CCA may extract incorrect features from a short TW (e.g. less than 2 s) since its recognition strategy is only based on the maximum of the canonical correlation coefficient; 2. It has been reported that sparsity constraint can handle the signal corruption [20,37] and image occlusion [24,38,39], such as noises, missing data and outliers. In our study, the sparsity constraint of the LASSO encouraged a sparse LASSO estimator that helped the LASSO model robustly recognize the dominant frequency component in EEG signals. Hence, the target and non-target stimulus frequencies could be classified more accurately.

6. Conclusions

In this study, the proposed LASSO based linear regression model can recognize the stimulus frequency of SSVEP-based BCI in shorter time without sacrificing the classification accuracy in contrast to the CCA. This frequency recognition model can be further used in online SSVEP-based BCIs. Our future work will focus on designing an online BCI system based on this frequency recognition model and testing its real-time performance in real world settings.

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References


