Multicriteria decision-making based on goal programming and fuzzy analytic hierarchy process: An application to capital budgeting problem

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Abstract

Our objective in this paper is to develop a decision-making model to assist decision-makers and researchers in understanding the effect of multiple criteria decision-making on a capital budgeting investment. This decision-making model helps decision-makers with reducing decision-making time and choosing a suitable decision alternative for a capital budgeting investment within the companies’ goals, constraints and strategies. The methods utilized in this paper are goal programming (GP) and fuzzy analytic hierarchy process (FAHP). We demonstrate a case study of the capital budgeting investment by using these two methods in a small car rental company.

Keywords:
Capital budgeting
Fuzzy analytic hierarchy process
Goal programming
Decision-making
Sensitivity analysis

1. Introduction

Capital budgeting decision-making is one of the most demanding responsibilities of top management [21,12]. An increasing number of companies have struggled to justify strategic technology investments using traditional capital budgeting systems [2]. The existing accounting-based decision-making models (such as discounted cash flow (DCF)) are said to be no longer adequate to help evaluate investments in technological innovation, mainly because of the strategic, intangible nature of the benefits involved [13,22].

When business decisions are made, they involve not only consideration of information which is quantifiable in numerical terms (e.g. financial information), but also consideration of subjective (e.g. non-financial information) opinions [27,2,1]. Such subjective considerations are naturally expressed in linguistic rather than in numerical terms [14]. Therefore, we realized that non-financial information needs to be quantified in order to integrate it with numerical information.

This research will focus on how to integrate financial and non-financial information in the company’s constraints, goals and strategies. The methodologies presented within this research are goal programming (GP) and fuzzy analytic hierarchy process (FAHP) which address the problem of capital budgeting in uncertain environments.

Capital budgeting is primarily concerned with sizable investments in long-term assets. Investment decisions deal with the funds raised in financial markets which are employed in productive activities to achieve the firm’s overall goal, in other words, how much should be invested and what assets should be invested are the main objectives. Therefore within this research it is assumed that the objective of the investment or capital budgeting decision is to achieve the company’s goals and to stay within its constraints.

GP normally deals with conflicting objective measures. Each of these measures is given a goal or target value to be achieved. FAHP provides a relatively more complete description of decision-making process involving the subjective and imprecise judgments of decision makers [4]; [17]; [11].

The methods are divided into two steps. Firstly, financial and other objectives along with a company’s goals, constraints and strategies are formulated as important selection criteria. A set of decision alternatives (DAs) as preliminary outcomes will be sifted by using GP from financial information. Secondly, subjective opinions elicited from decision-makers (DMs) are transformed into fuzzy comparison matrices (for the details of FAHP also refer to Chang [8], Tang [23]). A simple practical preference ranking method (synthetic extent method) is investigated to rank alternatives in a multiplicative aggregation process.

The extent analysis method has been employed in quite a number of applications, such as capital measurement [5], budget allocation [23], assets selections and investment [6,7,24] and for more detail refer to Wang et al. [26]. However, disadvantages have also
been pointed out, such as an inability to derive the true weights from a fuzzy or crisp comparison matrix [26]. This research will utilize the formulation of a degree of possibility for comparing two triangular fuzzy numbers as proposed in Zhu et al. [30].

One aspect of the FAHP method within this research is the prevalence of and allowance for incompleteness in the judgements made by DMs. For example, if a DM is not willing or is unable to specify the preference judgements, s/he is able to omit a judgment in the form of a pairwise comparison between two DAs. The rest of the paper is organized as follows. Section 2 describes the details of the used cars selection problem. Section 3 proposes GP procedures and the synthetic extent method of the FAHP. Section 4 illustrates the results of the used cars selection problem. The conclusion is provided in Section 5.

2. Identification of the used cars selection problem

The case study concerns a small car hiring company and their choice of type of fleet cars to be adopted. This choice is an important investment decision, with a large proportion of their budgets being tied up in their final choice. In order to find out which are the most important criteria used by the DMs, two interview phases are being tied up in their final choice. In order to find out which are the most important criteria used by the DMs, two interview phases are conducted.

The semi-structured interview within this research was designed to identify all the relevant issues affecting the decision-making (e.g., the company’s goals, constraints, and objectives, etc.). It was difficult to discover all these issues in the beginning until the DMs were reassured. In this phase, the semi-structured interview shows that there are two constraints (i.e., five cars can be chosen each time and the total size of engines are limited to under 8000cc); two goals (i.e., to minimize the cost of suitable cars and to minimize the cars’ age and mileage); four pieces of objective information (i.e., size of engine, price, car age and mileage). There are six subjective criteria identified by this company’s DMs, i.e., “Equipment, Comfort, Car Parts and Components, Customer Demand, Safety, and Image”, denoted as $C_1, C_2, \ldots, C_6$, respectively. Apart from those constraints, goals and criteria, the semi-structured interviews also identified ten type of most commonly used cars, denoted herein $A_1', A_2', \ldots, A_{10}'$, respectively (see Appendix A). They are the initial DAs in this case study.

After we identified their constraints, goals, and objectives, we utilized the GP methodology to obtain the DAs. With respect to the AHP’s pairwise comparison method, we still needed to explain a lot to the DMs as it is difficult for people to understand the judgment matrices in the first time. Therefore, in the second phase of interviews, the DMs were asked to indicate their preferences between pairs of criteria, and then between pairs of DAs over the different criteria. In this phase, a DM can leave a judgement out without giving any judgement on the fuzzy comparison matrix. The results from the second phase of interview are shown in Tables 1 and 2.

3. Proposition of GP procedures and the synthetic extent method of the FAHP

3.1. First step: introduction of the procedure of GP

GP is an important technique for allowing DMs to consider several objectives in finding a set of acceptable solutions. It has been accomplished with various methods such as Lexicographic (Preemptive), Weight (Archimedean), and MINIMAX (Chebyshev) achievement functions [18]. It can also be said that GP has been, and still is, the most widely used technique for solving multi-criteria decision-making problems. The purpose of GP is to minimize the deviation between the achievement of goals, $f_i(Y)$, and their acceptable aspiration levels, $g_i$. A mathematical expression for the standard version of GP is given below.

\begin{equation}
\text{(GP) method}
\end{equation}

Minimize $\sum_{i=1}^{n} |f_i(Y) - g_i|$, 
subject to $Y \in F, \quad (F$ is a feasible set);

where $f_i(Y)$ is the linear function of the $i$th goal, $Y$ is a $1 \times N$ vector of decision variables and $g_i$ is the aspiration level of the $i$th goal.

The oldest and still most widely used form of achievement function for GP is represented as follows.

\begin{equation}
\text{(GP-achievement)}
\end{equation}

Minimize $\sum_{i=1}^{n} d_i^+ + d_i^-$, 
subject to $f_i(Y) - d_i^- + d_i^+ = g_i$, \quad for $i = 1, 2, \ldots, n$.

$Y \in F, \quad (F$ is a feasible set);

where $d_i$ ($i = 1, 2, \ldots, n$) are additional continuous variables.

3.2. Second step: construction of the FAHP comparison matrices

The aim of any FAHP method is to elucidate an order of preference on a number of DAs, i.e., a prioritised ranking of DAs. Central to this method is a series of pairwise comparisons, indicating the relative preferences between pairs of DAs in the same hierarchy. It is difficult to map qualitative preferences to point estimates, hence a degree of uncertainty will be associated with some or all pairwise comparison values in an FAHP problem [28]. By using triangular fuzzy numbers (TFNs), via the pairwise comparisons made, the fuzzy comparison matrix $X = (x_{ij})_{n \times m}$ is constructed.

The pairwise comparisons are described by values taken from a pre-defined set of ratio scale values as described in Saaty [19]. The ratio comparison between the relative preference of elements

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Pairwise comparisons between criteria based on the DM’s opinions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>1</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1/3</td>
</tr>
<tr>
<td>$C_3$</td>
<td>9</td>
</tr>
<tr>
<td>$C_4$</td>
<td>-</td>
</tr>
<tr>
<td>$C_5$</td>
<td>-</td>
</tr>
<tr>
<td>$C_6$</td>
<td>1/3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>$a$ to $f$: Comparisons between DAs over the different criteria.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $C_1$</td>
<td>$A_1$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>1</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1/7</td>
</tr>
<tr>
<td>$A_3$</td>
<td>3</td>
</tr>
<tr>
<td>(c) $C_1$</td>
<td>$A_1$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>1</td>
</tr>
<tr>
<td>$A_2$</td>
<td>8</td>
</tr>
<tr>
<td>$A_3$</td>
<td>7</td>
</tr>
<tr>
<td>$A_4$</td>
<td>-</td>
</tr>
</tbody>
</table>

The elements are compared with each other, and the result is shown in the table.
indexed $i$ and $j$ on a criterion can be modeled through a fuzzy scale value associated with a degree of fuzziness. Then an element of $X$, $x_l$ (comparison of $l$th DA with $j$th DA with respect to a specific criterion) is a fuzzy number defined as $x_l = (l_l, l_m, l_u)$, where $l_m$, $u_l$ and $l_j$ are the modal-value, the upper bound and the lower bound values of a fuzzy number $x_l$, respectively. To keep the reciprocal nature of the fuzzy comparison matrix $X$, the fuzzy number is also satisfied with $l_l = \frac{1}{l_l}$, $m_l = \frac{1}{m_l}$, $u_l = \frac{1}{u_l}$ [24].

There are various operations upon TFNs, the relevant reference can be referred to in Kaufmann and Gupta [15].

### 3.2.1. Value of fuzzy synthetic extent

Let $C = \{C_1, C_2, \ldots, C_n\}$ be a criteria set, where $n$ is the number of criteria and $A = \{A_1, A_2, \ldots, A_m\}$ is a DA set where $m$ is the number of DAs. Let $M_1^C, \ldots, M_n^C$ be values of extent analysis of $i$th criteria for $m$ DAs where $i = 1, 2, \ldots, n$ and all the $M_i^C (j = 1, 2, \ldots, m)$ are TFNs. The value of fuzzy synthetic extent ($S_i$) with respect to the $i$th criteria is defined as:

$$S_i = \sum_{j=1}^{m} M_i^C \cdot \left[ \sum_{j=1}^{m} M_i^C \right]^{-1}$$

(1)

where represents fuzzy multiplication and superscript $-1$ represents the fuzzy inverse. The concepts of synthetic extent can also be found in Chang [8], Tang [23], Tang and Beynon [24].

The degree of possibility of two fuzzy numbers $M_1 \geq M_2$ is defined as:

$$V(M_1 \geq M_2) = \sup_{x,y} \left[ \min(\mu\min(x), \mu\max(y)) \right]$$

(2)

$$V(M_1 \geq M_2) = 1 \text{ iff } m_1 \geq m_2$$

$$V(M_2 \geq M_1) = hgt(M_1 \land M_2) = \mu\max(x_2)$$

where ifff represents "if and only if" and $d$ is the ordinate of the highest intersection point between the $\mu\min$ and $\mu\max$ TFNs (see Fig. 1) and $x_2$ is the point on the domain of $\mu\max$ and $\mu\min$ where the ordinate is defined. The term $\mu\min$ is the height of fuzzy numbers on the intersection of $M_1$ and $M_2$. For $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$, the possible ordinate of their intersection is given by the expression (2).

The degree of possibility for a convex fuzzy number can be obtained from the use of Eq. (3)

$$V(M_2 \geq M_1) = hgt(M_1 \land M_2) = \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} = d$$

(3)

The degree of possibility for a convex fuzzy number $M$ to be greater than the number of $k$ convex fuzzy numbers $M_i (i = 1, 2, \ldots, k)$ can be given by the use of the operation max and min [10] and can be defined by:

$$V(M \geq M_1, M_2, M_3, \ldots, M_k) = \max(\min(V(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots \text{ and } (M \geq M_k)))$$

(4)

Assume that $d(A_i) = \min(V(S_i \geq S_i))$, where $k = 1, 2, \ldots, n$; $k \neq i$ and $n$ is the number of criteria as described previously. Then a weight vector is given by:

$$W = (d(A_1), d(A_2), \ldots, d(A_n))$$

(5)

The vector $W$ is normalised and denoted by:

$$W = (d(A_1), d(A_2), \ldots, d(A_n))$$

One point of concern, highlighted in this paper is when two elements (fuzzy numbers $- M_1$ and $M_2$) say $(l_l, m_l, u_l)$ and $(l_2, m_2, u_2)$ in a fuzzy comparison matrix satisfy $l_1 - u_2 > 0$ then $V(M_2 \geq M_1) = hgt(M_1 \land M_2) = \mu\max(x_2)$, with $\mu\min(x_2)$ given by Zhu et al. [30]:

$$\mu\min(x_2) = \left\{ \begin{array}{ll} \frac{l_l - u_2}{(m_2 - u_2) - (m_1 - l_1)} & l_1 \leq u_2; \\
0 & \text{others.} \end{array} \right.$$
expression minimum workable degree of fuzziness is defined as the largest of the values of \( \delta \) at the various appearance points of criteria (or DAs) on the \( \delta \)-axis.

When considering the final results, the domain of workable \( \delta \) is expressed as \( \delta_t \), and is defined by the maximum of the various minimum workable degrees of fuzziness throughout the problem, that is here \( \delta_t = \max (\delta_1, \delta_2, \delta_3, \ldots, \delta_n, \delta_t) \) where the subscript \( T \) is the maximum of the minimum workable \( \delta \) values in the \( n + 1 \) (T, matrix) fuzzy comparison matrices.

4. The results of the used cars selection problem

4.1. The results of first step obtained from the GP

Two goals of the problem are given as to minimize the cost of suitable cars and to minimize the cars’ age and mileage. Other constraints are given as: (i) to find five suitable cars out of these ten cars; (ii) the total size of engine is limited to under 8000cc; (iii) the total price is limited to under 12,000 pounds. From these two goals we know that the smaller the value of goals, the better the achievement. Thus, we can set two goals of the problem \( g_1 \) and \( g_2 \) to be zero corresponding to the above goals

\[
\begin{align*}
\text{Minimize } & d_1^* + d_2^*, \\
\text{subject to } & A_1, A_2, A_3, \ldots, A_{10}, \\
& 2A_1 + 1.5A_2 + 3A_3 + 2A_4 + 2A_5 + 3A_6 + 3A_7 + 1.5A_8 + 3.5A_9 \\
& + 3A_{10} - d_1^* = g_1 (\text{Age}), \\
& 9500A_1 + 4000A_2 + 45000A_3 + 30000A_4 + 25000A_5 + 25000A_6 \\
& + 35000A_7 + 40000A_8 + 28000A_9 + 9300A_{10} - d_2^* = g_2 (\text{Mileage}), \\
& A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10} = 5 (5 \text{ cars}), \\
& 1995A_1 + 1995A_2 + 1600A_3 + 1600A_4 + 1000A_5 + 1600A_6 \\
& + 1600A_7 + 1400A_8 + 1995A_9 + 1600A_{10} < 8000 (\text{Size of engine}), \\
& 19995A_1 + 45000A_2 + 2000A_3 + 3000A_4 + 1500A_5 + 1850A_6 \\
& + 2500A_7 + 35000A_8 + 14000A_9 + 5000A_{10} <= 12000 (\text{Price}).
\end{align*}
\]

where \( g_i^* (i = 1, 2) \) is the lower bound of the \( i \)-th goal.

Assume that \( g_1^* = 6 \) and \( g_2^* = 100,000 \). This problem is solved using LINGO [20] to obtain the solutions as \( (A_1', A_2', A_3', A_4', A_5', A_6', A_7', A_8', A_9', A_{10}') = (0.0, 1, 1, 1.0, 1.0, 0.0) \), and the objective values as goal \( g_1^* = 11.5 \) (age) and goal \( g_2^* = 129,000 \) (mileage).

After the first step of sieving by using the GP method, there are five types of used cars which satisfy the company’s constraints, goals and strategic policy. They are Vauxhall, Volkswagen, Daewoo, Ford, and Proton, hereafter denoted as \( A_1, A_2, \ldots, A_5 \), respectively.

4.2. The results of second step obtained from the FAHP

Utilizing the expression given in Section 3.2, we apply the modified FAHP extent analysis method to the five used cars on capital budgeting case study previously described. The degree of fuzziness will be obtained from Figs. 2 and 3a to f by using sensitivity analysis as described previously.

4.2.1. Sensitivity analysis of resultant weight values

For the comparisons between DAs with respect to individual criterion fuzzy comparison matrices (on \( C_1, C_2, C_3, C_4, C_5 \) and \( C_6 \)) their minimum workable \( \delta \) values are \( \delta_1 = 3.15, \delta_2 = 4.5, \delta_3 = 2.8, \delta_4 = 2.4 \) and \( \delta_5 = 4.5 \), respectively (see Fig. 3a–f).

When considering the final results, the domain of workable \( \delta \) is expressed as \( \delta_T = \max (\delta_6, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6) = 4.5 \) where the subscript \( T \) is the maximum of the minimum workable \( \delta \) values in the seven fuzzy comparison matrices. It follows that for this problem the workable region of \( \delta \) is \( \delta > 4.5 \) and the results on weights should possibly only be considered in the workable \( \delta \) region. The use of minimum workable degree of fuzziness is intended to exclude values of \( \delta \) at which there are no positive weights for the DAs. However the use of a workable value of \( \delta \) is not to be strictly enforced.

In Table 3, the final results show that the most preferred car is the Ford (\( A_4 \)), and then the Volkswagen (\( A_2 \)), the Daewoo (\( A_3 \)), the Proton (\( A_5 \)) and the Vauxhall (\( A_1 \)), which is the least preferred car in the DM’s mind. From the comparison between the criteria in Table 3, the most preferred criterion out of the six criteria is Safety. It means that the DMs care about safety more than other criteria.

The results in Table 3 also can be compared with Tables 1 and 2a–f. For example, the weight value of \( C_6 \) is the least of the weights in Table 3. Comparing with Table 1, apart from \( C_6 \) having extreme preferences to \( C_4, C_5 \) has no preference in relation to other criteria.
In this research, the goals are represented as numerical data. The scales used within this FAHP are verbal definitions and explanations of the integer (1–9) scale. If we use fuzzy AHP methodology then all the numerical data will be assessed into verbal assessment inputs. Ghosh and Roy [11] mentioned that AHP does not consider the relevant constraints and multiple conflicting goals. The verbal data's limitations also can be referred to Cho [9]. They also mentioned that GP combined with the AHP can prove to be a flexible tool to reach the company’s goals and constraints. This research is focused on integrating the numerical and non-numerical information. Therefore, either FAHP or GP cannot stand alone to deal with both numerical and non-numerical information. This also can be referred to Badri [3]. Badri pointed out the drawbacks of using AHP alone and proposed that AHP and GP are combined to cover the limitations. Relevant references also can be referred to Tsai [25], Ravisankar and Ravi [16].

From the academic point of view, the model provided in this research can deal with the DMs consideration of the company’s goals, constraints, strategies and with imprecise judgements simultaneously. It also provides a high tolerance for ambiguity and a well-ordered sense of priorities. From the practical point of view, this research involved a field study and collection of data from semi-structured interviews, structured interviews and a questionnaire. It displays the integration of theory and practicality. The other contributions of this paper are saving decision-making time for decision makers – by using GP, and eliciting the subjective opinions from decision makers – by using FAHP, etc.

Future research associated with the FAHP include, from the MCDM point of view, those developments with the traditional AHP. These include the appropriateness of the 9–unit scale (integer values one to nine), which within AHP is still an ongoing issue. The effect of using a different 9-unit scale within FAHP would further elucidate the sensitivity analysis issues.

5. Conclusions

The aim of this research is to investigate the application of the methods of the goal programming (GP) and the fuzzy analytic hierarchy process (FAHP) of multi-criteria decision making (MCDM), within a capital budgeting case study. In this research, GP is proposed to solve the financial information, such as the company goals, constraints, or other company’s strategy. FAHP is to deal with the imprecise judgements made by decision makers (DMs). The redefinition of the degree of fuzziness associated with the preference judgements made allows the change of imprecision (fuzziness) to be succinctly reported. The issue of imprecision is reformulated in this study, which further allows a sensitivity analysis on the preference weights evaluated to changes in the levels of imprecision.

## Appendix A

1. Information table for car selection case study

<table>
<thead>
<tr>
<th>Type</th>
<th>BMW 3 Series Touring</th>
<th>Honda Civic</th>
<th>Vauxhall Merit</th>
<th>Volkswagen Polo</th>
<th>Daewoo Lanos</th>
<th>Ford Fusion</th>
<th>Honda New Civic</th>
<th>Proton Persona</th>
<th>Toyota Celica</th>
<th>Fiat Punto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central locking</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✔</td>
<td>✔</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Electric windows</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✔</td>
<td>ṭ</td>
<td>✓</td>
<td>✓</td>
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</tr>
<tr>
<td>Power steering</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✔</td>
<td>✔</td>
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<td>✓</td>
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</tr>
<tr>
<td>Antilock braking system</td>
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<td>✓</td>
<td>✔</td>
<td>✔</td>
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Appendix A (continued)

<table>
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<tr>
<th>Type</th>
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<th>Vauxhall Merit</th>
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<th>Proton Persona</th>
<th>Toyota Celica</th>
<th>Fiat Punto</th>
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<tbody>
<tr>
<td>Impact protection system</td>
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</tr>
<tr>
<td>Image</td>
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<td>Red</td>
<td>Metallic blue</td>
<td>Red</td>
<td>Black</td>
<td>Blue</td>
<td>Silver</td>
<td>Green</td>
<td>Metallic blue</td>
<td>Black</td>
</tr>
<tr>
<td>Size of engine (cc)</td>
<td>1995</td>
<td>1995</td>
<td>1600</td>
<td>1600</td>
<td>1000</td>
<td>1400</td>
<td>1600</td>
<td>1600</td>
<td>1995</td>
<td>1600</td>
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<td>Insurance</td>
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<td>8</td>
<td>7</td>
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<td>9</td>
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<td>Price (£)</td>
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<td>4500</td>
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<td>1500</td>
<td>3500</td>
<td>2500</td>
<td>1850</td>
<td>14,000</td>
<td>5000</td>
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<td>Car age (years)</td>
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<td>1.5</td>
<td>3</td>
<td>2</td>
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<td>1.5</td>
<td>3</td>
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<tr>
<td>Car mileage</td>
<td>9500</td>
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<td>45,000</td>
<td>30,000</td>
<td>25,000</td>
<td>4000</td>
<td>35,000</td>
<td>25,000</td>
<td>28,000</td>
<td>9300</td>
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</table>

The insurance has groupings from 1-20, depending on each car, for example, a Proton Persona is in group 11. A lower insurance grouping attracts a lower insurance premium cost. The higher the grouping the more expensive the insurance cost.

References