Enhanced Russell measure in fuzzy DEA

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Abstract: The radial measures of classical DEA models (CCR, BCC) are incomplete, they are only separate measures of input and output efficiency and their efficiency index omit the non-zero input and output slacks. Enhanced Russell graph measure (ERM) eliminates these deficiencies. All of the existing fuzzy DEA models are extension of CCR or BCC model, efficiencies of DMUs, ultimately, are solution of CCR or BCC model. Based on ERM model, a fuzzy DEA model is proposed to deal with the efficiency evaluation problem with the given fuzzy input and output data, by using a ranking method based on the comparison of $\alpha$ -cuts. The proposed framework is illustrated through an application to performance assessment of flexible manufacturing system and comparative results are presented. The efficiency measure of the proposed approach is relatively more reasonable than those of fuzzy DEA models based on CCR or BCC model and represents some real-life processes more appropriately.

Keywords: non-radial; DEA; data envelopment analysis; fuzzy; FMS; flexible manufacturing system.


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1 Introduction

Data envelopment analysis (DEA) is a non-parametric method for evaluating the relative efficiency of decision-making units (DMUs) on the basis of multiple inputs and outputs. Traditional DEA models such as CCR model (Charnes et al., 1978), BCC model (Banker et al., 1984) do not deal with imprecise data and assume that all inputs and outputs are exactly known; a key to the success of the DEA approach is the accurate measure of all factors, including inputs and outputs. In real-world situation, however, input and output data of DMUs evaluated often are not exactly known but can be characterised by fuzzy number which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved.

Several approaches have been proposed to deal with fuzzy data in the framework of DEA (Wang and Liang, 2009). Kao and Liu (2000) suggested transforming fuzzy data into interval data by applying the $\alpha$-cuts so that a family of conventional crisp DEA models could be utilised, but this approach is not efficient from a computational point of view. Lertworasirikul et al. (2003a,b) proposed a possibility approach in which fuzzy constraints were treated as fuzzy events and fuzzy DEA model was transformed into possibility DEA model by using possibility measures on fuzzy events. This approach is not a general way to solve all fuzzy DEA models – using possibility approach for solving the fuzzy BCC model may result in an unbounded optimal value for DMU, and cannot be adapted for other models. Guo and Tanaka (2001) proposed a fuzzy CCR model in which fuzzy constraints including fuzzy equalities and fuzzy inequalities were all converted into crisp constraints by predefining a possibility level and using the comparison rule for fuzzy numbers. Their method is excellent, but it is not a general approach, because their model has an optimal solution under a specific restrictive condition. In a similar manner, Leon et al. (2003) also proposed some fuzzy BCC models, but exploiting the use of the envelopment formulation of the BCC model instead of the dual multiplier one. Their fuzzy DEA models take the form of fuzzy linear programmes which require ranking of fuzzy sets, but ranking of fuzzy sets may lead to a problem of using different ranking methods can result in different results. Both the approach based on the comparison rule for fuzzy numbers and the approach transforming fuzzy data into interval data by applying the $\alpha$-cuts are commonly used approaches in real application.

All of the above fuzzy DEA models are extension of CCR or BCC model, efficiencies of DMUs, ultimately, are solution of CCR or BCC model. But the measures of these two models are incomplete, they are only separate measures of input and output efficiencies which have resulted from the radial measure depending on whether an ‘output-oriented’ or ‘input-oriented’ approach is used; and that their efficiency index omit the non-zero input and output slacks and, thus, fail to account for all inefficiencies that the model can identify. The Russell graph efficiency measure (Fare et al., 1985) eliminates these deficiencies but its programme is difficult to solve. Enhanced Russell graph efficiency measure (ERM) (Pastor and Ruiz, 1999) utilises a ratio measure in place of the weighted average of arithmetic and harmonic means of Russell graph efficiency measure, and it can be transformed into an ordinary linear programme that generate an optimal solution for the corresponding ERM model. The ERM model jointly determines input and output efficiencies that represent separate estimates of input and output efficiency, and unify the ratio efficiency and slacks into a scalar measure. This paper develops a new non-radial DEA model with fuzzy input–output data by using a method based on the comparison of
A flexible manufacturing system (FMS) is designed to combine the efficiency of a mass-production line and the flexibility of a job shop to produce work pieces on a group of machines (Karsak, 2008; Karsak and Kuzgunkaya, 2002; Karsak and Tolga, 2001). Recently, the use of DEA has been recommended as a discrete alternative multiple criteria tool for evaluation of manufacturing technologies and FMS (Karsak, 2008; Karsak and Kuzgunkaya, 2002; Karsak and Tolga, 2001). But a robust DEA procedure used for FMS selection should be able to incorporate quantitative as well as qualitative data. An efficient way to express factors such as work in process (wip) level, flexibility and quality of the products, which can neither be assessed by crisp values nor random processes, is using linguistic variables or fuzzy numbers. The proposed fuzzy non-radial DEA model is applied to performance assessment of FMS, it enables the decision makers to deal with imprecision inherent in the expression of each criteria by translating vague data into numerical ones.

The rest of this paper is organised as follows: Section 2 briefly introduces the original ERM model. Section 3 contains some results of fuzzy analysis based on the comparison of $\alpha$-cuts that will be used, then, develops a fuzzy non-radial DEA model. Section 4 illustrates our proposed fuzzy DEA method with an application to FMS. Finally, Section 5 concludes this paper.

2 Enhanced Russell graph efficiency measure

ERM (Pastor and Ruiz, 1999) represents in the following model:

$$
\begin{align*}
\text{Min } R_e(X_0,Y_0) &= \frac{(1/m)\sum_{i=1}^{m} \theta_i}{(1/s)\sum_{r=1}^{s} \phi_r} \\
\text{s.t.} & \\
\sum_{j=1}^{n} \lambda_j x_{ij} &\leq \theta_i X_{i0}, \quad i = 1, \ldots, m \\
\sum_{j=1}^{n} \lambda_j y_{rj} &\geq \phi_r Y_{r0}, \quad r = 1, \ldots, s \\
\theta_i &\leq 1 \quad \forall i \\
\phi_r &\geq 1 \quad \forall r \\
\lambda_j &\geq 0, \quad j = 1, \ldots, n
\end{align*}
$$

The objective in (1) jointly minimises the input and output efficiencies, the denominator is maximised to give value $\phi_r$ for any choice of $\theta_i$ in the numerator, simultaneously, the numerator is minimised to give value $\theta_i$ for any choice of $\phi_r$ in the denominator, the numerator and denominator are jointly optimised to achieve a minimum value for their ratio, it can be interpreted as the ratio between the average efficiency of inputs and the average efficiency of outputs (Cooper and Huang, 2007). On the other hand, by means of...
the following change of variables: \( \theta_i = 1 - \frac{s_{i0}}{x_{i0}}, \quad i = 1, \ldots, m \) and \( \phi_r = 1 + \frac{s_{r0}}{y_{r0}}, \quad r = 1, \ldots, s \),
it is easy to interpret formulation of \( R_e \) in terms of total slacks, this provides an
alternative expression of the ERM, the ultimate model is slacks-based measure (SBM)
(for further details about SBM, please see Tone, 2001).

The following is true for \( R_e \) (Pastor and Ruiz, 1999):

1. \( 0 < R_e \leq 1 \)
2. \( R_e = 1 \Leftrightarrow \text{DMU}_0 \text{ being evaluated is Koopmans-efficient} \)
3. \( R_e \leq \theta \), where \( \theta \) is the corresponding radial efficiency measure.

Model (1) can be transformed into a linear programme using the Charnes–Cooper
transformation (Charnes and Cooper, 1962) in the similar way as the CCR model. Let us
multiply a scalar variable \( \beta (>0) \) to both the denominator and the numerator of (1). This
causes no change in \( R_e \). We adjust \( \beta \) so that the denominator becomes 1. Then this term
is moved to constraints. The objective is to minimise the numerator. Let
\( u_i = \beta \theta_i, v_r = \beta \phi_r, \quad t_j = \beta \alpha_j \) then we have:

\[
\text{Min } \frac{\sum_{i=1}^{m} u_i}{m} \\
\text{s.t.}
\sum_{r=1}^{S} v_r = s \\
\sum_{j=1}^{n} x_{ij} t_j \leq u_i x_{i0}, \quad i = 1, \ldots, m \\
\sum_{j=1}^{n} y_{rj} t_j \geq v_r y_{r0}, \quad r = 1, \ldots, s \\
u_i \leq \beta, \quad i = 1, \ldots, m \\
\beta \leq v_r, \quad r = 1, \ldots, s \\
0 \leq t_j, \quad j = 1, \ldots, n \\
0 \leq \beta \leq 1
\]

3 Non-radial fuzzy DEA model and its solution

3.1 Non-radial fuzzy DEA model

Suppose that we are interested in evaluating the relative efficiency of \( n \) DMUs which use
\( m \) inputs to produce \( s \) outputs, and the data of inputs and outputs cannot be precisely
measured but can be expressed as fuzzy numbers. Let us assume that model (2) is used to
evaluate the relative efficiency of this set of DMUs. Then, the extended model of model
(2) can be expressed as the following fuzzy LP problem.
In model (3), \( \succ \) is defined as \( \succ^h \), where \( 0 \leq h \leq 1 \) is a pre-defined possibility level by decision maker. The relationship between efficiency score of a DMU and possibility level \( h \) in model (3) is:

**Proposition 1:** Efficiency score of a DMU is a decreasing function of the possibility level \( h \).

This conclusion is obvious. If a feasible solution be a optimal solution of model (3) at \( h = h_0 \), it must be a feasible solution of model (3) for \( \forall h \geq h_0 \), so, \( \theta_{h_0} \geq \theta_h \), where \( h, h_0 \in [0, 1] \).

### 3.2 The solution to the non-radial fuzzy DEA model

To get the solution of model (3), we are recalling how to perform the basic operations of arithmetic and the comparison of fuzzy intervals for ranking purpose.

#### 3.2.1 Comparison of fuzzy intervals based on \( \alpha \)-cuts

To be more precise, we deal with LR-fuzzy numbers whose definition is as follows (Wang and Liang, 2009):

**Definition 1:** Suppose that \( \tilde{M} \) is a fuzzy number, \( \mu_M(r) \) is its membership function. Let \( f : [0, +\infty] \to [0, 1] \) be a mapping and satisfy following conditions:

1. strictly decreasing in \( \text{supp}(\tilde{M}) = \{r : \mu_M(r) > 0\} \)
2. upper semi-continuous
3. \( f(0) = 1 \).

\( f \) is said to be a reference function of fuzzy number \( \tilde{M} \).
Definition 2: A fuzzy number $\hat{M}$ is said to be a LR-fuzzy number, if its membership function has the following form:

$$
\mu_{\hat{M}}(r) = \begin{cases} 
L \left( \frac{m^L - r}{\alpha^L} \right), & r \leq m^L \\
1, & m^L \leq r \leq m^R \\
R \left( \frac{r - m^R}{\alpha^R} \right), & r \geq m^R 
\end{cases}
$$

where $L$ and $R$ are reference function of fuzzy number $\hat{M}$. LR-fuzzy number $\hat{M}$ is denoted by $\hat{M} = (m^L, m^R, \alpha^L, \alpha^R)_{L,R}$.

For a given set of LR-fuzzy numbers

$$
\hat{M}_j = (m^L_j, m^R_j, \alpha^L_j, \alpha^R_j)_{L,R}, \quad x_j \in R, \quad j = 1, \ldots, n
$$

and some scalars $x_j \geq 0$, $j = 1, \ldots, n$, we have that

$$
\sum_{j=1}^{n} \hat{M}_j x_j = \left( \sum_{j=1}^{n} m^L_j x_j, \sum_{j=1}^{n} m^R_j x_j, \sum_{j=1}^{n} \alpha^L_j x_j, \sum_{j=1}^{n} \alpha^R_j x_j \right)_{L,R} \quad (4)
$$

We have following proposition with respect to comparison of fuzzy number.

Theorem 1: (Ramik and Rimanek, 1985) Let $\hat{M}, \hat{N}$ be two fuzzy numbers, and $h$ be a real number, $h \in [0,1]$. Then, $\hat{M} \leq h \hat{N}$ if and only if $\forall k \in [h,1]$ the two statements below hold:

$$
\inf \{ s : \mu_{\hat{M}}(s) \geq k \} \geq \inf \{ t : \mu_{\hat{N}}(t) \geq k \}, \quad \sup \{ s : \mu_{\hat{M}}(s) \geq k \} \geq \sup \{ t : \mu_{\hat{N}}(t) \geq k \}
$$

Corollary 1: Let $\hat{M}, \hat{N}$ be two LR-fuzzy number, $\hat{M} = (m^L, m^R, \alpha^L, \alpha^R)_{L,R}$

$$
\hat{N} = (n^L, n^R, \beta^L, \beta^R)_{L,R}, \quad \text{then}
$$

$$
\hat{M} \leq h \hat{N} \iff m^L - L^*(k) \alpha^L \geq n^L - L^*(k) \beta^L \quad \forall k \in [h,1] \\
m^R + R^*(k) \alpha^R \geq n^R + R^*(k) \beta^L \quad \forall k \in [h,1]
$$

where $L^*(k) = \sup \{ z : L(z) \geq k \}$, $L^*(k) = \sup \{ z : L'(z) \geq k \}$, $R^*(k) = \sup \{ z : R(z) \geq k \}$, $R^*(k) = \sup \{ z : R'(z) \geq k \}$

Corollary 2: Let $\hat{M}, \hat{N}$ be two LR-fuzzy number, $\hat{M} = (m^L, m^R, \alpha^L, \alpha^R)_{L,R}$

$$
\hat{N} = (n^L, n^R, \beta^L, \beta^R)_{L,R}, \quad \text{have bounded support, } L = L' \quad \text{and } R = R', \quad \text{then } \hat{M} \leq h \hat{N} \quad \text{if, and only if}
$$

$$
\begin{align*}
m^L &\geq n^L \quad m^L - L^*(h) \alpha^L \geq n^L - L^*(h) \beta^L \\
m^R &\geq n^R \quad m^R + R^*(h) \alpha^R \geq n^R + R^*(h) \beta^R
\end{align*}
$$

(5)
3.2.2 Transformation of model (3)

Suppose that the data of inputs and outputs can be expressed as LR-fuzzy numbers with bounded support

\[
\bar{x}_{ij} = \left( x_{ij}^L, x_{ij}^R, \alpha_{ij}, \alpha_{ij}^R \right)_{L_i, R_i}, \quad i = 1, \ldots, m, \ j = 1, \ldots, n
\]

\[
\bar{y}_{rj} = \left( y_{rj}^L, y_{rj}^R, \beta_{rj}, \beta_{rj}^R \right)_{L_r, R_r}, \quad r = 1, \ldots, m, \ j = 1, \ldots, n
\]

Satisfying

\[
L_{i1} = \cdots = L_{in} = L_i, \quad i = 1, \ldots, m, \quad L_{i1}^L = \cdots = L_{in}^L = L_i^L, \quad r = 1, \ldots, s
\]

\[
R_{i1} = \cdots = R_{in} = R_i, \quad i = 1, \ldots, m \quad R_{i1}^L = \cdots = R_{in}^L = R_i^L, \quad r = 1, \ldots, s
\]

that is, requiring that for all inputs and outputs, the corresponding n data can be described by means of LR-fuzzy numbers of the same type.

By using (4) and (5), model (3) can be transformed to be:

\[
\text{Min} \quad \sum_{i=1}^{m} \frac{u_i}{m}
\]

s.t.

\[
\sum_{r=1}^{s} v_r = s
\]

\[
\sum_{j=1}^{n} t_j x_{ij}^L \leq u_i x_{i0}^L, \quad i = 1, \ldots, m
\]

\[
\sum_{j=1}^{n} t_j x_{ij}^R \leq u_i x_{i0}^R, \quad i = 1, \ldots, m
\]

\[
\sum_{j=1}^{n} t_j x_{ij}^L - L_i^L (h) \sum_{j=1}^{n} t_j \alpha_{ij}^L \leq u_i x_{i0}^L - L_i^L (h) u_i \alpha_{i0}^L, \quad i = 1, \ldots, m
\]

\[
\sum_{j=1}^{n} t_j x_{ij}^R + R_i^R (h) \sum_{j=1}^{n} t_j \alpha_{ij}^R \leq u_i x_{i0}^R + R_i^R (h) u_i \alpha_{i0}^R, \quad i = 1, \ldots, m
\]

\[
\sum_{j=1}^{n} t_j y_{rj}^L \geq v_r y_{r0}^L, \quad r = 1, \ldots, s
\]

\[
\sum_{j=1}^{n} t_j y_{rj}^R \geq v_r y_{r0}^R, \quad r = 1, \ldots, s
\]

\[
\sum_{j=1}^{n} t_j y_{rj}^L - L_r^L (h) \sum_{j=1}^{n} t_j \beta_{rj}^L \geq v_r y_{r0}^L - L_r^L (h) v_r \beta_{r0}^L, \quad r = 1, \ldots, s
\]

\[
\sum_{j=1}^{n} t_j y_{rj}^R + R_r^R (h) \sum_{j=1}^{n} t_j \beta_{rj}^R \geq v_r y_{r0}^R + R_r^R (h) v_r \beta_{r0}^R, \quad r = 1, \ldots, s
\]
In practice, triangular fuzzy numbers (a special case of LR-fuzzy number) are very often used to model a wide variety of situations, they appear as useful means of quantifying the uncertainty in decision making due to their intuitive appear and computational-efficient representation (Wang and Liang, 2009). For instance, assuming that an expert’s estimate about a certain variable is ‘around 5’, it can be represented by a symmetrical triangular fuzzy number as $(5, 2)$, where 5 is the centre and 2 is the spread (the centre of a symmetrical triangular fuzzy number represents the most general case and the spread reflects some possibilities). If inputs and outputs are now assumed to be symmetrical triangular fuzzy numbers (a special case of LR-fuzzy number) denoted by the pairs consisting of the corresponding centres and spreads,

$$
\tilde{x}_{ij} = (x_{ij}, \alpha_{ij}), \quad i = 1, \ldots, m, \quad j = 1, \ldots, n; \quad \tilde{y}_{ij} = (y_{ij}, \beta_{ij}), \quad i = 1, \ldots, m, \quad j = 1, \ldots, n.
$$

In this special situation,

$$
x_{ij}^L = x_{ij}^R = x_{ij}, \quad y_{ij}^L = y_{ij}^R = y_{ij}; \quad \alpha_{ij}^L = \alpha_{ij}^R = \alpha_{ij}, \quad \beta_{ij}^L = \beta_{ij}^R = \beta_{ij}
$$

and

$$
L^*_r(h) = L^*_r(h) = R^*_r(h) = R^*_r(h) = 1 - h, \quad 0 \leq h \leq 1; \quad i = 1, \ldots, m, \quad r = 1, \ldots, s, \quad j = 1, \ldots, n
$$

Model (6.1) can be simplified, it becomes (6.2)

$$
\text{Min} \sum_{i=1}^{m} \frac{u_i}{m}
$$

s.t.

$$
\sum_{r=1}^{s} v_r = s
$$

$$
\sum_{j=1}^{n} t_j x_{ij} \leq u_i x_{i0}, \quad i = 1, \ldots, m
$$

$$
\sum_{j=1}^{n} t_j x_{ij} - (1 - h) \sum_{j=1}^{n} t_j \alpha_{ij} \leq u_i x_{i0} - (1 - h) u_i \alpha_{i0}, \quad i = 1, \ldots, m
$$

$$
\sum_{j=1}^{n} t_j x_{ij} + (1 - h) \sum_{j=1}^{n} t_j \alpha_{ij} \leq u_i x_{i0} + (1 - h) u_i \alpha_{i0}, \quad i = 1, \ldots, m
$$

$$
\sum_{j=1}^{n} t_j y_{ij} \geq v_r y_{r0}, \quad r = 1, \ldots, s
$$
\[ \sum_{j=1}^{n} t_j y_{rj} - (1-h) \sum_{j=1}^{n} t_j \beta_{ij} \geq \nu_r y_{r0} - (1-h)\nu_r \beta_{r0}, \quad r = 1, \ldots, s \]
\[ \sum_{j=1}^{n} t_j y_{rj} + (1-h) \sum_{j=1}^{n} t_j \beta_{ij} \geq \nu_r y_{r0} + (1-h)\nu_r \beta_{r0}, \quad r = 1, \ldots, s \]  
(6.2)

\[ u_i \leq \beta, \quad i = 1, \ldots, m \]
\[ \beta \leq v_r, \quad r = 1, \ldots, s \]
\[ 0 \leq t_j, \quad j = 1, \ldots, n \]
\[ 0 \leq \beta \leq 1 \]

Note that in model (6.1), if we add constraint (4) by constraint (3), we can get constraint (2); if we add constraint (7) by constraint (6), we can get constraint (5). Constraint (2) and constraint (5) can be eliminated since they are redundant. Then, model (6.1) can be further simplified to be

\[ \text{Min } \sum_{i=1}^{m} \frac{u_i}{m} \]

s.t.
\[ \sum_{r=1}^{s} v_r = s \]
\[ \sum_{j=1}^{n} t_j x_{ij} - (1-h) \sum_{j=1}^{n} t_j \alpha_{ij} \leq u_i x_{i0} - (1-h)u_i \alpha_{i0}, \quad i = 1, \ldots, m \]
\[ \sum_{j=1}^{n} t_j y_{ij} - (1-h) \sum_{j=1}^{n} t_j \beta_{ij} \geq \nu_r y_{r0} - (1-h)\nu_r \beta_{r0}, \quad r = 1, \ldots, s \]
\[ \sum_{j=1}^{n} t_j y_{rj} + (1-h) \sum_{j=1}^{n} t_j \beta_{ij} \geq \nu_r y_{r0} + (1-h)\nu_r \beta_{r0}, \quad r = 1, \ldots, s \]  
(7)

\[ u_i \leq \beta, \quad i = 1, \ldots, m \]
\[ \beta \leq v_r, \quad r = 1, \ldots, s \]
\[ 0 \leq t_j, \quad j = 1, \ldots, n \]
\[ 0 \leq \beta \leq 1 \]

By similar transformation, fuzzy DEA model based on input-oriented CCR can also be established (inputs and outputs are also assumed to be symmetrical triangular fuzzy numbers).
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\[ \min \theta_0 \]
\[ \sum_{j=1}^{n} \lambda_j x_{ij} - (1-h) \sum_{j=1}^{n} \lambda_j \alpha_j \leq \theta_0 x_{i0} - (1-h)\theta_0 \alpha_{i0}, \quad i = 1,\ldots,m \]
\[ \sum_{j=1}^{n} \lambda_j x_{ij} + (1-h) \sum_{j=1}^{n} \lambda_j \alpha_j \leq \theta_0 x_{i0} + (1-h)\theta_0 \alpha_{i0}, \quad i = 1,\ldots,m \]
\[ \sum_{j=1}^{n} \lambda_j y_{ij} - (1-h) \sum_{j=1}^{n} \lambda_j \beta_j \geq y_{r0} - (1-h)\beta_{r0}, \quad r = 1,\ldots,s \]
\[ \sum_{j=1}^{n} \lambda_j y_{ij} + (1-h) \sum_{j=1}^{n} \lambda_j \beta_j \geq y_{r0} + (1-h)\beta_{r0}, \quad r = 1,\ldots,s \]
\[ \lambda_j \geq 0, \quad j = 1,\ldots,n \]

A comparative result will be given between model (8) and model (7) Section 4, to illustrate the advantages of our non-radial approach over previous approach based on radial model.

Theoretically, fuzzy non-radial DEA model can deal with LR-fuzzy number, the fuzzy non-radial DEA model formulations which can deal with LR-fuzzy number are just like model (6). But in practice, it is difficult to obtain the membership functions of all LR-fuzzy inputs and outputs, therefore, model (6) and alike models have only theoretical significance. With a view to intuition and computational-efficiency, symmetrical triangular fuzzy numbers and model (7) are highlighted in this paper. In fact, literature on fuzzy DEA such as (Guo and Tanaka, 2001; Leon et al., 2003; Lertworasirikul, 2003a,b) do like this way; especially their number example, almost all designed only to deal with symmetrical triangular fuzzy number (Wang and Liang, 2009).

4 FMS performance evaluation

In this section, we will employ the proposed approach (model (7)) for selecting a FMS. The data utilised for the illustrative analysis is slightly modified from the study of Karsak and Kuzgunkaya (2002). In their research, the input items include capital and maintenance cost and floor space used, the output items include reduction in work-in-process, reduction in setup cost, improvement in quality, increase in market response and reduction in labour cost. The values for each FMS DMU are expressed using triangular fuzzy numbers. The FMS DMUs criteria values for capital and maintenance cost and floor space used, reduction in work-in-process, reduction in setup cost, reduction in labour cost are expressed by quantitative data. However, FMS DMUs values for improvement in quality, increase in market response are in qualitative terms that are represented by linguistic expressions such as ‘weak’, ‘fair’ and ‘good’ (numerical estimates require more mental effort than linguistic descriptors, people are more likely to bias their evaluations if they are forced to provide numerical estimates of vague or imprecise items). The dataset used in the DEA is given in Table 1. The membership functions of the linguistic variables used to represent improvement in quality and increase in market response are defined in the Figure 1 (Karsak and Kuzgunkaya, 2002).
Table 1  Data used in the analysis

<table>
<thead>
<tr>
<th>FMS</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Capital and maintenance cost ($)</td>
<td>Reduction in setup cost (%)</td>
</tr>
<tr>
<td></td>
<td>Floor space used (sq.ft)</td>
<td>Improvement in quality</td>
</tr>
<tr>
<td></td>
<td>Reduction in labour cost (%)</td>
<td>Increase in market response</td>
</tr>
<tr>
<td>DMUs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>(14, 15, 18)</td>
<td>(25, 30, 35)</td>
</tr>
<tr>
<td>B</td>
<td>(11, 13, 15)</td>
<td>(16, 18, 20)</td>
</tr>
<tr>
<td>C</td>
<td>(7.5, 9.5, 11.5)</td>
<td>(10, 15, 20)</td>
</tr>
<tr>
<td>D</td>
<td>(8, 12, 13)</td>
<td>(23, 25, 27)</td>
</tr>
<tr>
<td>E</td>
<td>(8.5, 9.5, 10.5)</td>
<td>(12, 14, 16)</td>
</tr>
<tr>
<td>F</td>
<td>(10, 12.5, 15)</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(15, 8, 12)</td>
</tr>
</tbody>
</table>

Notes: Weak: (0, 0.2, 0.4), fair: (0.3, 0.5, 0.7), good: (0.6, 0.8, 1).

Figure 1  Membership functions for linguistic variables
The efficiencies of DMUs obtained by using model (7) for the different $h$ values are illustrated in Table 2. Table 2 shows that as the value of $h$ increases, the efficiency score of a DMU decreases. Extreme situations $h = 0$ represents all the possible production scenarios are considered, $h = 1$ represents crisp case. DMU $D$, $E$, $G$ all are efficient at any $h$ value. DMU $H$ are inefficient at any $h$ value, so $D$, $E$, $G$ are best choice of all eight FMS alternatives. DMU $F$ is, particularly, sensitive to variable measurement, it is not efficient in crisp case; but when input and output are fuzzy ($0 < h < 1$), it becomes efficient. So, we can conclude that it is important to use a fuzzy approach to evaluate the efficiency with DEA model when inputs and outputs are fuzzy. Obviously, the reason why the number of efficient DMUs at given $h$ value is relatively large comparing to the number of DMUs is that the number of DMUs is small compared with the number of criteria employed for evaluation.

The efficiencies of DMUs obtained by using model (8) for the different $h$ values are illustrated in Table 3. DMU $D$, $E$ and $G$ all are efficient in Table 3 just as in Table 2 at any possible $h$ value. When DMUs are all inefficient in Tables 2 and 3, the efficiencies of these DMUs in Table 2 are smaller than the efficiencies of corresponding DMUs in Table 3 at given $h$ value, this is because efficiency index in Table 3 is only input efficiency and omit the non-zero input slacks and, thus, fail to account for all inefficiencies that the model can identify. It is worth noting that, when $h = 1$ (crisp case), DMU $F$ is inefficient in Table 2 but is efficient in Table 3; it is the relationship $R_i \leq \theta$ between ERM model and CCR model lead to this difference of efficiency (see Section 2), and omitting the non-zero input slacks lead to DMU $F$ which should be identified to be inefficient is identified to be efficient in Table 3.

We can observe that the number of efficient DMUs in Table 2 is relatively larger than the number of efficient DMUs in Table 3 at given $h$ value, this is because the number of constraints in model (7) is larger than the number of constraints in model (8).
Table 3  The efficiencies of DMUs obtained by using the model (8) for the different $h$ values

<table>
<thead>
<tr>
<th>Possibility level ($h$)</th>
<th>DMU</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>$h = 0$</td>
<td>1</td>
<td>0.81818</td>
<td>0.72321</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>$h = 0.1$</td>
<td>1</td>
<td>0.83036</td>
<td>0.73247</td>
<td>1</td>
<td>1</td>
<td>0.9673</td>
<td>1</td>
<td>0.8108</td>
<td></td>
</tr>
<tr>
<td>$h = 0.2$</td>
<td>0.97971</td>
<td>0.84211</td>
<td>0.74131</td>
<td>1</td>
<td>1</td>
<td>0.9375</td>
<td>1</td>
<td>0.79773</td>
<td></td>
</tr>
<tr>
<td>$h = 0.3$</td>
<td>0.95964</td>
<td>0.85345</td>
<td>0.74977</td>
<td>1</td>
<td>1</td>
<td>0.93396</td>
<td>1</td>
<td>0.78208</td>
<td></td>
</tr>
<tr>
<td>$h = 0.4$</td>
<td>0.94308</td>
<td>0.87108</td>
<td>0.76116</td>
<td>1</td>
<td>1</td>
<td>0.94444</td>
<td>1</td>
<td>0.76765</td>
<td></td>
</tr>
<tr>
<td>$h = 0.5$</td>
<td>0.93202</td>
<td>0.89066</td>
<td>0.7802</td>
<td>1</td>
<td>1</td>
<td>0.95455</td>
<td>1</td>
<td>0.75429</td>
<td></td>
</tr>
<tr>
<td>$h = 0.6$</td>
<td>0.92307</td>
<td>0.90874</td>
<td>0.8069</td>
<td>1</td>
<td>1</td>
<td>0.96429</td>
<td>1</td>
<td>0.74188</td>
<td></td>
</tr>
<tr>
<td>$h = 0.7$</td>
<td>0.92008</td>
<td>0.92546</td>
<td>0.83153</td>
<td>1</td>
<td>1</td>
<td>0.97368</td>
<td>1</td>
<td>0.73033</td>
<td></td>
</tr>
<tr>
<td>$h = 0.8$</td>
<td>0.92579</td>
<td>0.94091</td>
<td>0.85429</td>
<td>1</td>
<td>1</td>
<td>0.98276</td>
<td>1</td>
<td>0.71955</td>
<td></td>
</tr>
<tr>
<td>$h = 0.9$</td>
<td>0.94158</td>
<td>0.9552</td>
<td>0.87531</td>
<td>1</td>
<td>1</td>
<td>0.99153</td>
<td>1</td>
<td>0.70946</td>
<td></td>
</tr>
<tr>
<td>$h = 1$</td>
<td>0.95819</td>
<td>0.96842</td>
<td>0.89474</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>

By using comparative results which are presented above, we cannot conclude that our approach (model (7)) is more efficient than fuzzy DEA model based on CCR model (model (8)). But we must point out that the efficiency measure of our approach (model (7)) is relatively more reasonable than that of fuzzy DEA model based on CCR model (model (8)), in that the former jointly determines input and output efficiencies that represent separate estimates of input and output efficiency, and unify the ratio efficiency and slacks into a scalar measure; while the later is only input efficiency and omit non-zero input slacks and, thus, fail to account for all inefficiencies; therefore, the proposed approach represents some real-life processes more appropriately.

There are several efficient DMUs in our example, it is the number of input and output criteria, that is, relatively large comparing to the number of DMUs brings this situation. This may bring difficulty to decision maker’s choice, but this is a ubiquitous problem inherent in DEA. There are two ways to deal with this problem, one is to make great efforts to reduce the number of input criteria and output criteria, the other is to take account of the ranking of fuzzy DMUs.

5 Conclusions

In this paper, by using a ranking method based on the comparison of $\alpha$-cuts, we transform the fuzzy versions of ERM non-radial DEA model into equivalent crisp LP formulations. The proposed fuzzy DEA model extends ERM model to a more general form in which crisp, fuzzy and hybrid data can be handled easily. Uncertainty is incorporated in the model formulation. The efficiency measure of our approach is relatively more reasonable than those of fuzzy DEA models based on CCR or BCC model, in that the former jointly determines input and output efficiencies that represent separate estimates of input and output efficiency, and unify the ratio efficiency and slacks into a scalar measure; while the later is only input efficiency and omit non-zero input slacks and thus fail to account for all inefficiencies. The proposed approach represents some real-life processes more appropriately.
The fuzzy DEA approach that this paper presents can facilitate decision making in the selection of a FMS. In Section 4, linguistic variables and triangular fuzzy numbers are used to quantify the vagueness inherent in decision parameter. The proposed decision-making framework enables intangible as well as tangible aspects to be taken into account in the technology selection process. It is a viable decision-making tool by organisations considering technology investments and can determine the most appropriate FMS alternatives.

It is obvious that the decision framework presented in this paper is equally applicable to diverse decision-making problems encountered in management science that incorporate vagueness.

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References


