An FFT-based Multiuser Detection for Asynchronous Block-Spreading CDMA Ultra Wideband Communication Systems

Yang Tang, Student Member, IEEE, Branka Vucetic, Fellow, IEEE, and Yonghui Li, Member, IEEE,

Abstract—A novel multiuser detection scheme for asynchronous Ultra-Wideband (UWB) impulse radio systems is proposed in this paper. A block-spreading code-division multiple-access (BS-CDMA) with zero correlation window (ZCW) is designed and applied in the UWB systems. It is shown that the multiple access interference is completely removed by maintaining the code orthogonality even with the asynchronous reception. Furthermore, the UWB channel can be modelled as a block circulant matrix by re-ordering the received sequence. With the advantage of circulant matrix, the Fast Fourier Transform (FFT) based minimum mean square error equalization scheme is applied to eliminate the ISI to such an extent that the system performance approaches the AWGN channel. As the large number order of the UWB channel, the FFT based equalization scheme also dramatically decreases the computation complexity.

I. INTRODUCTION

As the commercial Ultra Wideband (UWB) communications systems are required to operate in the multiple user environments, some recent research papers addressed multiuser detection schemes, which are mainly classified into two categories: time-hopping (TH) [1],[2],[3],[4] and DS-CDMA [5],[6]. The TH-based schemes distinguish the users by assigning them into different time slots. In DS-CDMA based schemes, each users is assigned a unique spreading sequence. The spreading sequences of various users are selected to be orthogonal to each other.

In the DS-CDMA based UWB system, the code orthogonality is hard to be maintained due to the inter-symbol interference, multiple access interference and the asynchronous reception at the receiver. The sub-optimal multiuser detection techniques, such as decorrelating detection and linear minimum mean square error (LMMSE), can be used to suppress interference [5]. However, the computation complexity of MMSE DS-CDMA multiuser detection scheme is very high due to the large UWB channel order. These sub-optimal interference cancellation schemes also require the knowledge of the channel state information for all users. This assumption is unrealistic for the UWB communication systems.

In order to reduce the computation complexity and the need for channel estimation, the research on MAI-free reception design is motivated. In [7] and [1], a chip-interleaved block-spreading CDMA (CIBS-CDMA) system and multistage block-spreading (MS-BS) PPM are proposed to achieve a MAI-free feature. However, both CIBS-CDMA and MS-BS PPM work in the synchronous systems. As one of the most challenging, the synchronous reception is not easy to achieved, especially, in the high-speed UWB systems. Consequently, it is meaningful of this paper to extend the block-spreading CDMA into the asynchronous UWB systems.

This paper consists of two parts. In the first part, a set of spreading sequences with zero correlation window (ZCW) or zero correlation zone (ZCZ) [10], [11] are implemented. For the carrierless fashion of UWB system, we propose a set of analog ZCW sequences, which can achieve the same bandwidth efficiency as the binary ZCW when $M = 2^N$. With the property of the ZCW sequences, the MAI-free reception is satisfied in the asynchronous UWB systems.

The second part of this paper addresses the channel equalization to eliminate the ISI. The UWB channel of each user can be represented as a block circulant matrix, so that the Fast Fourier Transform (FFT) based equalization scheme is applied with the minimum mean square error criteria. The proposed equalization scheme not only achieves the better performance but also dramatically decrease the computation complexity compared with the conventional multiuser detection (MUD) strategies for UWB systems.

Notation: Bold upper (lower) letters denote matrices (column/raw vectors); $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote the conjugate, transpose and Hermitian transpose, respectively; $I_k$ denotes the identity matrix of size $k$; $0_{k \times p}$ denotes an all-zero matrix with size $k \times p$ and $0_k$ represents an all-zero square matrix with size $k$. $[\cdot]$ stands for integer floor and $\otimes$ denotes the Kronecker product.

II. SYSTEM DESCRIPTION

In this paper, an asynchronous UWB communication system with $K$ users is considered. The users employs direct-sequence spread-spectrum (DSSS) modulation. The block diagram of the system is illustrated in Fig. 1.

A. Transmitter Structure

For each user, the data stream at the transmitter is divided into data blocks, which consists of $N_s$ symbols. The data block
for user \(k, k \in [1, K]\) is defined as

\[
b_k = \{b_k(i)\}, i \in [1, N_c];
\]

(1)

The period of the data block is \(N_c T_s\), where \(T_s\) is the symbol duration. At the transmitter side, the data block \(b_k\) is modulated with a spreading sequence \(s_k = [s_{k1}, s_{k2}, \ldots, s_{kN_f}]\), where \(N_f\) is the spreading gain. Consequently, the transmitted data block generated by the \(k\)th user is given by

\[
x_k = s_k \otimes b_k.
\]

(2)

**B. Channel Model**

The physical multipath channel due to user \(k\) is denoted in terms of multipath gain and delay by \(h_k(t)\) and given as

\[
h_k(t) = \sum_{i=1}^{N_{m,k}} h_{ki} \cdot \delta(t - (i - 1)T_r),
\]

(3)

in which \(N_{m,k} T_r\) is the number of resolvable multipath within the maximum delay spread for the \(k\)-th user’s channel. \(T_{m,k}\) denotes the maximum delay spread of the multipath channel corresponding to user \(k\) and \(T_r\) is the resolution of the multipath channel.

Consequently, the chip-sampled discrete time equivalent channel corresponding to the \(k\)-th user can be expressed in the vector form as

\[
h_k = [h_{k1}, h_{k2}, \ldots, h_{kN_{m,k}}]^T;
\]

(4)

**C. Receiver Structure**

The receiver structure for each user includes an MAI-elimination function and FFT-based channel equalization function, which are shown in the receiver end of Fig. 1. All MAI from the asynchronous users can be removed with the MAI-elimination function. Consequently, the remained challenge becomes single-user channel equalization to mitigate the ISI across the symbols within the same data block. This is considered in the FFT-based channel equalizer design.

**III. MAI-ELIMINATION FUNCTION FOR ASYNCHRONOUS DS-CDMA UWB**

In the synchronous block-spreading CDMA designs [1], [7], the MAI-free reception can only be achieved with the orthogonal spreading sequences employed to reduce the MAI, when all symbols are received simultaneously. This assumption is not realistic for the high speed UWB systems. As the frequency selectivity of the propagation channel and the asynchronous multiple users, the orthogonality of the spreading sequences cannot be maintained.

In this section, a novel MAI-free multiuser detection strategy for asynchronous UWB systems is developed based on the designed spreading sequences with zero correlation zone.

With a quasi-static asynchronous multipath UWB channel, the received sampled sequences from all users can be illustrated in Fig. 2.

The shaded parts represent the so-called data block interchip interference, which are due to the multipath spread coming from the previous data block chip. \(\tau_k, k \in [1, K]\), denotes the arrival time delay of the \(k\)-th user compared to the first arrived one. Without loss of generality, we assume \(\tau_K \geq \tau_{(K-1)} \geq \cdots \geq \tau_1 = 0\). By choosing the block size, in which \(N_c = [T_c / T_r]\) denotes the number of samples within one symbol duration. The matrix \(\mathbf{H}_{k,N_f}\) consists of \(N_c N_f\) channel impulse responses vectors \(\mathbf{h}_k\), which is defined in Eq. 4.

In order to facilitate the analysis, the received sequence \(\mathbf{R}\) can be divided into \(N_f + 1\) parts as shown in Fig. 2. Each of the first \(N_f\) parts consists of \(N_c N_f\) samples and the last part contains \(N_{\tau_K} + N_{m,k} - N_r\) samples, where \((N_{m,k} - N_r)\) represents the number of the samples in the shaded part.

By combining the Part \(N_f + 1\) and the Part 1 in Fig. 2, the received sequence can be depicted in Fig. 3.

This procedure will enhance the noise power spectral density from \(\sigma^2\) into \((1 + \tau_K + T_{m,k} - T_s / N_f T_s T_r)\sigma^2\). As \(N_f T_s \gg \tau_K + T_{m,k} - T_s\), its effect can be neglected. Consequently, the revised received sequence is given as
Fig. 3. Converted DS-CDMA based UWB data structure by adding the Part $N_f + 1$ into Part 1

\[
\hat{\mathbf{r}} = \sum_{k=1}^{K} (\mathbf{I}_{N_f} \otimes \hat{\mathbf{H}}_{k1}) (\mathbf{s}_k \otimes \mathbf{b}_k) + \sum_{k=1}^{K} (\mathbf{I}_{N_f} \otimes \hat{\mathbf{H}}_{k2}) (\mathbf{s}_k \otimes \mathbf{b}_k) + \mathbf{n}
\]

\[
= \sum_{k=1}^{K} \mathbf{s}_k \otimes (\hat{\mathbf{H}}_{k1} \mathbf{b}_k) + \sum_{k=1}^{K} \mathbf{s}_k \otimes (\hat{\mathbf{H}}_{k2} \mathbf{b}_k) + \sum_{k=1}^{K} \mathbf{s}_k \otimes (\hat{\mathbf{H}}_{k3} \mathbf{b}_k) + \mathbf{n};
\]

in which $\mathbf{s}_k = [s_{k,N_f}, s_{k,N_f-1}, \ldots, s_{k}]$ is one bit shifted version of $\mathbf{s}_k$ to the left and

\[
\hat{\mathbf{H}}_{k1} = [0_{N_x \times N_c}, \hat{\mathbf{H}}_{k1}],
\]

\[
\hat{\mathbf{H}}_{k2} = [\hat{\mathbf{H}}_{k2}; 0_{(N_c - N_x) \times N_c}],
\]

\[
\hat{\mathbf{H}}_{k3} = [0_{N_x \times N_c}; 0_{((N_c + 1)N_c - N_m, k - N_x) \times N_c}]
\]

are $N_c \times N_c$ matrices. By defining the channel matrix $\mathbf{H}_k$ as

\[
\mathbf{H}_k = \begin{pmatrix}
\hat{\mathbf{H}}_{k1} \\
\hat{\mathbf{H}}_{k2} \\
\hat{\mathbf{H}}_{k3}
\end{pmatrix}
\]

(11)

$\mathbf{H}_{k1}$, $\mathbf{H}_{k2}$ and $\mathbf{H}_{k3}$ actually are the sub-matrices of $\mathbf{H}_k$ and can be illustrated as

\[
\mathbf{H}_k = \begin{bmatrix}
\mathbf{H}_{k1} \\
\mathbf{H}_{k2} \\
\mathbf{H}_{k3}
\end{bmatrix}
\]

(12)

In Eq. 7, $\sum_{k=1}^{K} \mathbf{s}_k \otimes (\hat{\mathbf{H}}_{k2} \mathbf{b}_k)$ is the interferences due to the arrival time delay and $\sum_{k=1}^{K} \mathbf{s}_k \otimes (\hat{\mathbf{H}}_{k3} \mathbf{b}_k)$ is the interferences due to the multipath spread.

In order to remove the interferences from all other users in Eq. 7, the spreading codes have to be selected as:

\[
\mathbf{s}_i = \{s_i | s_i \mathbf{s}_j^T = N_f \delta(i - j); s_i \mathbf{s}_j^T = 0, \forall i, j \in [1, K]\}
\]

(13)

This family of spreading sequences defined in Eq.13 is actually an orthogonal sequence set with zero correlation window of 3. When the number of users is even, The binary sequences with ZCW of 3 is given in [11]. For the odd number of users, there is no binary spreading sequences existing to achieve the bandwidth efficiency as same as what has been achieved in [11].

$b_k$ The modified sequence $\hat{\mathbf{r}}$ in Eq. (7) is de-spread in two parallel stages by $(\mathbf{s}_k)^T$ and $(\mathbf{s}_k^T)^T$ as depicted in Fig. 4. The output of the first stage de-spreading, denoted by $\mathbf{d}_1$, $k \in [1, K]$, is given by

\[
\mathbf{d}_1 = \mathbf{s}_k^T \hat{\mathbf{r}}
\]

(7)

\[
= \sum_{i=1}^{K} (\mathbf{s}_i^T \mathbf{s}_i) \otimes (\hat{\mathbf{H}}_{i1} \mathbf{b}_i) + \sum_{i=1}^{K} (\mathbf{s}_i^T \mathbf{s}_i) \otimes (\hat{\mathbf{H}}_{i2} \mathbf{b}_i)
\]

\[
+ \sum_{i=1}^{K} (\mathbf{s}_i^T \mathbf{s}_i) \otimes (\hat{\mathbf{H}}_{i3} \mathbf{b}_i) + \mathbf{s}_k^T \mathbf{n};
\]

(14)

At the second stage, the received sequences $\hat{\mathbf{r}}$ is de-spread by the sequence $\mathbf{s}_k^T$ and the output, $\mathbf{d}_2$, $k \in [1, K]$, is obtained as

\[
\mathbf{d}_2 = \mathbf{s}_k^T \hat{\mathbf{r}}
\]

(15)

\[
= \sum_{i=1}^{K} (\mathbf{s}_i^T \mathbf{s}_i) \otimes (\hat{\mathbf{H}}_{i1} \mathbf{b}_i) + \sum_{i=1}^{K} (\mathbf{s}_i^T \mathbf{s}_i) \otimes (\hat{\mathbf{H}}_{i2} \mathbf{b}_i)
\]

\[
+ \sum_{i=1}^{K} (\mathbf{s}_i^T \mathbf{s}_i) \otimes (\hat{\mathbf{H}}_{i3} \mathbf{b}_i) + \mathbf{s}_k^T \mathbf{n};
\]

(16)

where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the corresponding additive noise vectors. From Eq.14 and Eq.15, by re-ordering the detected sequences, the MAI-free reception for the $k$-th user is represented by $\mathbf{d}_k$ and given as

\[
\mathbf{d}_k = \begin{bmatrix}
\mathbf{H}_{k1} \\
\mathbf{H}_{k2} \\
\mathbf{H}_{k3}
\end{bmatrix} \mathbf{b}_k + \mathbf{n}_k
\]

(17)

where $\mathbf{n}_k$ is the corresponding additive noise.

IV. FFT-BASED CHANNEL EQUALIZATION FUNCTION

As the multiple access interference has been completely removed with the MAI-elimination function in the previous
section, an FFT-based channel equalization scheme is investigated in this section to mitigate inter-symbol interference within the data block for a single user. Due to the large multipath channel order, the main challenge of the equalizer design in the UWB system is to deliver high-performance with low computation complexity.

The proposed FFT-based channel equalizer is depicted in Fig. 5.

The MAI-free sequence \(d_k\) in Eq. 16 is multiplied by a matrix \(H_{\text{tran}}\), which is given as

\[
H_{\text{tran}} = \begin{bmatrix}
1 & 0 & \\
0 & 1 & \\
0 & 0 & 0'
\end{bmatrix}
\]

in which \(0'\) is a \(((N_c + 1)N_r - N_{m,k}) \times (N_{m,k} - N_r)\) zero matrix and \(H_{\text{tran}}\) is \(N_cN_r \times ((N_c - 1)N_r + N_{m,k})\) matrix. As a result, the transformed MAI-free sequence is denoted as \(d_{k}^{\text{tran}}\) and given by

\[
d_{k}^{\text{tran}} = H_{\text{tran}}(H_k b_k + n_k)
\]

where \(n_k\) is the additive noise and this procedure is equivalent to overlap the last \(N_{m,k} - N_r\) vectors of \(H_k\) on its first \(N_{m,k} - N_r\) vectors. The noise \(n_k\) is colored by the matrix \(H_{\text{tran}}\), however the noise color can be negligible when the number of symbols per data block is increased.

Consequently, for the \(k\)-th user, the equivalent channel matrix \(\tilde{H}_k\) is a block-circulant matrix and given as

\[
\tilde{H}_k = \begin{bmatrix}
h_k & \\
h_k & \\
h_k & \end{bmatrix}
\]

From the matrix theory [12], for a \(nq \times np\) block-circulant matrix \(D\), we have

\[
D = F_{(n,q)}^{H} \Lambda_b F_{(n,p)}
\]

where \(F_{(a,b)} := F_{a} \otimes I_{b}\), \(F_{a}\) stands for a \(a \times a\)-sized Fourier matrix, where \(|F_{a}|_{ij} = a^{-\frac{1}{2}} e^{-(\pi i l/\alpha)}\), \(i, l \in [0, a - 1]\).

For the equivalent block-circulant channel matrix \(\tilde{H}_k\), we have

\[
\tilde{H}_k = F_{(N_c,N_r)}^{H} \Delta_k F_{(N_c,1)}
\]

\(\Delta_k\) is a block diagonal matrix with

\[
\Delta_k = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{N_r})
\]

where \(\lambda_n, n \in [1, N_c]\), is the eigenvalue of Hermitian matrix \(H_k^{H}H_k\).

The MMSE equalization filter taps \(\Gamma_k^{M\text{MSE}}\) for user \(k\) are given by

\[
\Gamma_k^{M\text{MSE}} = \arg \min_{\Gamma_k \in C^{N_c \times N_c N_r}} E\{||b_k - \Gamma_k (H_k b_k + n_k)||\}
\]

The solution to Eq. 23 is given as

\[
\Gamma_k^{M\text{MSE}} = H_k^T (PH_k^T H_k + \sigma^2 I)^{-1};
\]

By substituting Eq. 21 into Eq. 24, the MMSE equalizer taps is given in Eq. 25. As the FFT-based block circulant matrix decomposition scheme is involved, the MMSE equalization scheme becomes easy to implement by inverting the diagonal matrix \((\Delta_k^T \Delta_k + \sigma^2 I)\) in Eq. 25 instead of general square matrix \((H_k^T H_k + \sigma^2 I)\) in Eq. 24, which will decrease the computation complexity dramatically with the large matrix \(H_k^T H_k\) in UWB systems.

Consequently, the output of the FFT-based equalizer \(\tilde{b}\) is obtained as

\[
\tilde{b}_k = \Gamma_k^{M\text{MSE}} (H_k b_k + n_k)
\]

\[
= F_{(N_c,1)}^{T} (P \Delta_k \Delta_k^T + \sigma^2 I)^{-1} \Delta_k F_{(N_c,1)} b_k^T + F_{(N_c,1)}^{T} (P \Delta_k \Delta_k^T + \sigma^2 I)^{-1} F_{(N_c,1)} n_k
\]

\[
= F_{(N_c,1)}^{T} (P \Delta_k \Delta_k^T + \sigma^2 I)^{-1} \Delta_k^T \Delta_k F_{(N_c,1)} b_k^T + F_{(N_c,1)}^{T} (P \Delta_k \Delta_k^T + \sigma^2 I)^{-1} F_{(N_c,1)} n_k
\]

\[
= F_{(N_c,1)}^{T} \Phi_k F_{(N_c,1)} b_k^T + F_{(N_c,1)}^{T} \Psi_k n_k;
\]

where \(P\) is the transmit power and the element of \(n_k = F_{(N_c, N_r)}^{T} n_k\) is independent zero mean Gaussian distributed variable with variance \(\sigma_n^2\). \(\Phi_k\) is defined as

\[
\Phi_k = \begin{bmatrix}
\frac{\lambda_1}{P \lambda_1 + \sigma_n^2} & 0 & \cdots & 0 \\
0 & \frac{\lambda_2}{P \lambda_2 + \sigma_n^2} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \frac{\lambda_{N_r}}{P \lambda_{N_r} + \sigma_n^2}
\end{bmatrix}
\]

and

\[
\Psi_k = \begin{bmatrix}
\frac{\sqrt{\lambda_1}}{P \lambda_1 + \sigma_n^2} & 0 & \cdots & 0 \\
0 & \frac{\sqrt{\lambda_2}}{P \lambda_2 + \sigma_n^2} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \frac{\sqrt{\lambda_{N_r}}}{P \lambda_{N_r} + \sigma_n^2}
\end{bmatrix}
\]

V. SIMULATION RESULTS

In this section, the simulation results given illustrate the performances of the proposed multiuser detection scheme for UWB systems. All simulation results are obtained based on the Intel UWB channel model in [9]. The channel impulse responses for different users are independently generated.

In our simulation, the UWB communication system with 8 coexisting users is considered. Equal transmit power for all users is assumed and the channel state information of each user is assumed to be perfectly known for the corresponding receivers. The binary phase-shift keying (BPSK) is selected as the modulation scheme and the arrival delay for each user is randomly generated between \(0ns\) to \(40ns\).

Fig. 6 illustrates the BER performance comparison between the proposed scheme and the CIBS-CDMA [7] and MS-BS PPM (TH) [1] schemes with the asynchronous reception. The BER performance in the AWGN channel is also given as a lower bound. For the orthogonality of the spreading sequences
in the proposed scheme is maintained, the simulation shows
that the performance of the proposed ZCW-based BS-CDMA
is very robust to the asynchronous reception compared with
the other designs. The performance in the AWGN channel
is superior by about 1.25dB to the proposed scheme when
the BER is around $10^{-4}$ and that is much better than the
conventional BS-CDMA with MMSE receiver.

In Fig. 7, the performance of the proposed scheme are
evaluated with three different transmission rates. We adopt
Intel NLOS UWB channel model, the length of data block, $N_r$, is 100; When transmission rate is $10 Mbps$, the corresponding period of chip is $6.25ns$ and $N_r$ is 31. The performance gap between AWGN channel bound and the proposed scheme, shown in Fig. 7, is very trivial. With the same length of data block, when the transmission rate is increased to $50 Mbps$ or the period of chip is $1.25ns$, the performance of AWGN channel is superior $1.25dB$ to the proposed scheme.

VI. Conclusion

In this paper, we develop a new ZCW-based block-spreading
CDMA multiuser detection scheme for the asynchronous
UWB communication systems. For spreading code design with
zero correlation window, the orthogonality of spreading codes
are maintained through the frequency-selective UWB channel
with asynchronous reception, such that the MAI free reception
is achieved. In the part of channel equalization, the channel
matrix is modelled as a block-circulant matrix with detected
samples reordered. With this advantage feature, a FFT-based
MMSE equalization scheme for UWB system is developed.
For the uncoded UWB system, when the transmission rate is
50MHz and number of users is 8, the proposed scheme can
approach the BER performance within 1.25dB from what is
achieved in the AWGN channel with single user when SNR is
around $10^{-4}$. It is also noted that the computation complexity
of proposed schemes is much simpler than the conventional
DS-CDMA UWB system.

REFERENCES

Impulse Radio Multiple Access Through ISI Channel", IEEE J. Select. Areas
2, Toronto, ON, Canada, pp. 775-779, 1998.
DS-CDMA Ultra-Wideband Communication Systems", in Proc. Joint
UWST and IWUWBST, Kyoto, Japan, WA4-5 May, 2004.
University Press, 1996
[9] Jeff Foerster and Qinghua Li, "UWB Channel Modeling Contribution
from Intel", IEEE P802.15-02/0279r0-SG3a
[10] Pingshi Fan, "New direction in Spreading Sequence Design and the
Related Theoretical Bounds", Communications, Circuits and Systems and
West Sino Expositions, IEEE 2002 International Conference on, Volume: 1 ,
Pages: xliii - xlviii vol.1, 29 June-1 July 2002
[11] Shaojun Xu, Duaben Li "Ternary complementary orthogonal sequences
with zero correlation window", "Personal. Indoor and Mobile Radio Com-
ee.stanford.edu/ gray/toeplitz.pdf

\[ \Gamma_{MMSE}^k = \mathbf{F}_{(N_c,1)}^T \Delta_k^T \mathbf{F}_{(N_c,1)} (\mathbf{P} \mathbf{F}_{(N_c,1)}^T \Delta_k \mathbf{F}_{(N_c,1)} + \sigma^2 \mathbf{I})^{-1} \mathbf{F}_{(N_c,1)}^T \Delta_k^T \mathbf{F}_{(N_c,1)}^T + \sigma^2 \mathbf{I})^{-1} \]

\[ \mathbf{F}_{(N_c,1)}^T \Delta_k^T (\mathbf{P} \Delta_k^T + \sigma^2 \mathbf{I})^{-1} \mathbf{F}_{(N_c,1)}^T \Delta_k^T \mathbf{F}_{(N_c,1)}^T \]

\[ \text{Fig. 6. The BER performance in the NLOS UWB channels in the presence}
\text{of 7 users with the same transmit power; Transmission rate is 50MHz;}
\text{Each data block consists of 100 symbols; The sampling rate is 5GHz.}

\[ \text{Fig. 7. The BER performance in the NLOS UWB channels with different}
\text{transmission rates} \]