Outage Performance of DF Network Coded (DFNC) Multi-User Cooperative Diversity in Orthogonal Uplink Channels

Bin Guo, Kanchan Vardhe, Yu Liu, Chi Zhou, and Yu Cheng
Department of Electrical and Computer Engineering,
Illinois Institute of Technology, Chicago, IL, USA
Emails: bguo2@iit.edu, kanchan.vardhe@ieee.org, {yliu77,zhou,cheng}@iit.edu

Abstract—In this paper, we propose a Decode-and-Forward Network Coded (DFNC) multi-user cooperation scheme for orthogonal uplink channels in a wireless network. The network consists of $m$ peers having independent information transmitting to a common destination. Each peer transmits its own data in the first time phase and serves as a relay in the second time phase to transmit the network coded information combining its own and others. Assuming block fading with independent fading coefficients, we evaluate the outage probability performance of the proposed scheme using high-SNR approximation. Particularly, we develop an upper bound on outage probability of DFNC, and compare it with Space-Time-Coded (STC) cooperation protocol and Repetition-based (REP) cooperation protocol. From theoretical analysis and numerical results, we show that the outage performance of proposed scheme outperforms both STC and REP at least in high-SNR region, regardless of total number of cooperating peers and provides diversity order of $m+1$ in contrast with diversity order of $m$ provided by STC and REP.

I. INTRODUCTION

Cooperation communication technique, by which multiple users share their physical resources to create a virtual MIMO system for transmitting their information, has been widely acknowledged as a promising diversity technique to combat wireless fading [1]–[4]. In cooperative communications, two or more users form partners to help each other transmitting their information typically in two phases. In the first phase, the source broadcast the message to both the destination and relays; in the second phase, the relays forward the message to the destination. The forwarding strategy performed by the relays could be either amplify-and-forward (AF) or decode-and-forward (DF). Particularly Lameman et al., in [4] develop a space-time-coded (STC) DF cooperation protocol and theoretically analyze the outage probability, comparing with repetition-based (REP) cooperation protocol for multi-user wireless networks. However, these cooperation protocols may be inefficient in a large-scale network. When the network size increases, it becomes technically challenging to design an appropriate space-time code required for cooperation, and the repetition protocol suffers from low bandwidth efficiency.

Cooperation communication achieves better diversity performance at the price of extra bandwidth resource, which implies the loss of system throughput. On the other hand, network coding [5], [6] has attracted a lot of interest in wireless networks, since it could leverage the inherent broadcast characteristic of the wireless channels and thus yield better throughput [7]. Hence it would be a natural idea to integrate these two techniques in order to achieve better performance. Actually, network coded cooperation communication has been studied in recently works [8]–[14]. In [8], [9], network coding is used for two-user cooperative communications, and simple XOR operation is utilized as the network coding scheme. Yet, it may not be optimal and may be difficult to generalize in multi-user network. The authors in [10] consider the scenario where multiple source-destination pairs exchange their information via several common relays, on which network coding is applied. The assumption that each destination can reliably overhear the data from other sources is too weak to be practical. In [12], [13], the authors propose a AF transmission protocol utilizing physical-layer network coding for the scenario that $M$ users transmit to a common destination with the help of $L$ relays. A multi-timeslots network coded scheme is proposed in [14] for a single-source and multiple-relays network. All these schemes don’t fully exploit the benefit of network coding in multi-user cooperation network.

In this paper, we propose a Decode-and-Forward Network Coded (DFNC) multi-user cooperation scheme for orthogonal uplink channels in a wireless network. We consider a wireless network with $m$ peers transmitting independent information on orthogonal subchannels (achieved for example though TDMA, FDMA or CDMA) to a single destination $d$. These peers form a cooperation set to help transmit each other’s information. The protocol consists of two transmission phases, which are similar to normal cooperation protocols. During the first phase, each peer transmits its own information to the destination, other peers overhear the information. In the second phase, each peer will linearly combine all the information it possessed using coding coefficients similar to the idea of linear network coding [6] and then transmits. The coding coefficients are designed according to the decoding set, i.e., the set of other users that receive one’s message correctly in the first phase. Each peer chooses the coding coefficients based only on the information about its decoding set. With
low complexity design of coding coefficients, the protocol can be easily implemented. Then we theoretically analyze the outage probability of DFNC protocol. Specifically, we develop an upper bound on the outage probability of the proposed protocol, and compare it with STC and REP. We show that the proposed scheme achieves diversity order of \( m + 1 \) in contrast with STC and REP which can provide diversity order of \( m \). Using numerical results, we observe that the proposed scheme outperforms the other two protocols in terms of lower outage probability at least in high-SNR region, regardless of the size of cooperation set \( m \).

The rest of the paper is organized as follows. Section II introduces the system model and describes the DFNC cooperation protocol. Section III theoretically analyzes the diversity performance of the proposed protocol in term of information outage probability. We provide numerical results and compare the proposed DFNC with other cooperation protocols in section IV and finally we provide the conclusion in section V.

II. SYSTEM MODEL AND DESCRIPTION OF DFNC PROTOCOL

A. System Model

We consider a wireless network with \( m \) peers transmitting different information on orthogonal subchannels to a single destination \( d \). Each peer transmits its own information and potentially relays other’s information. Thus, these \( m \) peers form a cooperation set \( \mathcal{M} \), and \( d \notin \mathcal{M} \). Here we use the term “peer” to emphasize the fairness of the contribution for each user. As in [3], [4], the system consists of two transmission phases. In the first phase, each peer broadcasts its information to the destination and other peers. During the second phase, all peers transmit the network coded information as shown in Fig. 1, the details will be explained in the sequel. The DF scheme, as we mentioned, is adopted as the forwarding technique, thus we define the decoding set \( \mathcal{D}(p) \) for a specific peer \( p \) to denote the set of other peers that received \( p \)'s information successfully in the first transmission phase, thus we have \( \mathcal{D}(p) \subseteq \mathcal{M} \setminus \{p\} \). In contrast, the retrieve set \( \mathcal{R}(p) \) denotes the set of peers whose information has been successfully received by \( p \).

We assume that the transmissions suffer from frequency nonselective Rayleigh fading and additive white Gaussian Noise (AWGN), which is modeled as a complex Gaussian random variable with variance \( N_0 \). We model the channel fading coefficient \( \alpha_{i,j,k} \) as zero-mean, independent, circularly symmetric complex Gaussian random variables with variance \( 1/\lambda_{i,j,k} \), so that \( |\alpha_{i,j,k}|^2 \) are exponentially distributed with parameter \( \lambda_{i,j,k} \), where \( i \in \mathcal{M} \) denotes the transmitting nodes, \( j \in \mathcal{M} \cup \{d\} \) denotes the receiving nodes, \( k = 1, 2 \) indicates the time phase index. We further model the channel between peer and destination as block fading with independent fading coefficients, \( \alpha_{p,d,1} \neq \alpha_{p,d,2} \). The channels between peers are assumed to be symmetrical, i.e., \( \alpha_{i,j,k} = \alpha_{j,i,k} \) where \( i, j \in \mathcal{M} \), hence we have \( \mathcal{D}(p) = \mathcal{R}(p) \) for peer \( p \). Note that it only holds on average in practice, since each receiver observes different noise. It is also assumed that the receivers know perfectly the Channel State Information (CSI), thus the receiver is able to correctly estimate the received information in high probability.

Finally, we discuss the required transmit SNR for different protocols in order to fairly compare with each other. The SNR in absence of fading in noncooperative protocol is referred to as \( \gamma = \frac{P}{N_0} \), where \( P \) is discrete-time transmit power constraint. The above defined SNR will be used as a baseline for comparison. Following the definitions in [4], each peer employing STC, REP, DFNC utilizes \( \frac{1}{m}, \frac{1}{m}, \frac{1}{m} \) of total degrees of freedom in the channel, respectively. Therefore the corresponding required SNR for STC, REP, DFNC are \( \frac{2}{m}, \gamma, \gamma, \gamma \), respectively. These standardized SNRs are important in calculations of the mutual information and outage probability in the sequel.

B. Description of DFNC Protocol

All the peers are orthogonally allocated the same portion of the total channel, hence we focus on the transmission of peer \( p \) with other peers as potential relays.

In the first phase, the destination and each relay will receive

\[
\begin{align*}
y_{p,d,1} &= \alpha_{p,d,1}x_p + n \\
y_{p,r,1} &= \alpha_{p,r,1}x_p + n
\end{align*}
\]

where \( x_p \) is the transmitted information from source \( p \); \( \alpha_{p,d,1} \) and \( \alpha_{p,r,1} \) are the channel gains defined in section II-A; \( y_{p,d,1} \) and \( y_{p,r,1} \) are the received information at destination \( d \) and relay \( r \), respectively; and \( n \) is the AWGN noise. The sufficient condition for \( r \) to be in the decoding set of \( p \) (i.e., \( r \in \mathcal{D}(p) \)) is that the SNR is large enough for \( r \) to decode the information transmitted from \( p \). When the cooperation relationship between \( p \) and \( r \) is established, the cooperation scheme will remain steady toward the end of the transmission. It is possible if we consider the inter-peer channels are stationary. This consideration is important to design the structure of network coding, since our network coding coefficients are designed according to the decoding set of \( p \). While the information of \( p \) is received by other peers in its decoding set, \( p \) will receive several independent information \( x_r, r \in \mathcal{R}(p) \). The symmetry of the inter-peer channels implies \( r \in \mathcal{D}(p) \).

During the second phase, if the decoding set is empty, it implies that \( p \) does not successfully receive information from any of other peers. In this situation, \( p \) repeats transmitting \( x_p \), then the destination performs diversity combining (e.g., MRC) to decode. If decoding set exists, \( p \) will combine all the information it possessed \((x_p \text{ and } x_r, r \in \mathcal{R}(p)) \) with certain
coding coefficients by linear network coding, and then transmit the network coded information to the destination. Hence the information received at the destination in the subchannel of \( p \) during the second phase is:

\[
y_{p,d,2} = \alpha_{p,d,2} \sum_{r \in R(p) \cup \{p\}} \beta_r^p x_r + n
\]  

(3)

where \( \beta_r^p \) is the network coding coefficients assigned by \( p \) to the different information \( x_r \) in the combination. The coding coefficients need to satisfy the following constraints:

\[
\begin{align*}
\sum_i |\beta_r^p|^2 &= 1. & \text{(a)} \\
E(|\beta_r^p|^2) &= \frac{1}{|R(p)|+1}. & \text{(b)} \\
Var(|\beta_r^p|^2) &= \sigma. & \text{(c)}
\end{align*}
\]

where \( r \in R(p) \cup \{p\} \), \( E(\cdot) \) is the expectation value, \( Var(\cdot) \) is the variance, \( \sigma \) is a relative small number compared to the expectation. The constraint (a) guarantees that the received information is within the power requirement at the destination; constraint (b) and (c) ensure that each information \( x_r \) contributes similar contribution in the combination with different coefficients.

In general, the destination receives \( m \) versions of network coded information from \( m \) orthogonal subchannels. Assuming the destination has full CSI, the received information after estimation is written as below:

\[
\hat{y} = Bx
\]  

(4)

where \( x = [x_1 \ldots x_m]^T \) is the vector of transmitted information from all peers, \( \hat{y} = [\hat{y}_1 \ldots \hat{y}_m]^T \) is the vector of estimated received information. \( B = [B_1 \ldots B_m]^T \) is the network coding coefficient matrix, where \( B_i = [\beta_j^i \ldots \beta_m^i] \) is the network coding coefficient vector assigned by peer \( i \) to all the information components involved in the combination information. The coefficient \( \beta_j^i \) \((i, j = 1 \ldots m)\) in \( B_i \) is assigned to be 0 if peer \( i \) is not able to decode the corresponding information from peer \( j \). Since the inter-peer channels remain stationary during the entire transmission, the network coding coefficient matrix \( B \) is considered to be unchanged. In reality, we need extra time and bandwidth resource to transmit \( B \) to the destination. However, the cost can be neglected if the information content is large. Furthermore, the coding coefficients could be selected from a large Galois Field to guarantee that the coefficient vectors are linearly independent.

During the second phase of DFNC, \( x_p \) is potentially involved in \( m \) network coded transmissions. These \( m \) network coded information can be equivalent to \( m \) different independent codewords, and they are subject to independent fading. Intuitively, \( x_p \) undergoes \( m \) independent fading channel during two transmission phases, therefore we expect DFNC protocol to offer a diversity order of \( m \). In summary, we expect DFNC protocol to offer a diversity order of \( m \) during two transmission phases, therefore we expect DFNC protocol to offer a diversity order of \( m \) during two transmission phases, therefore we expect DFNC protocol to offer a diversity order of \( m \).

III. THEORETICAL ANALYSIS OF OUTAGE PERFORMANCE

The outage probability is defined as the probability that the average mutual information between source and destination falls below a fixed spectral efficiency \( R \). The system is said to be in outage, when the decoding error probability cannot be made arbitrarily small, no matter what kind of code used by the transmitter [16]. In DFNC protocol, the average mutual information \( I_{DFNC} \) between peer \( p \) and the destination \( d \) is related to the decoding set. Since the decoding set \( D(p) \) is a random variable, the outage probability is mathematically described as below by using the total probability law:

\[
P_{out,DFNC} = \sum_{D(p)} P_r[I_{DFNC} < R|D(p)]P[D(p)]
\]  

(5)

**Theorem 1**: In the proposed \( m \)-peers cooperative network, DFNC protocol achieves a diversity order of \( m + 1 \).

**Proof**: We analyze the mutual information and outage probability of DFNC protocol from two cases. The first case is that the decoding set is not empty \( |D(p)| \neq 0 \); the second one is that no other peers receive the information of \( p \) correctly, i.e., \(|D(p)| = 0 \).

**Case I**: Outage on non-empty decoding set. In this case, \( x_p \) is broadcast in the first phase to both the destination and other peers; during the second phase, \( x_p \) is involved in \(|D(p)| + 1 \) network coded version, transmitted by \( p \) and \( D(p) \). The network coded information can be regarded as different independent codewords for \( p \). Consequently, \( p \)’s information is carried across \(|D(p)| + 1 \) parallel independent channels, resulting in the total mutual information being the sum-log form with all the involved peers’ instantaneous SNRs. The conveyed network coded information is shared by \(|D(p)| + 1 \) peers, hence a factor of \( \frac{1}{|D(p)|+1} \) is needed. Furthermore, the factor of \( \frac{1}{2} \) accounts to the fact that the total time is divided into two phases. In summary, conditioned on the decoding set \( D(p) \), the mutual information between \( p \) and \( d \) is derived as:

\[
I_{DFNC|D(p)|\neq0} = \frac{1}{2} \log(1 + \gamma |\alpha_{p,d,1}|^2) + \frac{1}{2} \log(1 + \gamma |\beta_{p,d}^p|^2) + \frac{1}{|D(p)|+1} \sum_{r \in \{p\} \cup D(p)} \log(1 + \gamma |\alpha_{r,d,2}|^2).
\]

(6)

Now we look at the second term in (6). Since log function is a convex function, we have the following by Jensen’s Inequality \( (\rho = \frac{1}{|D(p)|+1} ) \) is defined for convenience):

\[
\rho \sum_{r \in \{p\} \cup D(p)} \log(1 + \gamma |\alpha_{r,d,2}|^2) \geq \log(1 + \rho \gamma \sum_{r \in \{p\} \cup D(p)} |\alpha_{r,d,2}|^2)
\]

(7)

By using such a fact, the mutual information can be lower bounded as:

\[
I_{DFNC|D(p)|\neq0} \geq \frac{1}{2} \log(1 + \gamma |\alpha_{p,d,1}|^2) + \frac{1}{2} \log(1 + \rho \gamma \sum_{r \in \{p\} \cup D(p)} |\alpha_{p,d,2}|^2) \triangleq I_{up}
\]

(8)

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Using this lower bounded mutual information, the outage probability conditioned on the decoding set is upper bounded as:

\[ P_r[I_{DFNC,|D(p)|\neq 0} < R|\mathcal{D}(p)|] \leq P_r[I_{up} < R|\mathcal{D}(p)|] \]  

(9)

Now we consider the upper bounded outage probability in (9). From (6), we show that the information of \( p \) suffers \( |\mathcal{D}(p)| + 2 \) independent fading coefficients: one from the first phase, \( |\mathcal{D}(p)| + 1 \) from the second phase. Therefore we expect the outage probability to decrease asymptotically proportional to \( \epsilon = \frac{2^2R-1}{\rho \gamma} \). We define \( \epsilon = \frac{2^2R-1}{\rho \gamma} \), then we have:

\[
\lim_{\epsilon \to 0} \frac{P_r[I_{up} < R|\mathcal{D}(p)|]}{\epsilon^{|\mathcal{D}(p)|+2}} = \rho \times \lambda_{p,d,1} \prod_{r \in \mathcal{D}(p)} \lambda_{r,d,2} \times A_{|\mathcal{D}(p)|+1}(2^2R-1) \]

(10)

where \( A_p(t) = \frac{1}{m(n-w)} \int_0^{1-w} \frac{1}{1+\epsilon w} dw \) \( n > 0 \). The proof is similar to [4] and omitted here due to the limited space. Consequently, the high-SNR approximation for the upper bounded outage probability conditioned on the decoding set is given by:

\[
P_r[I_{up} < R|\mathcal{D}(p)|] \approx \left( \frac{2^2R-1}{\rho \gamma} \right)^{|\mathcal{D}(p)|+1} \times \lambda_{p,d,1} \prod_{r \in \mathcal{D}(p)} \lambda_{r,d,2} \times A_{|\mathcal{D}(p)|+1}(2^2R-1) \]

(11)

Then we consider the probability of decoding set \( P_r[|\mathcal{D}(p)|] \).

The probability that the terminal \( r \) belongs to \( \mathcal{D}(p) \) can be derived from the mutual information between \( p \) and \( r \) is:

\[
P_r[r \in \mathcal{D}(p)] = P_r\left[ \frac{1}{2} \log(1 + \gamma |\alpha_{p,r,1}|^2) > R \right]
\]

\[
= P_r[|\alpha_{p,r,1}|^2 > \frac{2^2R-1}{\gamma}] = e^{-\lambda_{p,r,1}(2^2R-1)} \]

(12)

For a specific decoding set \( \mathcal{D}(p) \), the probability is obtained from (12) and further approximated in high-SNR region:

\[
P_r[|\mathcal{D}(p)|] = \prod_{r \in \mathcal{D}(p)} \left( e^{-\lambda_{p,r,1}(2^2R-1)} \right) \prod_{r \notin \mathcal{D}(p)} (1-e^{-\lambda_{p,r,1}(2^2R-1)})
\]

\[
\approx \left( \frac{2^2R-1}{\gamma} \right)^{|\mathcal{D}(p)|-1} \times \prod_{r \notin \mathcal{D}(p)} \lambda_{p,r,1}. \]

(13)

Combining (11) and (13) into (5), we obtain the upper bounded outage probability approximated in high-SNR region when the decoding set is not empty:

\[
P_{out,|\mathcal{D}(p)|\neq 0} \approx \left( \frac{2^2R-1}{\gamma} \right)^{|\mathcal{D}(p)|-1} \times \prod_{D(p)} \lambda_{r,d,2} \times \prod_{r \notin \mathcal{D}(p)} \lambda_{p,r,1} \times A_{|\mathcal{D}(p)|+1}(2^2R-1) \]

(14)

From (14), we obtain that the diversity order achieved in DFNC protocol is \( m+1 \).

**Case II: Outage on empty decoding set.** When the decoding set is empty, no other peers correctly receives the information from \( p \). According to the symmetry of inter-peer channels, \( p \) receive nothing from other peers as well. In this situation, \( p \) retransmits its own message in the second phase. Assuming the destination performs MRC to decode, the mutual information is:

\[
I_{DFNC,|\mathcal{D}(p)|=0} = \log(1 + \gamma |\alpha_{p,d,1}|^2 + |\alpha_{p,d,2}|^2). \]

(15)

Similar to (11), the outage probability conditioned on the empty decoding set is approximately expressed as:

\[
P_r[I_{DFNC} < R|\mathcal{D}(p)| = 0] \sim \frac{1}{2} \left( \frac{2^2R-1}{\gamma} \right)^2 \times \lambda_{p,d,1} \lambda_{p,d,2}. \]

(16)

and the probability that the decoding set is empty is given by:

\[
P_r[|\mathcal{D}(p)| = 0] = \prod_{r \notin \mathcal{M}, r \neq p} (1-e^{-\lambda_{p,r,1}(2^2R-1)})
\]

\[
\sim \left( \frac{2^2R-1}{\gamma} \right)^{-m-1} \times \prod_{r \in \mathcal{M}, r \neq p} \lambda_{p,r,1}. \]

(17)

Combining (16) and (17), we obtain:

\[
P_{out,|\mathcal{D}(p)|=0} \sim \left( \frac{2^2R-1}{\gamma} \right)^{-m-1} \left( \frac{2^2R-1}{\gamma} \right)^2
\]

\[
\times \prod_{k=1,2} \lambda_{p,d,k} \prod_{r \in \mathcal{M}, r \neq p} \lambda_{p,r,1}. \]

(18)

From (18), it is also shown that the outage probability decays asymptotically proportional to \( 1/\gamma^{m+1} \), which implies \( m+1 \) diversity order. Again, this is because all the other peers contribute to the diversity performance as potential relays, although they are not in the decoding set, as we mentioned in Section II.

Finally, the total outage probability is obtained by summing (14) and (18) together according to (5). We conclude that \( m+1 \) diversity order is offered by DFNC, **Theorem 1** is proved. \( \Box \)

**IV. NUMERICAL RESULTS**

In this section, we show the numerical results for outage performance of DFNC protocol, and then compare it with non-cooperation transmissions, STC protocol and REP protocol. The outage probability of STC and REP cooperation protocols have been derived in [4], please refer to the reference for details. In all the figures, the outage probability curves are plotted for \( m=1 \), assuming that the threshold spectral efficiency is \( \rho = 1 \) bit/sec/Hz.

In Fig. 2, the upper bounded outage probability of DFNC protocol with different \( m \) are plotted under high-SNR approximation, and the outage probability without high-SNR approximation are also plotted for comparison. With the increment of \( m \), the outage probability becomes lower, and the slope of outage probability is large, which indicates a higher diversity order is achieved. Moreover, it is observed that the gap between the outage curve without high-SNR approximation and that of upper bounded approximation becomes larger when \( m \) increases.
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Fig. 2. Upper bounds of outage probability performance (under high-SNR approximation) of DFNC protocol with different size of cooperation set \( m=2, 5, 8, 10, 15, 20 \) (from right to left along x axis). The outage probability without high-SNR approximation are also plotted (dashed line) for comparison.

Fig. 3. Outage probability performance (under high-SNR approximation) of DFNC protocol compared with other cooperation protocols: STC, REP and noncooperation on different \( m \).

V. CONCLUSION

In this paper, a Decode-and-Forward Network Coded (DFNC) cooperation protocol is presented for multi-user orthogonal unlink channels. In the proposed \( m \)-peers cooperation network, each peer transmits its own information in the first phase and transmits the network coded information in the second phase. Focus on the metric of outage probability, the diversity performance of DFNC is evaluated both by theoretical analysis and numerical results. Particularly, we derive a upper bound on outage probability for DFNC to compare with STC and REP protocols. Assuming block fading with independent fading coefficients, it is concluded that DFNC offers a diversity order of \( m+1 \), and the outage probability of DFNC outperforms others at least in high-SNR region, regardless of the size of cooperation set \( m \). The performance of DFNC on different modulation schemes and the performance in non-orthogonal subchannels could be considered in the future work.

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