On the Performance of A Simple Adaptive Relaying Protocol For Wireless Relay Networks

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Abstract—Distributed coding has been shown to be an effective scheme to explore cooperative spatial diversity in wireless relay networks. To date, the distributed coding schemes employ two major relaying protocols, decode and forward (DAF) and amplify and forward (AAF). They suffer from a disadvantage of either noise amplification or error propagation. In this paper, we propose a simple adaptive relaying protocol (ARP) for general relay networks. For the proposed approach, all relays are included into one of two relay groups, referred to as a DAF relay group and an AAF relay group. All relays, which decode correctly, are included in the DAF relay group, and other relays, which could not decode correctly, are included in the AAF relay group. Performance analysis of the proposed adaptive relaying protocol is carried out, and compared with other relaying protocols. It is shown that the proposed adaptive relaying protocol benefits from a significant coding gain contributed from the DAF relay group compared to a pure AAF relay protocol and simultaneously circumvent the detrimental effects of error propagation due to the imperfect decoding at relays in a DAF relay protocol, thus always outperforming both the AAF and DAF relaying protocols in all SNR regions.

I. INTRODUCTION

In wireless relay networks (WRNs) signals are transmitted from one terminal to another through a number of relays. The main advantage of doing so is a reduced signal transmit power. Design of efficient relaying protocols and distributed coding schemes in WRNs has recently attracted a lot of attention. Some cooperative or distributed coding schemes [3], [4], [7]–[9] have been proposed to explore the cooperative spatial diversity and cooperative coding gains in WRNs. However, most of the existing distributed coding schemes was based on the amplify and forward (AAF) and decode and forward (DAF) relaying protocols [1], [10], which suffer from a disadvantage of either noise amplification or error propagation. It is therefore very important to develop a simple but robust relaying protocol.

In this paper, we propose a simple adaptive relaying protocol (ARP), which takes advantages of both DAF and AAF protocols and minimize their disadvantages. For the proposed scheme, all relays are included into one of two relay groups, which we called a DAF relay group, and an AAF relay group. All relays, which can correctly decode the signals transmitted from the source, are included in the DAF relay group and the rest of relays, which fail to make a correct decoding, are included in the AAF relay group. All the relays in the AAF relay group amplify the received signals from the source and forward it to the destination, while the relays in the DAF relay group decode the received signals, re-encode and forward them to the destination. The processing at the destination for the ARP is the same as for the AAF and DAF. All signals received at the destination, forwarded from the relays in both the AAF and DAF groups are combined together into one signal. A Viterbi decoding algorithm can then be used to recover the source information. The performance of the proposed ARP is analyzed and compared with other relaying protocols. It is shown that the proposed ARP scheme considerably outperforms the AAF scheme and circumvents the error propagation due to the imperfect decoding at relays in a DAF protocol, thus considerably outperforming both the AAF and the DAF protocols. This performance gain grows as the number of relays increases and it approaches the perfect DTC at the high signal to noise ratios (SNR). The above analytical results are also validated by simulation results.

In a practical system, in order to determine whether a relay can decode correctly or not, some CRC bits can be appended to each information block (frame). After decoding each frame, the relay can examine the CRC checks to determine if the received signals are decoded correctly or not.

One important feature of the proposed ARP scheme in a practical application is that the relay can automatically adapt to the channel quality by simply switching between the AAF and the DAF without any need for the channel state information (CSI) to be fed back from the destination to the relays or the source. This feature is very important in practical relay networks, especially in a multi-hop large network, in which the feedback of CSI for adaptation is very expensive. Another important feature is that the processing at relays and destination for the ARP scheme is the same as for the AAF and DAF and it does not add any complexity to the system performance.

II. SYSTEM MODEL

For simplicity, in this paper we consider a general 2-hop relay network, consisting of one source, n relays and one destination. Fig. 1 gives a block diagram of a 2-hop relay system with a direct link from the source to the destination.

We assume that the source and relays transmit data through orthogonal channels. For simplicity, we will concentrate on a
time division multiplex [1], for which the source and relays transmit in the separate time slots.

The source first broadcasts the information to both the destination and relays. The received signals at the relay \( i \) and destination, at time \( k \), denoted by \( y_{sr,i}(k) \) and \( y_{sd}(k) \), respectively, can be expressed as

\[
y_{sr,i}(k) = \sqrt{P_{sr,i}} h_{sr,i} s(k) + n_{sr,i}(k) \quad (1) \\
y_{sd}(k) = \sqrt{P_{sd}} h_{sd} s(k) + n_{sd}(k) \quad (2)
\]

where \( s(k) \) is the symbol transmitted at time \( k \), \( P_{sr,i} = P_s \cdot (G_{sr,i})^2 \), \( P_{sd} = P_s \cdot (G_{sd})^2 \) are the received signal power at the relay \( i \) and destination, respectively, \( P_s \) is the source transmit power, \( G_{sr,i} \) and \( G_{sd} \) are the channel gains between the source and relay \( i \) and that between the source and destination, respectively. Also \( h_{sr,i} \) and \( h_{sd} \) are the fading coefficients between the source and the relay \( i \) and between the source and destination, respectively. In this paper, we consider a quasi-static fading channel, for which the fading coefficients are constant within one frame and change independently from one frame to another. Furthermore, \( n_{sr,i}(k) \) and \( n_{sd}(k) \) are zero mean complex Gaussian random variables with two sided power spectral density of \( N_0/2 \) per dimension.

The relays then process the received signals and send them to the destination. Let \( x_{r,i}(k) \) represent the signal transmitted from the relay \( i \) at time \( k \). It satisfies the following transmit power constraint,

\[
E(|x_{r,i}(k)|^2) \leq P_{r,i} \quad (3)
\]

where \( P_{r,i} \) is the transmitted power limit at the relay \( i \).

The corresponding received signal at the destination at time \( k \), denoted by \( y_{rd,i}(k) \), can be written as

\[
y_{rd,i}(k) = G_{rd,i} h_{rd,i} x_{r,i}(k) + n_{rd,i}(k) \quad (4)
\]

where \( G_{rd,i} \) is the channel gain between the relay \( i \) and destination, \( h_{rd,i} \) is the fading coefficient between the relay \( i \) and destination and \( n_{rd,i}(k) \) is a destination noise with two sided power spectral density of \( N_0/2 \) per dimension.

III. ADAPTIVE RELAYING PROTOCOL (ARP)

In this section, we propose an adaptive relaying protocol. Before describing the proposed relaying protocol, we first define an AAF and DAF relay group.

A. AAF Relay Group

An AAF relay group, denoted by \( \Omega_{AAF} \), consists of all the relays, which could not decode correctly. Upon receiving signals from the source, each relay in the AAF relay group will simply amplify the received signals from the source. Let \( x_{r,i}(k), i \in \Omega_{AAF}, \) represent the signal transmitted from the relay \( i \) at time \( k \), then it can be expressed as

\[
x_{r,i}(k) = \mu_i y_{sr,i}(k), i \in \Omega_{AAF} \quad (5)
\]

where \( \mu_i \) is an amplification factor such that \( x_{r,i}(k) \) satisfies the power constraint in (3) and it can be calculated as

\[
\mu_i \leq \sqrt{\frac{P_{r,i}}{|h_{sr,i}|^2 P_{sr,i} + N_0}} \quad (6)
\]

By substituting (5) and (1) into (4), the received signal at the destination, transmitted from \( i \)-th relay, become

\[
y_{rd,i}(k) = G_{rd,i} h_{rd,i} y_{sr,i}(k) + n_{rd,i}(k) + n_{rd,i}(k), i \in \Omega_{AAF} \quad (7)
\]

B. DAF Relay Group

The DAF relay group, denoted by \( \Omega_{DAF} \), consists of all the relays, which can make an error-free decoding. Since all the relays in the DAF relay group can decode correctly, each relay in the DAF relay group can accordingly recover the binary information stream \( B \). \( B \) is then encoded into \( C \) and modulated into \( S \).

The relay \( i \) in the DAF relay group will then forward the modulated symbols \( S \) with power \( P_{r,i} \) to the destination,

\[
x_{r,i}(k) = \sqrt{P_{r,i}} s(k), i \in \Omega_{DAF} \quad (8)
\]

The received signals at the destination, transmitted from the relay \( i \) in the DAF relay group, become

\[
y_{rd,i}(k) = G_{rd,i} h_{rd,i} \sqrt{P_{r,i}} s(k) + n_{rd,i}(k), i \in \Omega_{DAF} \quad (9)
\]

C. Adaptive Relaying Protocol

For the proposed adaptive relaying protocol (ARP), in each transmission, based on whether relays can make correct decoding or not, each relay is included into one of two groups, an AAF relay group and a DAF relay group. All the relays in the AAF relay group amplify the received signals from the source and forward it to the destination, while all the relays in the DAF relay group will decode the received signals, re-encode and forward them to the destination. All signals received at the destination, forwarded from the source, the AAF and DAF groups, are combined as follows,

\[
y_{rd-ARP}(k) = w_{sd} y_{sd}(k) + \sum_{i \in \Omega_{AAF}} w_{r,i} y_{rd,i}(k) + \sum_{i \in \Omega_{DAF}} w_{r,i} y_{rd,i}(k) \quad (9)
\]
The optimal values of $w_{sd}$ and $w_{ri}$ are given by [5]

$$
w_{sd} = \frac{\sqrt{P_{rd}h_{sd}^*}}{N_0}, \quad w_{ri} = \frac{h_{rd,i}^*\sqrt{P_{rd,i}}}{N_0}, \quad i \in \Omega_{DAF}
$$

$$
w_{ri} = \frac{\mu_iG_{rd,i}\sqrt{P_{rd,i}h_{rd,i}^*}}{(\mu_i^2G_{rd,i}h_{rd,i}^*)^2 + 1}N_0, \quad i \in \Omega_{AAF}
$$

By substituting (10) into (9), the destination SNR after combination, denoted by $\gamma_{ARP}$, can be approximated as

$$
\gamma_{ARP} = \gamma_{AAF} + \gamma_{DAF}
$$

(11)

where

$$
\gamma_{AAF} = \gamma_{sd}|h_{sd}|^2 + \frac{1}{2} \sum_{i \in \Omega_{AAF}} H_i^2
$$

(12)

$$
\gamma_{DAF} = \sum_{i \in \Omega_{AAF}} \gamma_{rd,i}|h_{rd,i}|^2
$$

(13)

where $\gamma_{sd} = \frac{P_{sd}}{N_0}$, $\gamma_{rd,i} = \frac{P_{rd,i}}{N_0}$, $\gamma_{sr,i} = \frac{P_{sr,i}}{N_0}$, $H_2^2 = (\frac{1}{2} \sum_{p=1}^{n} \lambda_{pi}^2)^{-1}$ is called the Harmonic Mean of variables $\lambda_{pi}$, $p = 1, 2$, $\lambda_{i,1} = |h_{sr,i}|^2 \gamma_{sr,i}$, and $\lambda_{i,2} = |h_{rd,i}|^2 \gamma_{rd,i}$.

IV. PERFORMANCE ANALYSIS OF ADAPTIVE RELAYING PROTOCOL

In this section, we analyze the performance of the ARP and compare with other relaying protocols. For simplicity of calculation, we assume that $\gamma_{sr,i} = \gamma_{sr}$ and $\gamma_{rd,i} = \gamma_{rd}$ for all $i = 1, \cdots, n$.

A. Error probability of the ARP

Let us first calculate the PEP for a scenario where the AAF relay group consists of $q$ relays numbered from 1 to $q$ and the DAF relay group consists of $(n - q)$ relays numbered from $(q + 1)$ to $n$.

Let $\gamma_{AAF,(q)}$ and $\gamma_{DAF,(n-q)}$ represent the instantaneous received SNR of the combined signals in the AAF and DAF relay groups, then we have from Eqs. (12) and (13) that

$$
\gamma_{AAF,(q)} = \gamma_{sd}|h_{sd}|^2 + \frac{1}{2} \sum_{i=1}^{q} H_i^2
$$

(14)

$$
\gamma_{DAF,(n-q)} = \gamma_{rd} \sum_{i=q+1}^{n} |h_{rd,i}|^2
$$

(15)

Let $P_{F_{sr}}^i(d_{sr}, \gamma_{sr,i}|h_{sr,i})$ be the conditional pair-wise error probability (PEP) of incorrectly decoding a codeword into another codeword with Hamming distance of $d_{sr}$ in the channel from the source to the relay $i$. Since we assume that $\gamma_{sr,i} = \gamma_{sr}$ for all $i = 1, \cdots, n$, then we have

$$
P_{F_{sr}}^i(d_{sr}, \gamma_{sr,i}|h_{sr,i}) = Q\left(\sqrt{2d_{sr} \gamma_{sr}|h_{sr,i}|^2}\right)
$$

Let $P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i})$ represent the conditional word error probability in the channel from the source to the i-th relay,

$$
P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i}) = \sum_{d_{sr}=d_{sr,min}}^{2l} \overline{A}(d_{sr})P_{F_{sr}}^i(d_{sr}, \gamma_{sr}|h_{sr,i})
$$

where $d_{sr,min}$ is the code minimum Hamming distance, $\overline{A}(d_{sr}) = \sum_{i=1}^{l} \binom{l}{i}p(d_{sr}|i)$, $\binom{l}{i}$ is the number of words with Hamming weight $i$ and $p(d_{sr}|i)$ is the probability that an input word with Hamming weight $i$ produces a codeword with Hamming weight $d_{sr}$.

Then the conditional PEP at high SNR for this scenario, denoted by $P_{ARP}^i(d|h_{sd}, h_{sr}, h_{rd})$, can be calculated as

$$
P_{ARP}^i(d|h_{sd}, h_{sr}, h_{rd}) \leq \prod_{i=1}^{q} P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i}) \prod_{k=q+1}^{n} \left(1 - P_{F_{sr}}^i(\gamma_{sr}|h_{sr,ik})\right) Q\left(\sqrt{2d_{\gamma_{AAF,(q)}}, 2d_{\gamma_{DAF,(n-q)}}}\right)
$$

The probability that the AAF relay group consists of any $q$ relays and the DAF relay group consists of the rest $(n - q)$ relays is given by

$$
\sum_{(i_1, \cdots, i_q) \in \{1, \cdots, n\}} \prod_{i=1}^{q} P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i}) \prod_{k=q+1}^{n} \left(1 - P_{F_{sr}}^i(\gamma_{sr}|h_{sr,ik})\right)
$$

Due to the uniform distribution of relays and assumption of $\gamma_{sr,i} = \gamma_{sr}$ for all $i = 1, \cdots, n$, the average PEP at high SNR, denoted by $P_{ARP}(d)$, can be calculated as

$$
P_{ARP}(d) \leq \sum_{q=0}^{n} \binom{n}{q} \overline{A}(d_{sr})P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i}) \prod_{i=1}^{q} P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i}) \prod_{i=q+1}^{n} \left(1 - P_{F_{sr}}^i(\gamma_{sr}|h_{sr,ik})\right) Q\left(\sqrt{2d_{\gamma_{AAF,(q)}}, 2d_{\gamma_{DAF,(n-q)}}}\right)
$$

$$
\sum_{q=0}^{n} \binom{n}{q} \overline{A}(d_{sr})P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i}) \prod_{i=1}^{q} P_{F_{sr}}^i(\gamma_{sr}|h_{sr,i}) \prod_{i=q+1}^{n} \left(1 - P_{F_{sr}}^i(\gamma_{sr}|h_{sr,ik})\right) Q\left(\sqrt{2d_{\gamma_{DAF,(n-q)}}}\right)
$$

$$
\leq \sum_{q=0}^{n} \binom{n}{q} (f(d))^q \left(1 - P_{F_{sr}}(\gamma_{sr}|h_{sr,i})\right)^{n-q}
$$

$$
= (\overline{A}(d_{sr})d_{sr, min})^{-1}\sum_{d_{sr}=d_{sr, min}}^{2l} \overline{A}(d_{sr})\overline{A}(d_{sr})d_{sr, min}
$$

$$
= (d_{sr})^{-1}(f(d)) + \frac{1}{d_{sr}} \overline{A}(d_{sr})d_{sr, min}
$$

where $f(d) = E\left(P_{F_{sr}}(\gamma_{sr}|h_{sr,i})Q\left(\sqrt{dH_2}\right)\right)$ and $P_{F_{sr}} = \gamma_{sr}^{-1}\sum_{d_{sr}=d_{sr, min}}^{2l} \overline{A}(d_{sr})d_{sr, min}$.

The exact closed form expression of $f(d)$ is too complex to be presented here. At high SNR, $f(d)$ can be approximated as

$$
f(d) \leq \sum_{d_{sr}=d_{sr, min}}^{2l} \overline{A}(d_{sr})\gamma_{sr}^{-1}
$$

$$
\left(1 + \frac{1}{d_{sr} + \overline{A}(d_{sr})d_{sr, min} + \overline{A}(d_{sr})d_{sr, min}}\right)
$$

(16)

By substituting (16) into $P_{ARP}(d)$, we have

$$
P_{ARP}(d) \leq \gamma_{sr}^{-1}\overline{A}(d_{sr})d_{sr, min}^{-1}(1 + \overline{A}(d_{sr})d_{sr, min})^{2l} \left(1 + \frac{1}{d_{sr} + \overline{A}(d_{sr})d_{sr, min} + \overline{A}(d_{sr})d_{sr, min}}\right)^n
$$

(17)
where \( \phi(d) = \sum_{d=0}^{2^j} \binom{d}{j} \) solely depends on the channel code used at the source. It can be proved that \( \phi(d) < 1 \).

Similarly, for a perfect DAF, in which all relay are assumed to decode correctly, the average PEP of incorrectly decoding to a codeword with weight \( d \), denoted by \( P_{DAF}^{\text{Perfect}}(d) \), can be calculated as

\[
P_{DAF}^{\text{Perfect}}(d) = (\gamma_{sd})^{-1}d^{-n}d^{-(n+1)} \tag{18}
\]

Eq. (17) can be further expressed as

\[
P_{ARP}(d) \leq \left( \frac{\gamma_{rd}}{\gamma_{sr}} \phi(d) + 1 \right)^n P_{DAF}^{\text{Perfect}}(d)
\]

\[
= G_{ARP} P_{DAF}^{\text{Perfect}}(d) \tag{19}
\]

where

\[
G_{ARP} = \left( \frac{\gamma_{rd}}{\gamma_{sr}} \phi(d) + 1 \right)^n > 1 \tag{20}
\]

represents the performance loss of the ARP compared to the perfect DAF.

It can be noted from Eq. (20) that as \( \gamma_{sr} \to \infty, G_{ARP} \to 1 \), \( P_{ARP}(d) \to P_{DAF}^{\text{Perfect}}(d) \) and the performance of the ARP approaches the perfect DAF.

Let \( P_{b,ARP} \) be the average upper bound on the bit error rate (BER) for the ARP. At high SNR, it can be approximated as

\[
P_{b,ARP} \approx 4 \sum_{d=d_{\min}}^{2^j} \sum_{j=1}^{\lfloor \frac{d}{d_{\min}} \rfloor} \binom{j}{l} p(d,j) P_{ARP}(d) \tag{21}
\]

where \( \binom{j}{l} \) is the number of words with Hamming weight \( j \) and \( p(d,j) \) is the probability that an input word with Hamming weight \( j \) produces a codeword with Hamming weight \( d \).

From Eq. (21), we can also observe that a diversity order of \( (n+1) \) can be achieved for an ARP scheme in a relay networks with \( n \) relays.

B. Error probability of the AAF

Following the similar analysis, the average PEP of the AAF of incorrectly decoding to a codeword with weight \( d \), denoted by \( P_{AAF}(d) \), can be similarly calculated as

\[
P_{AAF}(d) = Q(\sqrt{2d}\gamma_{AAF}) \leq (\gamma_{sd})^{-1}\left( \frac{1}{\gamma_{sr}} + \frac{1}{\gamma_{rd}} \right)^{d}d^{-(n+1)}
\]

Compared to \( P_{AAF}(d) \), the average PEP of the ARP in Eq. (17) can also be expressed as

\[
P_{ARP}(d) \leq G_{ARP/AAF} P_{AAF}(d) \tag{22}
\]

where

\[
G_{ARP/AAF} = \left( 1 - \frac{1 - \phi(d)}{1 + \gamma_{sr} \gamma_{rd}} \right)^n < 1
\]

represents the performance gain of the ARP over the AAF.

It can be proved that \( G_{ARP/AAF} \) is a decreasing function of \( \gamma_{sr} \). As \( \gamma_{sr} \) increases, the ARP can achieve a considerable error rate reduction compared to the AAF under the same \( \gamma_{sr} \) and \( \gamma_{rd} \) values. This error rate reduction exponentially grows as the number of relays increases.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we provide simulation results comparisons for various relaying schemes with various numbers of relays. All simulations are performed for a BPSK modulation and a frame size of 130 symbols over quasi-static fading channels. We use a 4-state recursive systematic convolutional code (RSC) with the code rate of 1/2 and the generator matrix of \((1, 5/7)\). For simplicity, we assume that \( \gamma_{sr,i} = \gamma_{sr} \) and \( \gamma_{rd,i} = \gamma_{rd} \) for all \( i = 1, \cdots, n \), and \( \gamma_{rd} \) and \( \gamma_{sd} \) are the same.

As discussed in [1], the DAF relay protocol requires the relays to fully decode the source information and this limits the performance of DAF to that of direct transmission between the source and relays, so it does not offer a diversity gain [1].
As we can see from Section IV that other relaying protocols, including the AAF, ARP, can all achieve a full diversity order for large SNR. Considering this, in this paper, we will not compare other relaying protocols with the DAF, simply because that it cannot achieve a full diversity. Instead, we compare with a perfect DAF, where relays are always assumed to decode correctly.

Figs. 2-5 compare the performance of the AAF, the ARP and the perfect DAF for various numbers of relays. It can be noted that as the number of relays increases, the ARP significantly outperforms the AAF in all SNR regions, and perform very close to the perfect DAF as $\gamma_{sr}$ increases. This conclusion is consistent with the analysis in Section IV.

The above results can be explained from the analytical results in Section IV. It can be noted from Eq. (23) that $G_{ARP/AAF}$ is a decreasing function of $\gamma_{sr}$, therefore, as either $\gamma_{sr}$ or $n$ increases, $G_{ARP/AAF}$ is exponentially decreasing, and the ARP can achieve considerable performance gain compared to the AAF. This can be easily explained in the following way. For low $\gamma_{sr}$ values, the channel from the source to the relays are very noisy and probabilities of decoding errors at each relay are very high, so most of relays cannot correctly decode the received signals. In this case, as the number of relays increases, the probability that the DAF relay group contains 2404
at least one relays also increases and the contribution of coding gain from the DAF relay group become significant. This explain the reason why the ARP can provide a significant coding gain over AAF, even at low $\gamma_{sr}$ values, as the number of relays increases.

The above simulation results confirm that the ARP can provide a significant coding gain in all SNR regions and perform very closely to the perfect DAF as $\gamma_{sr}$ increases.

VI. Conclusion

In this paper, we propose a simple adaptive relaying protocol for a general two hop relay networks when imperfect decoding occurs at relays. Based on whether relays can correctly decode or not, we include all the relays into two groups, referred to as an AAF relay group and a DAF relay group. Our results reveal that the proposed scheme can provide a significant SNR gain compared to the pure AAF relay protocols due to the contribution of coding gain from the DAF relay group. This gain increases as the number of relays increases. The proposed scheme can also circumvent the error propagation due to imperfect decoding at relay, which usually occurs in a DAF relay protocol, and approach the perfect DAF scheme at a high SNR region.

References