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Abstract—This paper considers joint schedule and resource allocation in downlink cooperative OFDMA systems with multiple sources, multiple relays and a single destination. Each user has individual QoS requirements and the relays operate in amplify-and-forward and half-duplex mode. We try to find optimal power allocation, relay selection and subcarrier assignment to maximize overall system rates. The problem is formulated as a Mixed Integer Linear Programming (MILP). Although the original problem is comprehensive in nature, a novel two-level dual-primal decomposition based algorithm is proposed to tackle the problem. Optimal solutions are given in closed-form and the algorithm has a polynomial complexity in general. The efficiency of the algorithm is finally illustrated by numerical results.

I. INTRODUCTION

In the future 4th Generation (4G) wireless system, orthogonal frequency division multiple access (OFDMA) is one of the most promising solutions to provide a high-performance transmission in emerging cellular networks. OFDMA eliminates the frequency selectivity effect by transmitting the wideband signal on multiple orthogonal subcarriers as narrow-band signals. Also recently, cooperative relaying has emerged as a promising technology to achieve virtual spatial diversity in wireless communication networks [1]. Cooperative transmission by relay nodes has the potential to further improve the overall network performance, e.g., throughput increase, coverage extension, power saving, and interference mitigation. Cooperative OFDMA based broadband wireless access (BWA) networks are currently under standardization by the IEEE 802.16j task group [2].

Although the effectiveness of the new architecture has been demonstrated, analytical results have also indicated that such performance heavily depends on schedule and resource allocation, i.e., power allocation, relay selection and subcarrier allocation. Several fundamental questions need to be answered: In a cooperative OFDMA system, each user should communicate with the help of relay or not? Which relay and what relay strategy should be employed? How much power should be allocated? How to schedule the transmission and allocate subcarriers? And what the scenarios will be like when users have heterogenous QoS requirements? However, answers to these questions in a cooperative OFDMA system are generally complicated due to several reasons: (1) relay selection and subcarrier assignment are discrete in nature, (2) unlike systems without relays, the cooperative transmission rate is non-convex or even non-differential to the overall resource, and (3) the individual Quality of Service (QoS) constraints make the problem even less tractable.

For these reasons, the literature for cooperative OFDMA systems is still in a nascent state. A graph theory based schedule algorithm is proposed in [3] yet with fixed power allocation. Heuristic algorithms are discussed in [4], [5]. While enjoy a low complexity, the performance of the algorithms may not be guaranteed. Resource allocation for multi-hop cooperation is considered in [6] and [7], but they are generally not applicable for other cooperative modes. [8] focuses on a peer-to-peer relay network while no QoS requirements are incorporated.

In this paper, we first formulate a general optimization problem for schedule and resource allocation in downlink cooperative OFDMA systems. Both relay schemes of amplify-and-forward (AF) with and without diversity are considered in half-duplex mode. Each user is subject to individual QoS constraint. We try to find optimal power allocation, relay selection and subcarrier assignment to maximize overall system rates. Although the problem is comprehensive in its original form, which has nonconvex objective, integer and coupled constraints, we propose a novel two-level dual-primal decomposition based algorithm to tackle the problem. The problem is first decomposed into subproblems at each subcarrier and then we obtain optimal power allocation, relay selection and subcarrier assignment in closed-form.

The rest of this paper is organized as follows. Section II provides the cooperative OFDMA system model. Section III formulates the resource allocation problem. Section IV analyzes the optimization problem and proposes our algorithms. Section IV illustrates the efficiency of the algorithms by numerical results. Finally, we conclude the paper in Section VI.

II. SYSTEM MODEL

Consider a downlink wireless cellular network with one base station (BS), multiple relay stations (RSs) and multiple mobile stations (MSs) (Fig. 1). Each station is equipped with a single antenna. Full channel state information (CSI) is assumed to be available at the stations via channel estimation. RSs and
shown in Section III.

In the second time slot, the RSs forward signals to the MSs of RSs. We consider a TDD transmission mode while in the half-duplex operation, we assume the full-duplex operation, the orthogon al subcarriers with flat fading. The channel is also selective and OFDMA is employed to convert the channel into an effective channel. We assume the channel between stations is frequency selective and OFDMA is employed to convert the channel into orthogonal subcarriers with flat fading. The channel is also assumed to be static during the resource allocation.

Since there are many limitations in radio implementation for full-duplex operation, we assume the half-duplex operation of RSs. We consider a TDD transmission mode while in the first time slot, the BS transmits with RSs and MSs receive. In the second time slot, the RSs forward signals to the MSs by employing Amplify-and-Forward (AF) relay schemes as shown in Section III.

III. PROBLEM FORMULATION

Suppose there are one base station, M mobile stations denoted \( S = \{s_1, \ldots, s_M\} \) and K relay stations forming the set \( R = \{r_1, \ldots, r_k, \ldots, r_K\} \). All stations share a total number of N subcarriers in the cell denoted \( N = \{n_1, \ldots, n_N\} \). For the nth subcarrier, the channel coefficients between BS and mth MS, BS and kth RS, kth RS and mth MS are denoted by \( h_{d,m}^n \), \( h_{a,k}^n \), \( h_{b,km}^n \), respectively (Fig. 1).

A. Achievable Rate

The base station can communicate with mobile stations either in cooperative mode or non-cooperative mode. For the cooperative mode, both AF with and without diversity schemes are considered [1]. In the first time slot, BS sends data with power \( P_{r,km}^n \) on the nth subcarrier to \( r_k \). In the second time slot, \( r_k \) forward the received data to the mth user on the same subcarrier with power \( P_{s,k}^n \). The noise variances at RS and MS within one OFDM subchannel are denoted by \( \sigma_r^2 \) and \( \sigma_m^2 \), respectively.

1) AF with diversity: For AF with diversity scheme, signals of both time slots are combined with maximum-ratio combiner. The signal to noise ratio (SNR) at the destination for the relay pair \((k, m)\) on the nth subcarrier is given by

\[
\text{SNR}_{km,AFD}^n = \frac{P_{s,k}^n a_{k}^n \cdot P_{r,km}^n b_{km}^n}{1 + P_{s,k}^n a_{k}^n + P_{r,km}^n b_{km}^n}
\]

(1)

where \( a_{k}^n = |h_{a,k}^n|^2/\sigma_r^2 \), \( b_{km}^n = |h_{b,km}^n|^2/\sigma_m^2 \), and \( d_{km}^n = |h_{d,m}^n|^2/\sigma_m^2 \). The instantaneous rate of relay pair \((k, m)\) on the nth subcarrier for AF with diversity scheme is therefore given by

\[
c_{km,AFD}^n = \frac{1}{2} \log(1 + \text{SNR}_{km,AFD}^n)
\]

(2)

2) AF without diversity: When AF without diversity protocol is adopted, the destination only receives the signal from the relay in the second time slot, and the transmission rate can be similarly derived as

\[
c_{km,AF}^n = \frac{1}{2} \log(1 + \text{SNR}_{km,AF}^n)
\]

(3)

3) Direct Transmission: When the BS works in non-cooperative mode, it transmits directly to the mobile station over two time slots. Assume the BS transmits with power \( P_{s,0m}^n \) to \( s_m \) on subcarrier \( n \), the instantaneous rate can be written as

\[
c_{0m}^n = \log(1 + P_{s,0m}^n d_{km}^n)
\]

(4)

The schedule of transmission can be represented by binary assignment variables \( x_{km}^n \), with \( x_{km}^n = 1 \) indicating that the base communicates with \( s_m \) with the help of relay \( r_k \), utilizing subcarrier \( n \) and \( x_{km}^n = 0 \) otherwise. For mobile station \( m \), it works in non-cooperative mode on subcarrier \( n \) if \( x_{km}^n = 1 \), \( k = 0 \) and in cooperation with relay station \( k \) if \( x_{km}^n = 1 \), \( k = 1, 2, \ldots, K \). Then the aggregate system rate can be expressed in a compact form as

\[
C_{\text{sum}} = \sum_{k=0}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} c_{km}^n x_{km}^n
\]

(5)

B. QoS Constraint

QoS guarantees play an important role in the future mobile wireless networks. It is a challenging task to meet the diverse QoS requirements imposed by current and envisioned services. With various practical applications, QoS requirements can be modeled in many ways including e.g., minimum transmission rates, maximum tolerable error rates, maximum delay bounds. In this paper, we try to maximize system aggregate rates...
subject to individual minimum rate constraints, thus the QoS constraint can be described for each \( s_m \) as

\[
\sum_{k=0}^{K} \sum_{n=1}^{N} c_{km}^{n} x_{km}^{n} \geq \tau_m, \quad \forall m
\]  

(6)

where \( \tau_m \) is the minimum rate requirement for user \( m \).

Since the cooperative transmission rate is not joint convex in \( P_s \) and \( P_r \), we assume a pre-determined \( P_s \) and the relays have a sum power constraint

\[
\sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} P_{r,km}^{n} \leq \mathcal{P}.
\]  

(7)

Each MS can utilize multiple subcarriers to transmit, either cooperative or non-cooperative. However, each subcarrier cannot be shared by different MSs which means

\[
\sum_{k=0}^{K} \sum_{m=1}^{M} x_{km}^{n} \leq 1, \quad \forall n.
\]  

(8)

Therefore, the optimization problem for joint schedule and resource allocation in downlink cooperative OFDMA networks can be formulated as follows.

\[
\text{maximize} \quad C_{\text{sum}} = \sum_{k=0}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} c_{km}^{n} x_{km}^{n}
\]

subject to

\[
\sum_{k=0}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} c_{km}^{n} x_{km}^{n} \geq \tau_m, \quad \forall m
\]

\[
\sum_{k=0}^{K} \sum_{m=1}^{M} x_{km}^{n} \leq 1, \quad \forall n
\]

\[
x_{km}^{n} \in \{0, 1\}, \quad \forall k, m, n
\]

\[
\sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} P_{r,km}^{n} \leq \mathcal{P}
\]

\[
P_{r,km}^{n} \geq 0, \quad \forall k, m, n
\]

\( X \) is a \((K+1) \times M \times N \) matrix with elements \( x_{km}^{n} \). It should be noted that we can answer three questions simultaneously if optimal \( X \) is obtained: Cooperate or not? Cooperate with which relay? Transmit on which subcarrier? Meanwhile, optimal power control is realized by optimizing \( P_r \). Unfortunately, the optimization problem is a Mixed Binary Integer Programming (MBIP) problem which is hard to tackle. Since binary variable \( X \) and continuous variable \( P_r \) coexist, even an exhaustive search of the problem space is not applicable. The coupled QoS constraints make the problem even harder to solve. However, in the following section, we will propose a two-level primal-dual decomposition method and obtain optimal algorithm to solve (9) which can be proven to be optimal and efficient.

IV. JOINT OPTIMAL SCHEDULE AND RESOURCE ALLOCATION ALGORITHM

We first derive the Lagrangian function as follows

\[
L(X, P_r, \lambda, \mu) = \sum_{k=0}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} c_{km}^{n} x_{km}^{n} + \sum_{m=1}^{M} \lambda_m \left( \sum_{k=0}^{K} \sum_{n=1}^{N} c_{km}^{n} x_{km}^{n} - \tau_m \right)
\]

\[+ \mu \left( \mathcal{P} - \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} P_{r,km}^{n} \right) \]

\[= \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=0}^{K} c_{km}^{n} x_{km}^{n} + \sum_{m=1}^{M} \lambda_m \sum_{k=0}^{K} c_{km}^{n} x_{km}^{n} \]

\[- \mu \sum_{k=1}^{K} \sum_{m=1}^{M} P_{r,km}^{n} \right] - \sum_{m=1}^{M} \lambda_m \tau_m + \mu \mathcal{P}
\]  

(10)

where \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_M]^T \) is the vector of dual variables for the QoS constraints and \( \mu \) is single dual variable for sum power constraint. The Lagrangian dual function can therefore be obtained [11]

\[
g(\lambda, \mu) = \max_{X, P_r} L(X, P_r, \lambda, \mu)
\]

s.t.

\[
\sum_{k=0}^{K} \sum_{m=1}^{M} x_{km}^{n} \leq 1, \quad \forall n
\]

\[
x_{km}^{n} \in \{0, 1\}, \quad P_{r,km}^{n} \geq 0
\]

and dual problem is

\[
\min_{\lambda, \mu \geq 0} g(\lambda, \mu)
\]  

(12)

There are several things to be noted here. First, the duality gap is generally not zero due to the integer constraints. However, when time sharing condition is satisfied, it can be proved that the duality gap becomes asymptotically zero as \( N \) goes to infinity [9]. A rigorous investigation is available in [10]. Since time sharing condition is readily satisfied in our case, it follows that the duality gap for (12) is zero. Second, we have removed the coupling between subcarriers via lagrangian relaxation and \( g(\lambda, \mu) \) can be decomposed into \( N \) subproblem at each subcarrier which can be independently solved given \( \lambda, \mu \). The subproblem at subcarrier \( n \) is

\[
\text{maximize} \quad L_n(X^n, P_r^n) = \sum_{k=0}^{K} \sum_{m=1}^{M} c_{km}^{n} x_{km}^{n} + \sum_{m=1}^{M} \lambda_m \sum_{k=0}^{K} c_{km}^{n} x_{km}^{n} - \mu \sum_{k=0}^{K} \sum_{m=1}^{M} P_{r,km}^{n}
\]

subject to

\[
\sum_{k=0}^{K} \sum_{m=1}^{M} x_{km}^{n} \leq 1
\]

\[
x_{km}^{n} \in \{0, 1\}, \quad P_{r,km}^{n} \geq 0, \quad \forall k, m
\]

where \( X^n, P_r^n \) are vectors of \( x_{mk}^{n}, P_{r,km}^{n} \) on subcarrier \( n \). In order to get close form solution, we then try to solve the subproblem through a second level primal decomposition.
Note that the variables \( P_r^n \) and \( X^n \) are only coupled at the objective, we can first fix schedule variables \( X^n \) and have optimization over \( P_r^n \)

\[
f(X^n) = \begin{cases} 
\max_{P_r^n} & \sum_{k=0}^{K} \sum_{m=1}^{M} c_{km} x_{km} + \sum_{m=1}^{M} \lambda_m \sum_{k=0}^{K} c_{km} x_{km} \\
\quad \text{s.t.} & P_{r,km}^n \geq 0, \forall k, m
\end{cases}
\]

and then solve the master problem over \( X^n \)

\[
\text{maximize} \quad f(X^n) \\
\text{subject to} \quad \sum_{k=0}^{M} P^n_{r,km} x_{km} \leq 1, \quad x_{km} \in \{0, 1\}
\]

In general, the master problem can be solved by exhaustive search over feasible space of \( (X^n)^d \). However, observing the specific structure of the subproblem, we can obtain close-form solution for the primal problem. To do so, we first introduce the following lemma.

**Lemma 1:** The optimization problem (14) and (15) are equivalent\(^1\) to the following problem

\[
a_{km}^n = \begin{cases} 1, & (k, m) = (k^*, m^*) = \arg \max_{k,m} A(k, m) \\
0, & \text{otherwise}
\end{cases}
\]

where \( A \) is a matrix with elements \( A(k, m) \) and

\[
A(k, m) = \left\{ \begin{array}{lr} \max_{P_r^n} c_{km} + \lambda_m c_{km} - \mu P_{r,km}^n \\
\quad \text{s.t.} \quad P_{r,km}^n \geq 0, \forall k, m
\end{array} \right.
\]

**Proof:** By visiting the constraints in (15), we first note that \( X^n \) is a all-zero matrix except for one non-zero entry. Therefore, \( f(X^n) \) is equivalent to \( A(k, m) \) when transmission pair \((k, m)\) utilize the subcarrier. As a result, the optimization of problem (15) over feasible space of \( X^n \) can be cast as finding optimal \((k, m)\) that with maximal element in \( A \). This is the optimization in (16) and that concludes our proof. \( \blacksquare \)

Therefore, the difficulty of problem has been reduced to the optimization of (17). Since the rates are concave in relay powers, we can find closed-form solution by substituting (2), (3) into (17) and taking derivative with respect to \( P_{r,km}^n \). After some manipulations, the optimal power allocation for both with and without diversity can be expressed in a compact form as follows

\[
P_{r,km}^n = \left[ \frac{(\kappa_{km}^n)^2 + \frac{4\mu x_{km}^n}{\mu} (1 + \kappa_{km}^n)(1 + \lambda_m) - \kappa_{km}^n - 2}{2(\alpha_{km}^n + \beta_{km}^n)} \right]^{+}
\]

\(^{1}\)Technically, the equivalence of two optimization problems is quite complicated. A simple notation is applied here which means the two problem have the same solutions.

**TABLE I**

<table>
<thead>
<tr>
<th>JOINT OPTIMAL SCHEDULE AND RESOURCE ALLOCATION (JOSRA) ALGORITHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initialization: Initialize ( \lambda(0), \mu(0) )</td>
</tr>
<tr>
<td>2. Solve subproblem (13) at each subcarrier as follows</td>
</tr>
<tr>
<td>1) Calculate optimal ( P_{r,km}^n ) according to (18).</td>
</tr>
<tr>
<td>2) Obtain ( A(k, m) ) with optimal ( P_{r,km}^n ).</td>
</tr>
<tr>
<td>3) Solve optimal ( x_{km}^n ) via (16).</td>
</tr>
<tr>
<td>4. Updates dual variables ( \lambda, \mu ) according to (22).</td>
</tr>
<tr>
<td>4. Termination: Repeat 2,3 until convergence.</td>
</tr>
</tbody>
</table>

where

\[
\kappa_{km}^n = \begin{cases} 1 + \frac{P_{n,k}^n}{\alpha_{km}^n}, & \text{AF with diversity} \\
1 + P_{n,k}^n, & \text{AF without diversity}
\end{cases}
\]

\[
\alpha_{km}^n = k_{km}^n \beta_{km}^n
\]

\[
\beta_{km}^n = \frac{b_{km}^n}{1 + P_{n,k}^n}
\]

Finally, The dual problem (12) can be solved by subgradient method [11]. Dual variables \( \lambda, \mu \) are updated in parallel as follows

\[
\lambda_m(t + 1) = \left[ \lambda_m(t) + \alpha(t) \left( \sum_{k=0}^{K} \sum_{m=1}^{M} c_{km} x_{km}^n(t) - \tau_m \right) \right]^{+}
\]

\[
\mu(t + 1) = \left[ \mu(t) + \beta(t) \left( P - \sum_{k=0}^{K} \sum_{m=1}^{M} \sum_{n=1}^{N} P_{r,km}^n(t) \right) \right]^{+}
\]

where \( \alpha(t) \) and \( \beta(t) \) are proper step sizes. The above updated is guaranteed to converge to the optimal dual variable if the step sizes are chosen following a diminishing step size rule [12]. Since our problem has zero duality gap as mentioned before, the optimal primal variables can be obtained accordingly. The algorithm is finally presented in Table I.

**A. Complexity Analysis and Implementation Issues**

We now analyze the complexity of our algorithm. For all constraints, \( M(K + 1) \) computations are needed to calculate \( A \) and \( N \) assignments for all subcarriers. Therefore, the complexity for one iteration is \( O(MKN) \). Since the iteration number increases linearly with the dual variables and there are \( (M + 1) \) dual variables, the overall complexity is \( O(M^2KN) \).

With channel state information through feedback, which is the major overhead, the base station can easily perform the optimization. The polynomial complexity of the algorithm also greatly facilitates the implementation.

**V. NumerICAL RESULTS**

This section provides performance of our proposed algorithm by means of Monte-Carlo simulations. We consider an
OFDMA based cooperative cellular network with a single cell, $M$ mobile stations and $K$ relay stations sharing $N$ subcarriers. The base station located at the center of the cell with a radius of $r_1 = 300m$. Relay stations are placed on a circle with a radius of $r_2 = 150m$ at equal angular distance. MSs are randomly distributed in the cell coverage area with a radius between $r_1$ and $r_2$. Power level is determined to ensure SNR ratio $\rho_0 = \frac{\sigma^2}{\sigma^2(1+\alpha)} = \frac{\rho}{\sigma^2} \frac{L}{N}$ from cellular boundary to the base station. The channel is generated with number of channel taps $L = 4$, path loss exponent $\alpha = 3$ and noise ratio $\sigma^2 = \sigma^2_f$.

In Fig. 2, the average rates for two AF schemes with respect to number of subcarriers are obtained. With the increase of subcarriers, spectrum efficiency of all scenarios are improved. Since direct transmission is within the selection pool, our algorithm guarantee to outperform no relay cases. Moreover, we can observe that little rate is sacrificed to guarantee QoS performance which facilitate our implementations.

In Fig. 3, QoS guarantee results are shown in detail. We consider a four users case where the first two users have higher QoS requirements $\tau_m = 12$ and the other two need $\tau_m = 6$ as illustrated with dash line in the figure. Four schemes are plotted and we can observe that for scenarios without QoS constraints, the rates vary a lot between different users and sometimes users may achieve no rates (i.e., user 2) which means no fairness is considered in the system. With QoS constraints, however, the rates are reallocated and users not only have fair transmission but also meet their heterogenous requirements as well.

VI. CONCLUSION

In this paper we consider schedule and resource allocation in OFDMA based cooperative cellular networks with multiple sources, multiple relays and a single destination. We formulate the power allocation relay selection and subcarrier assignment problem subject to individual QoS constraints as a binary integer programming problem. We propose a two level dual-primal decomposition based algorithm to solve the problem. The efficiency of the algorithm is finally illustrated by numerical results.

REFERENCES


