CONTROL OF SPIRAL WAVE BREAKUP BY SPATIOTEMPORAL MODULATION

YU QIAN
Nonlinear Research Institute,
Baoji University of Arts and Sciences,
Baoji 721007, P. R. China
qianyu02720163.com

YUANYUAN MI* and XIAODONG HUANG†
Department of Physics, Beijing Normal University,
Beijing 100875, P. R. China
*miyuanyuan0102@163.com
†huangxd@mail.bnu.edu.cn

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The control of spiral wave breakup due to Doppler instability has been investigated. It is found that by applying the spatiotemporal modulation method with suitable control parameters, spiral wave breakup can be prevented. Further numerical simulations show that preventing spiral wave breakup is a result of the decrease of meandering motion of the spiral tip.

Keywords: Spiral wave; meandering; spatiotemporal modulation.

1. Introduction

Spiral waves are frequently observed in a large variety of excitable media, such as the Belousov–Zhabotinsky (BZ) reactions [Winfree, 1972; Ouyang & Flesselles, 1996], the catalytic surface processes [Jakubith et al., 1990], the cardiac tissue [Davidenko et al., 1992; Qu et al., 2000] and neural networks [Sbitnev, 1998]. One common feature of spiral waves is that with the variation of the environment, the dynamics of spiral waves will change dramatically. So spiral wave instability in various systems is a robust phenomenon observed in both experiments and numerical simulations [Bär & Eiswirth, 1993; Li et al., 1996; Ouyang et al., 2000; Tobia & Knobloch, 1998; Zhou & Ouyang, 2000; Sandstede & Scheel, 2000; Aranson & Kramer, 2002]. Phenomenologically, two different kinds of spiral wave breakup scenarios have been documented in experiments and numerical simulations: Doppler instability [Bär & Eiswirth, 1993; Li et al., 1996; Ouyang et al., 2000] and Eckhaus instability [Tobia & Knobloch, 1998; Zhou & Ouyang, 2000; Sandstede & Scheel, 2000; Aranson & Kramer, 2002]. The first instability usually occurs in excitable media and the spiral wave breaks up near the spiral core. In the second instability, which usually appears in oscillatory media, spiral wave breakup happens far from the spiral core, and the region near the core may remain practically unchanged.

Recently, researchers find that the features and the characteristic changes of the spiral waves are of crucial importance. For instance, spiral waves in cardiac muscle can be a cause of tachycardia. Repetitious breakup of spiral waves due to Doppler instability can lead to spatiotemporal chaos, which is believed as a mechanism of ventricular fibrillation (VF). VF and sudden cardiac death are the
leading cause of cardiovascular mortality in industrialized countries [Grayalife et al., 1998; Weiss et al., 1999; Gray, 2002]. Therefore, the problem of controlling spiral wave breakup due to Doppler instability in excitable media has attracted much attention. Up to now, various methods to control spiral waves and spiral turbulence have been put forward [Kim et al., 2001; Sakaguchi & Fujimoto, 2003; Zhang et al., 2003; Alonso et al., 2003; Zhang et al., 2005; Rapple et al., 1999; Xiao et al., 2005; Zhang et al., 2006]. There exist two major directions for these control methods: first, to kill spiral waves or spiral turbulence when they are harmful; second, to maintain well-behaved spiral waves and prevent them from breaking up into spatiotemporal chaos. The former control problem has been investigated extensively [Kim et al., 2001; Sakaguchi & Fujimoto, 2003; Alonso et al., 2003; Zhang et al., 2005; Rapple et al., 1999; Xiao et al., 2005; Zhang et al., 2006]. So it is necessary to develop some efficient and practical control schemes to eliminate such events.

The characteristic feature of the spiral wave breakup in excitable media due to Doppler instability is that the spiral tip moves toward its adjacent waves and compresses them. When the waves in the spiral core region are compressed too much and the heterogeneity increases beyond a threshold, waves in this region can collide with each other, and then spiral waves break up and evolve into spatiotemporal chaos. Thus it becomes a crucial problem to modulate the motion of the spiral wave tip to prevent spiral wave breakup. Spatiotemporal modulation is a useful method for spatiotemporal pattern control [Rapple et al., 1999; Alonso et al., 2003; Zhang et al., 2005; Wu et al., 2006]. In the present paper, we will suggest a simple and effective spatiotemporal modulation control method to prevent spiral wave breakup.

2. Model and Results

We now first begin with the Bär model [Bär & Eiseleith, 1993]:

\[
\frac{\partial u}{\partial t} = -\frac{1}{\epsilon}w(u-1)\left(u - \frac{v + b}{a}\right) + \nabla^2 u. \tag{1}
\]

\[
\frac{\partial v}{\partial t} = f(u) - v. \tag{2}
\]

Here variables \(u\) and \(v\) describe the activator and the inhibitor, respectively. The spatiotemporal dynamics, for two-dimensional case \(\nabla^2 = \partial^2_x + \partial^2_y\), is investigated by varying \(\epsilon\) by fixing \(a = 0.84, b = 0.07\), where the system is an excitable media. In the range \(0.010 < \epsilon < 0.060\), suitable initial condition will lead to steadily rotating spiral waves. At \(\epsilon = 0.060\), the spiral wave will undergo a transition from steady rotation to meandering. When \(\epsilon > \epsilon_c \approx 0.070\), spiral waves will break up and the system will quickly fall into spatiotemporal chaos. Figure 1(a) shows a stable spiral wave in an excitable media for \(\epsilon = 0.040\), and we will use this stable spiral wave as the initial condition throughout this paper. Our simulation is taken on a two-dimensional (2D) square sheet with 200 × 200 space sites and the no-flux boundary condition is used. The model is integrated by the second-order Runge–Kutta scheme with time step \(\Delta t = 0.031\) and spatial step \(\Delta x = \Delta y = 0.39\), respectively. Figures 1(b)–1(d) show the process of spiral wave breakup due to Doppler instability at \(\epsilon = 0.070\). From these figures we can see that spiral wave breakup first occurs in the spiral core region and is caused by the collision of the spiral tip with the adjacent waves.

To prevent spiral wave breakup due to Doppler instability, we introduce a spatiotemporal modulation term for the parameter \(b\), which determines the excitation threshold of the system, into the the Bär Model. Then Eq. (1) reads

\[
\frac{\partial u}{\partial t} = -\frac{1}{\epsilon}w(u-1)\left(u - \frac{v + b + E}{a}\right) + \nabla^2 u, \tag{4}
\]

where

\[
E = \frac{\gamma_1 v}{\gamma_2 + u}. \tag{5}
\]

It is obvious that the spatiotemporal modulation term \(E\) is zero at the steady state \((u = v = 0)\), and the spatiotemporal modulation functions effectively where the variable states are excited \((u > 0, v > 0)\). We use this kind of spatiotemporal modulation here as based on the fact that in the real reaction diffusion system the excitation threshold (i.e. parameter \(b\)) may be spatially
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Fig. 1. \( a = 0.84, b = 0.07 \). An excitable medium is considered. These two parameters will be used throughout this paper. (a) A stable spiral wave of Bär model at \( \epsilon = 0.040 \) and this pattern will be the initial condition as \( t = 0 \) for all the following figures. (b)–(d) The process of spiral wave breakup due to Doppler instability at \( \epsilon = 0.070 \). The state at (b) \( t = 31 \), (c) \( t = 34 \), (d) \( t = 310 \), respectively.

inhomogeneous and the evolution of the system is generally expected to be influenced by this spatial inhomogeneity. And the reasons why we choose this particular modulation form here are two-fold. On one hand, this form is very simple. On the other hand, most importantly, this modulation form has some possibly practical implications in describing cardiac systems, which has been used for a cardiac cell model [Aliev & Panfilov, 1996] (i.e., AP model) in order to recover approximately the actual cardiac-cell restitution curve with considerably simplified dynamic equations. For simplicity, we fix \( \gamma_2 = 1.0 \) and vary \( \gamma_1 \) to investigate the control of spiral wave breakup.

Without control, the spiral wave will break up and the system will evolve into a spatiotemporal chaotic state, just as indicated in Fig. 1(d). But the phenomenon is dramatically changed when the spatiotemporal modulation is applied to the system. Figure 2(a) shows the final control result after a long time evolution when this control method is used. From this figure we can see that the spiral wave is stabilized and the effect to prevent spiral wave breakup succeeds. The reason underlying this
control method can be intuitively understood as follows. When the spatiotemporal modulation is added to the system, parameter $b$ will be spatiotemporal perturbed by the modulation item $E$, which is shown in Fig. 2(b). And this gradient distribution of the excitation threshold along the normal direction of the spiral arm makes excitation threshold of excited region greater than the remaining region. The high excitation threshold in the excited region then makes the corresponding cells more difficult to be excited and more easy to relax to the rest state, which can cause the spiral wave to become thin [compare Fig. 2(a) to Fig. 1(b)]. Then the curvature near the spiral tip increases. According to the eikonal relation in an excitable media [Persov et al., 1997], this curvature increase leads to the decrease of the normal velocity of wave front near the spiral tip. Hence, the spiral tip leaves its original trajectory which will lead to the spiral wave breakup and follows a new trajectory. Figure 2(c) shows the corresponding orbit of the spiral tip when the spatiotemporal modulation is used. Here we define the spiral tip to be the intersection of the two contours $u = 0.50$ and $v = 0.35$. The flower petals indicate that the spiral wave is stabilized by this control method.

By using this spatiotemporal modulation control method, spiral wave breakup can be prevented and the choosing of the control parameter is very important. Many numerical simulations show that not all the control parameters of $\gamma_1$ can prevent the spiral wave breakup. Only for a suitable parameter of $\gamma_1$, spiral wave breakup can be prevented by this control method. With low or high $\gamma_1$ this control method fails, which are shown in Figs. 3(a) and 3(b) for $\gamma_1 = 0.01$ and $\gamma_1 = 1.00$, respectively.
Fig. 3. (a) and (b) The final control results of spiral wave breakup at $\epsilon = 0.070$ with low and high control parameter $\gamma_1$. (a) $\gamma_1 = 0.01$, (b) $\gamma_1 = 1.00$. (c) The controllable phase diagram in the $\epsilon - \gamma_1$ plane.

Fig. 4. The orbits of spiral tip in the meandering parameter region ($\epsilon = 0.065$) for different control parameters: (a) without control (i.e. $\gamma_1 = 0.00$), (b) $\gamma_1 = 0.10$, (c) $\gamma_1 = 0.13$ and (d) $\gamma_1 = 0.60$. 
The dependence of $\gamma_1$ on $\epsilon$ is presented in Fig. 3(c). For a given $\epsilon$, there exists two critical values of $\gamma_1$. For example, at $\epsilon = 0.070$, the low critical value is 0.02 and the high critical value is 0.61. One can see that the controllable region of $\gamma_1$ becomes small as the value of $\epsilon$ is increased, i.e. further deep in the turbulence parameter region, the harder this control method succeeds. Once $\epsilon$ is increased to $\epsilon_c = 0.075$, the spiral waves can no longer be stabilized by this control method.

In order to give an insight into the mechanism underlying this control method, we present the orbit of the spiral tip in the meandering region ($\epsilon = 0.065$) without control and that with spatiotemporal modulation for different $\gamma_1$ in Fig. 4. Figure 4(a) shows a hypocycloid trace of a meandering spiral tip without control (i.e. $\gamma_1 = 0.00$). After the spatiotemporal modulation control method is applied to the system, the spiral tip follows a hypocycloid trace with a considerable decrease of the region of the meandering motion. This meandering region decreases with increasing $\gamma_1$ [see Figs. 4(b) and 4(c)]. For even large control parameter, such as $\gamma_1 = 0.60$, a dramatic phenomenon has been found. The spiral wave which is in the meandering region has been controlled to a stable spiral wave whose tip follows a small circle [see Fig. 4(d)]. So we can say that spatiotemporal modulation control method can effectively decrease the region of meandering motion, which plays a key role in preventing spiral wave breakup.

In Fig. 5, we plot the radius of the region of meandering motion to make a quantitative analysis. We find that the region of the meandering motion increases as the parameter $\epsilon$ is increased without control (see the black squares). And the spiral wave will break up as the radius exceeds a critical value.
When the spatiotemporal modulation is applied to the system, a sudden decrease of the radius has been found (see the red circles). And the gap between the radius without control and that with control is remarkable. We conclude that it is the efficient decreasing of the highly meandering motion of the spiral tip controlled by spatiotemporal modulation that prevents the spiral wave breakup. And the parameter region which supports the spiral waves is extended.

3. Conclusion

In conclusion, we have proposed a simple and effective method to prevent spiral wave breakup due to Doppler instability in an excitable medium. With the suitable control parameter, we can stabilize the unstable spiral wave and prevent its breakup into spatiotemporal chaos. The changed flower petals of the tip motion after the spatiotemporal modulation is applied indicate that preventing spiral wave breakup is a result of the decrease of meandering motion of the spiral tip. We hope that our work can provide useful guidance for ventricular defibrillation.

References


