Learning multiplication through computer-assisted learning activities

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ABSTRACT
This study develops and implements a computer-assisted learning (CAL) program with both multiplicative facts practices and the instruction of meaning behind these facts. The effectiveness of CAL on the development of multiplication abilities is also explored. Eight CAL activities are developed to teach multiplication to second grade elementary school students. The CAL program is comprised of three stages of instruction that addressed in succession the basic concept of multiplication, the meaning and properties of multiplication and multiplication-related computation skills. Evaluation of the effects of the learning activities reveals that CAL activities are effective in improving comprehension of the concept of multiplication and the meaning and properties of multiplication for students who have lower prior knowledge of multiplication, but it does not significantly improve the development of multiplication-related computation skills. Nevertheless, CAL activities facilitate overall learning of multiplication.

1. Introduction

Computer technologies profoundly impact research and practice in mathematics education. The numerous studies (Chang, Sung, & Lin, 2006; Chang, Sung, & Lin, 2007; Kaput, 1986; Prensky, 2000) reporting that computer-assisted learning (CAL) helps students to comprehend mathematical concepts indicate the increasing importance of CAL. Therefore, teachers should be encouraged to continue...
using CAL or gaming software in the mathematics classroom (Becker, 1991; Manoucherihihi, 1999). However, studies of the effectiveness of CAL for teaching mathematics have produced diverse findings. Some claim that CAL practice is superior to traditional instructional approaches (Harrison & Van Derender, 1992; Williams, 2000). However, Bass, Ries, and Sharpe (1986) reported that learning effectiveness does not significantly differ between CAL and traditional materials. Indeed, some studies indicate that traditional instruction methods more effective (Wong & Evans, 2007).

Furthermore, this review of related research findings revealed that learning performance is enhanced if the learning environment facilitates development of both declarative and procedural knowledge of mathematical facts and skills. Cognitive psychologists agree that simultaneous development of declarative and procedural knowledge is required to solve mathematical learning problems (Gagne, 1993). However, the methods presently used to teach mathematics focus mainly on procedural knowledge and emphasize the learning of mathematical symbols and the practice of symbol-based rules. Such methods overlook the declarative knowledge underlying the symbol-based rules. Therefore, students become aware of the limitations of symbol-based rules instead of learning the declarative meanings behind these rules. Procedural knowledge unsupported by declarative knowledge is often constrained to specific scenarios in the learning process, which causes difficulties when referring such knowledge to a new scenario within which a problem must be solved (Hiebert & LeFevre, 1986). Greer (1987) believed that instruction focused on computation instead of concepts made learning more difficult for students who have difficulty with multiplication and division.

The standard mathematics curriculum proposed by the US National Council of Teachers of Mathematics (NCTM) in 2000 indicated that understanding basic multiplication benefits the development of higher-level mathematical abilities and achievements. The NCTM also asserted that declarative knowledge should be developed prior to computational fluency (National Council of Teachers of Mathematics, 2000). Hiebert and LeFevre (1986) suggested that the development of declarative and procedural knowledge should be linked and that developing problem-solving techniques should be balanced with comprehension of concepts when developing mathematical abilities. Kouba (1989) suggested that the concept of multiplication should be established before children are instructed to memorise multiplication tables and learn multiplication symbols.

To establish basic multiplication concepts prior to developing multiplication-related computation skills, Riedesel, Schwartz, and Clements (1996) suggested the following three-stage method of teaching multiplication.

1.1. Stage one: basic multiplication concepts

This stage introduces basic concepts of multiplication from different viewpoints. Greer (1992) proposed three types of problems to enable students to understand the concept of multiplication, including equivalent group problems, multiple comparisons and array-matching. An example of an equivalent group problem is the following: “If a bear has two ears, how many ears do two bears have?” An example of a multiple comparison problem is the following: “A steel ball weighs three times as much as a wooden block. If a wooden block weighs 2 kg, how many kilograms does a steel ball weigh?” An example of an array-matching problem is the following: “If a group of sheep lines up in three rows with four sheep in each row, how many sheep are in the group?”

1.2. Stage two: the meaning and properties of multiplication

Once students understand the basic concept of multiplication, which is established during stage one (see above), multiplication expressions are introduced to express the meaning and properties of a multiple. An important concept in multiplication is the multiple while multiplication expressions represent multiplication problems. The commutative law of multiplication is the most commonly applied property of multiplication. Therefore, three key concepts are introduced during this stage: the computation structure of multiplication (or the concept of a multiple), symbols of multiplication and the commutative law of multiplication.
1.3. Stage three: multiplication-related computation skills

During this stage, students are taught the basic multiplication facts, as well as reinforced with the reactions to multiplication facts. The basic multiplication facts are refer to multiplication tables, and reaction to multiplication means instant and accurate reaction to multiplication facts; training of the facts and reaction enables students to apply such facts when computing multiplication expression.

This research first consolidated theories for teaching multiplication and then structured a three-stage teaching model in which balances the development of declarative and procedural knowledge. This model facilitates the teaching of multiplication concepts to second grade students. According to the three-stage teaching model, the software contained eight learning activities for teaching multiplication to second grade elementary school students. In addition to establishing the systems, the effects of CAL on the development of multiplication abilities were also explored. The experiment was conducted to address the hypothesis that learning multiplication is more efficient when using computer-assisted learning with both multiplicative fact practice and instruction in the meaning behind these facts than when using traditional materials.

2. Learning activities

The teaching model included eight learning activities based on a three-stage instructional model. Table 1 lists the learning activities and the corresponding goals. Each of the eight learning activities is described in detail below.

2.1. Moving home

As Fig. 1 shows, a set of questions was presented in the upper-left corner of screen. For example, one question was, “A civet has four legs; how many legs do five civets have?” The students were required to move the civets on the screen from within the confines of the upper fence into the lower fence. At the same time, the results of the total number of legs of civets within the lower fence were presented on the lower portion of the screen. Fig. 1 illustrates the above process in the case of a student who had moved one civet into the lower fence; at this time, the total number of legs would be indicated on the lower portion of the screen by the message “There is one ‘4’”. The student was required to move four more civets into the lower fence to answer the question correctly. If the student answered the question correctly by moving the appropriate number of civets (in this example, five civets) into the lower fence, the result of the computation of the total number legs of civets (i.e., “20”) was displayed as “There are five ‘4’s”. Twelve such questions were presented during this stage to establish the concept of equivalent groups.

<table>
<thead>
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<th>Table 1</th>
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<tr>
<td>Three-stage instructional model and corresponding learning activities</td>
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<td>Stage</td>
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<tr>
<td>Basic concepts of multiplication</td>
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<td>Meaning and properties of multiplication</td>
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<td>Multiplication computation abilities</td>
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</table>
2.2. Object matching

As Fig. 2 shows, the questions and computational feedback were presented on the computer screen (on the upper-left corner and lower portion, respectively) during this activity. An example was the following: “A small goat eats three carrots and a big goat eats five times as many carrots as the small goat; how many carrots does a big goat eat?” Fig. 2 illustrates the location of the big goat. Students were provided a sufficient number of carrots to feed the big goat. For this example, a computation result of “15” was presented on the lower portion of the screen while “There are five ‘3’s” was displayed. Twelve such questions were presented during this stage to establish the concept of multiple comparisons.

2.3. Lining up

An example of questions presented during this stage (Fig. 3) was the following: “A group of cheetahs lines up in six columns with four cheetahs in each column; how many cheetahs are in the group?” During this activity, the student could move a mouse to make an array and obtain the numbers of columns and rows in the array. For example, Fig. 3 illustrates how a student could create an array with six columns and four rows; upon doing so, a computational result of “24” was displayed on the lower portion of the screen. Twelve such questions were presented during this stage to establish the concept of array-matching.

2.4. Balloon grabbing

Questions presented during this activity (Fig. 4) were in the format “1 time of 5”. The screen displayed balloons that started to float upwards within a chamber displayed on the right side of the screen. Each balloon was labeled with a multiple of a number. For example, if a balloon had five candies attached, the multiple of five represented 1 time of 5. At this time, students could move the balloons from the chamber into a box displayed in the upper-left corner. For example, Fig. 4 illustrates...
how a student would move one balloon into the box. Twelve such questions were presented during this activity to establish the concept of multiples.
2.5. Card matching

Questions presented during this activity (Fig. 5) were in the format “7 times of 2”. Students played a game in which they flipped cards over to find corresponding multiples and multiplication expres-
sions. For example, Fig. 5 illustrates that a student matched “7 times of 2” with the corresponding expression “$2 \times 7$”. This activity was comprised of thirty-two such questions to teach multiplication symbols.

2.6. Balancing

During this activity, the screen presented an image of a balance (Fig. 6) next to several weights on a tray. Students were required to select and move trays on either side. Fig. 6 shows an example of this activity. In this example, three 7-kg weights were placed into the tray on the left of the balance, and students were required to move seven 3-kg weights into the tray on the right to achieve a balanced load. Twelve such questions were presented to teach the commutative law of multiplication.

2.7. Link it

During this exercise, a multiplication expression such as “$3 \times 1 =$” appeared in the upper-left corner of the screen (see Fig. 7). Students were required to select from several dots on the screen, each of which represented an answer to the aforementioned expression. If the correct answer was selected, the system automatically drew a line that linked the expression to the answer. Once the student had selected the correct answer to a particular expression, the expression was replaced with a new expression that required the student to select another answer. Once all of the expressions had been paired with the correct answers, the lines connecting the answers formed the shape of an animal. In the example in Fig. 7, the student had identified the correct answer to four expressions (“$3 \times 1 =$”, “$3 \times 2 =$”, “$3 \times 3 =$” and “$3 \times 4 =$”), and the fifth expression was, “$3 \times 5 =$”. Eight sets of expressions were presented during this activity to establish the concept of multiplication-specific rules.

2.8. Shooting

As Fig. 8 shows, during this exercise, a number (e.g., “16”) was initially presented on the lower part of the screen. At the same time, four balloons appeared on the upper part of the screen, each
containing a different expression; one of these expressions corresponded to the number (i.e., the answer). A timer displayed the time remaining (e.g., 82 s) for the student to respond was presented in the upper-left corner of the screen. Students were required to move a gun to shoot the balloon.
containing the expression that matched the answer. The time limit for each session was 90 s. The student scored one point for each correct answer, and the students were required to score 20 points within the time limit to win the game. If a student made five incorrect selections or failed to score 20 points within the time limit, the game was restarted. The aims of this exercise were to allow students to become familiar with multiplication facts and to reinforce these facts through repeated practice.

3. Method

An experiment was conducted to determine whether the learning activities described above enhanced the learning of multiplication, including the basic concept of multiplication, the meaning and properties of multiplication and multiplication-related computation skills.

3.1. Subjects

The study population was comprised of forty-two second grade students from Taipei City (22 male, 20 female; the average age was 8 years). The students were assigned randomly to either a control or an experimental group, each of which comprised 21 subjects (11 boys and 10 girls). None of the students had received any instruction regarding multiplication prior to the study.

3.2. Experimental design

A quasi-experimental design was used in which the independent variable was the group (control group or experimental group). The dependent variable was the post-test score for each learning stage, and the pretest score was the covariance. Prior to the experiment, pretest scores were obtained using a 30-min multiplication assessment. The experimental phase was conducted over a 3-week-long period in three stages, each of which was carried out over a period of 120 min. During the experiment, a teacher implemented a 40-min-long classroom-based teaching session during which all students were instructed using the same lesson plan and materials. Thereafter, students in the control group continued to learn using traditional written materials while students in the experimental group learned using the above software-based exercises. This phase lasted 70 min. After each learning session, all students took a 10-min post-test. After completing the experiment, all students in the two groups completed a survey questionnaire.

3.3. Experimental tools

The design of the tools used for the pre- and post-tests were based on the guidelines for teaching multiplication and included assessments of the comprehension of basic multiplication concepts, the meaning and properties of multiplication and multiplication-related computation skills. The tools were tested on 50 third-grade students. The Cronbach $\alpha$ coefficient for the tests was 0.84, which indicated good internal consistency. Expert validity was used to enhance the validity of the tests: four senior second grade teachers provided expert advice regarding the tests.

4. Results and discussion

4.1. Results

Table 2 lists the pre- and post-test scores. One-way ANCOVA was used to identify significant differences between experimental and control group post-test scores for basic multiplication concepts, the meaning and properties of multiplication, multiplication-related computation skills and overall post-test scores after eliminating the effects of the pretest scores. First, the homogeneity of the regression coefficient of the total post-test score and post-test score for each stage were tested. The $F$ values for the homogeneity of the regression coefficients on the post-test scores for the basic concept of
multiplication, the meaning and properties of multiplication, and the total post-test score reached the level of significance \((F_{1,38} = 4.95, 4.45, 12.73; p < .05)\). These statistical results demonstrated that outcomes of the aforementioned tests did not match the basic hypothesis of the homogeneity of the regression coefficient; ANCOVA could not be used to examine these data. Instead, Johnson–Neyman method was used to examine interactions among these test scores. In addition, in post-test scores for multiplication-related computation skills, the homogeneity \(F\) value of within-class regression coefficient was 3.819, \(p > .05\), which did not reach statistical significance \((F_{95, (1,38)} = 4.10)\); the result was consistent with the hypothesized homogeneity of regression coefficient; therefore, ANCOVA could be performed.

Figs. 9–11 show the regression intersection point and significant difference point for post-test scores in basic multiplication concepts, the meaning and properties of multiplication and total post-test score derived by Johnson–Neyman method and the regression lines of the two groups.

Examination of the interactions revealed that the post-test scores (maximum score = 30) for basic multiplication concepts did not differ between the two groups for pretest scores between 17.94 and 30. However, for pretest scores lower than 17.94, the post-test performance of the experimental group was better than that of the control group, indicating that students with pretest scores lower than 17.94 were more likely to improve in comprehension of basic multiplication concepts after using the software.

The post-test scores for tests of meaning and properties of multiplication (maximum score = 30) did not significantly differ between the groups with pretest scores between 24.46 and 30, whereas the post-test performance of the experimental group was better than that of control group students with pretest scores lower than 24.46. This indicates that students with pretest scores lower than

<table>
<thead>
<tr>
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<th>N</th>
<th>Basic concept of multiplication Mean (SD)</th>
<th>Meaning and properties of multiplication Mean (SD)</th>
<th>Multiplication-related computation skills Mean (SD)</th>
<th>Total Score Mean (SD)</th>
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<tr>
<td>Pretest</td>
<td>21</td>
<td>18.24 (6.47)</td>
<td>20.29 (7.86)</td>
<td>12.29 (7.02)</td>
<td>50.82</td>
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<td>21</td>
<td>23.05 (3.86)</td>
<td>26.00 (3.41)</td>
<td>14.00 (6.09)</td>
<td>63.05</td>
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<td></td>
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<td><strong>Control group</strong></td>
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</tr>
<tr>
<td>Pretest</td>
<td>21</td>
<td>16.24 (5.40)</td>
<td>22.81 (5.67)</td>
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<tr>
<td>Post-test</td>
<td>21</td>
<td>18.52 (5.14)</td>
<td>25.29 (4.58)</td>
<td>11.19 (5.97)</td>
<td>55.00</td>
</tr>
</tbody>
</table>

Fig. 9. Regression lines of the experimental and control groups on the test scores for the basic concept of multiplication.
24.46 were more likely to exhibit improved comprehension of the meaning and properties of multiplication after using the software.

The overall post-test score (maximum score = 80) revealed that when total pretest score exceeded 76.6, the post-test performance of the control group was better than that of the experimental group. Conversely, when the total pretest score was lower than 41.53, the post-test performance of the experimental group was better than that of the control group. There was no difference between the two

Fig. 10. Regression lines of the experimental and control groups on the test scores for the meaning and properties of multiplication.

Fig. 11. Regression lines of the experimental and control groups on the total test scores.
groups when pretest scores were between 41.53 and 76.6, indicating that students with pretest scores higher than 76.6 responded well to traditional teaching methods while those with pretest scores lower than 41.53 were more responsive to software-based instruction.

After excluding the effects of the pretest scores, the one-way ANCOVA of the post-test scores for multiplication-related computation skills revealed that the post-test performance of the experimental group was not significantly better than that of the control group \( (F_{(1,39)} = 0.01, p > .01) \); within-class differences did not reach statistical significance \( (F_{99, (1,39)} = 7.335) \). This indicates that the instructional software was not significantly better than traditional instruction for improving multiplication-related computation skills.

4.2. Discussion

This study examined the development and implementation of a CAL program and explored the effectiveness of CAL on the development of multiplication abilities. Basic multiplication proficiency is essential for further advancement in mathematical abilities (Kilpatrick, Swafford, & Findell, 2001; Norbury, 2002). Students often use inaccurate counting methods and encounter difficulties in memorising basic multiplication facts (Kilpatrick et al., 2001). However, few studies have evaluated software specially designed to help students learn multiplication (Irish, 2002; Wong & Evans, 2007). Studies in multiplication instruction and learning usually involve two dimensions, namely comprehension of multiplication concepts and ability to solve multiplication-based problems; moreover, understanding the underlying concepts of multiplication is more important than rote memorization (Dempsey & Marshall, 2001).

This study revealed that CAL improved comprehension of basic multiplication concepts in students with relatively low pretest scores (lower than 17.94) whereas CAL did not improve this aspect beyond the improvement achieved using traditional materials for students with relatively high pretest scores (between 17.94 and 30). A similar outcome was observed in comprehension of the meaning and properties of multiplication, and, in this case, CAL failed to affect outcome in students scoring above 24.46. The questionnaire results revealed that array-matching activities might be too difficult for second grade students because the array-matching exercise lowered the overall scores of students in the test of basic multiplication concepts; nevertheless, the scores for the test of comprehension of the meaning and properties of multiplication were relatively higher. These test results indicate that the instructional software improved comprehension of the meanings and properties of multiplication in students who scored lower prior to the implementation of instruction. Restated, students with less prior knowledge of multiplication benefit more from CAL activities. The finding, which is consistent with an earlier report by Bass et al. (1986), may be attributable to the use of game-like scenarios, which are effective for engaging students with low motivation.

Experimental and control group tests scores for multiplication-related computation skills did not significantly differ in this study. One reason for this may be the fact that basic multiplication facts must be memorised; therefore, the two subject groups were unlikely to have developed significantly different multiplication-related computation skills given the relatively short duration of the experimental phase in the study. Therefore, we conclude that CAL does not improve multiplication skills when used for a relatively brief period of time.

In the study, the software for practicing multiplication had no effect on the overall outcome in students with pretest scores between 41.53 and 76.6. Students with pretest scores higher than 76.6 were more adept at practicing multiplication using traditional written materials while students with pretest scores lower than 41.53 were more adept at practicing multiplication using computer software. The likely explanation is that students with high pretest scores had superior learning skills and were more motivated than the low-scoring students. According to the questionnaire results, the instructional software was more effective for improving motivation than for actually improving multiplication skills. Therefore, low-scoring students who are poorly motivated are likely to exhibit improved comprehension of multiplication if instruction is based on CAL rather than on traditional materials.
5. Conclusions

This study developed a three-stage instructional model for teaching basic multiplication concepts, the meaning and properties of multiplication and multiplication-related computation skills. Based on this model, eight different learning activities were developed. The activities were effective for improving comprehension of multiplication concepts and the meaning and properties of multiplication for students who did not have high test scores prior to instruction. However, the activities failed to significantly improve the development of multiplication-related computation skills, which we attribute to the relatively brief period of instruction in this study. Irish (2002) showed that CAL can improve the ability of students to perform multiplication and that CAL-based instruction can complement traditional classroom-based teaching methods. Because remedial learning of multiplication is important to elementary school students, an important goal for future research is the best application of CAL.

References


