Abstract—This paper investigates effective SINR mapping (ESM) methods for link abstraction (or channel quality evaluation) used in system level performance simulation. Among all existing ESM methods, the mean mutual information per-bit (MMIB) ESM has been shown to be superior in terms of high accurate block error rate (BLER) prediction and supporting multiple modulation schemes within one data block. However, complicated multi-Gaussian distribution approximation is needed in MMIB-ESM when high modulation schemes such as 16QAM and 64QAM are considered. This paper proposes a new generalized bit-wise E-ESM (BE-ESM), which is shown to have similar performance as MMIB-ESM but with a very simple structure. Furthermore, it is also shown that BE-ESM and MMIB-ESM can be equivalent to each other in certain extents.

Index Terms—Link abstraction, Exponential ESM, Mutual Information ESM,

I. INTRODUCTION

In wireless communications, Monte Carlo simulation has been widely adopted to evaluate the link level error performance. However, in certain circumstances, it is desirable to evaluate the error performance (bit or block) analytically. For example, in system level computer simulation, link abstraction is used to provide link level bit/block error rate (BER/BLER) to evaluate the system level performance such as cell coverage, impact of high layer overhead, system effective throughput, and so on. Analytical link level BER/BLER prediction can greatly accelerate the system level simulation by avoiding the link level Monte Carlo simulation.

In system level simulations, there are two types of link abstraction, i.e., static and dynamic link abstractions. Static link abstraction is based on an averaged (static) SINR [1-2], where the topology and macro channel characteristics (propagation and shadowing) is used to compute the average SINRs of users distributed across the cell. Each user’s average SINR was then mapped to the highest modulation and coding scheme (MCS) that could be supported at the cost of satisfying certain BER/BLER requirement. And the achievable throughputs of these set of MCSs will be treated as the input parameters for system level simulation.

On the other hand, modern broadband wireless systems are designed to exploit instantaneous channel conditions to enhance the system performance. Channel dependent scheduling and adaptive modulation and coding (AMC) based on instantaneous channel state information are two examples. Therefore, link abstraction for those systems should be capable of capture the fast fading effect as well, which is known as dynamic link abstraction.

Without loss of generality, a channel-coded broadband OFDM system with block/frame data transmission is considered in this paper. The channel is assumed to be time-frequency selective, i.e., the channel gains on all sub-carriers within one OFDM data frame will change over sub-carriers and time as well. Defining one tone as one sub-carrier by one OFDM symbol, dynamic link abstraction needs to predict the achievable BLER given a set of SINR values of these tones. Theoretically, the BLER should be a function of this set of SINR values. However, it is very difficult, if not impossible, to get such a function analytically. Instead, an effective SINR mapping (ESM) method is widely used, where the set of tone SINR values is firstly compressed to a single effective SINR with a particular mapping function, then this effective SINR is expected to be with a one-to-one relationship with the BLER. In practice, an exact one-to-one relationship is not achievable since there is always diverseness because of the imperfect mapping function. The accuracy of this one-to-one relationship is then determined by the employed mapping function. In [3-5], the classic Shannon channel capacity formula was used as the mapping function (C-ESM). Later, a more accurate mapping function was introduced in [6-7], where symbol-level mutual information (MI) function (MI-ESM) was used, which can be taken as the constraint channel capacity. Meanwhile, an exponential effective SINR mapping (E-ESM) was presented in [8-11], which was derived from the Chernoff bound of channel decoder. Using adjustable scaling factors in the mapping functions, these methods can provide similar performance. In practice, an optimal scaling factor should be derived for each MCS with numerical method. Thus, when various modulation schemes are considered within one data block, a set of scaling factors are needed and should be optimized jointly. The complexity of this numerical joint optimization process increases exponentially with the number of modulation schemes. Therefore, it is difficult for the fore-mentioned methods to support various modulation schemes in one data block. In addition, scaling factors can only introduce linear effect whereas non-linear effect is more general in real situation. To solve these problems, the MI-ESM was recently extended to calculate the bit-level mutual information [12] and the resultant scheme was known as mean mutual information per-bit (MMIB) ESM. Although this method was simple for BPSK and QPSK, it will involve complicated multi-Gaussian distribution approximation for high order modulation schemes (16/64-QAM). This paper proposes a generalized bit-wise...
E-ESM (BE-ESM) which can perform as good as MMIB-ESM, but with very simple structure. By means of analysis, it will be shown that BE-ESM and MMIB-ESM could be equivalent to each other in certain extents.

The rest of the paper is organized as follows. Section II describes the basic idea of ESM. The BE-ESM method is proposed in section III. Section IV illustrates the equivalency between BE-ESM and MMIB-ESM. Then the performance of BE-ESM is shown in Section V. Finally, conclusions are drawn in the last section.

II. EFFECTIVE SINR MAPPING METHOD

To simplify the descriptions, a single antenna OFDM system is considered. The user data is firstly divided into many blocks. Each block is processed by a transmit signal processing chain including the channel encoder, bit interleaver, bit-to-symbol mapping and multi-carrier modulator. Finally, a frame of modulated OFDM symbols are generated and pass through a time-frequency selective channel. The channel is defined by a set of SINRs on totally \( L \) sub-carriers in \( T \) OFDM symbol duration, given by \( \gamma_{l,t} \), where \( l = 1 : L \), \( t = 1 : T \), and \( \gamma_{l,t} \) denotes the SINR on the \( l \)-th sub-carrier in the \( t \)-th OFDM symbol duration. At the receiver, the frame is passed through a receive signal processing chain composed by multi-carrier demodulator, symbol-to-bit demapping, de-interleaver and channel decoder. Thus, the transmitted data packet is recovered. The purpose of link abstraction is to estimate/predict the BLER based on above system information without doing real time Monte Carlo simulation. Obviously, the achievable BLER is affected by many elements, including block length, coding type, coding rate, modulation scheme, and most important of all, the experienced instantaneous channel condition \( \gamma_{l,t} \). Compared with channel condition \( \gamma_{l,t} \), the impact of other elements are relatively simple and easy to be formulated [12]. Thus, this paper focuses on the impact of \( \gamma_{l,t} \).

First of all, the achievable BLER can be written as a function of the experienced SINRs \( \gamma_{l,t} \), i.e.,

\[
\text{BLER} = f_{lh}\left(\gamma_{l,t}\right)_{l=1,\ldots,L,t=1,\ldots,T}
\]

(1)

However, in practice, it is almost impossible to derive such a function analytically. Instead, ESM method is widely adopted, which decouples the derivation of \( f_{lh}(\cdot) \) into two steps as follows

\[
\text{BLER} = f_{lh}\left(\gamma_{l,t}\right)_{l=1,\ldots,L,t=1,\ldots,T} = f_{l}\left(\gamma_{l,t}\right)_{l=1,\ldots,L,t=1,\ldots,T} + \varepsilon
\]

(2)

where \( f_{l}(\cdot) \) denotes the function of the first step compressing the set of SINR values \( \gamma_{l,t} \) to one single effective SINR value \( \gamma_{eff} \). \( \gamma_{eff} \) is then expected to be with a one-to-one relationship \( f_{l}(\cdot) \) with the BLER theoretically. However, in practice a residual random error \( \varepsilon \) should be considered. The accuracy of BLER prediction can be assessed in terms of the covariance of \( \varepsilon \). The compression function \( f_{l}(\cdot) \) is the key of ESM method and will be discussed later. Given \( f_{l}(\cdot) \), the function \( f_{l}(\cdot) \) can be obtained by numerical method as shown in Fig. 1 with the following steps:

Step 1. Randomly generate a time-frequency selective channel with SINR values \( \gamma_{l,t} \).

Step 2. Calculate \( \gamma_{eff} \) from \( \gamma_{l,t} \) by using \( f_{l}(\cdot) \).

Step 3. Conduct a Monte Carlo simulation for this instantaneous channel to obtain BLER of data block.

Step 4. Plot a dot of the obtained (BLER, \( \gamma_{eff} \) ) pair in the coordination.

Step 5. Repeat step 1 to step 4 until \( f_{l}(\cdot) \) can be obtained through curve fitting over the dots.

Note that the two dimensional \( \gamma_{l,t} \) is reordered as one dimensional variable \( \gamma_{n} \), \( n = 1 : N \), \( N = L \times T \) in Fig. 1 to facilitate the following discussion.

![Diagram of ESM method](image)

Figure 1. Diagram of ESM method

It can be seen that the accuracy of BLER prediction is determined by the compression function \( f_{l}(\cdot) \), which can be heuristically expressed as

\[
\gamma_{eff} = \Phi^{-1}\left(\frac{1}{N} \sum_{n=1}^{N} \Phi(\gamma_{n})\right)
\]

(3)

where \( \Phi(\cdot) \) is an invertible function called ESM mapping function. Various ESM schemes have been proposed according to different ESM mapping functions, such as C-ESM, MI-ESM, MMIB-ESM and E-ESM. Due to the limit of space, only E-ESM is discussed here. Readers can refer to relevant literatures for other schemes.

A. Exponential ESM (E-ESM)

The ESM mapping function of exponential ESM is given by

\[
\Phi(\gamma_{n}) = \exp\left(-\frac{\gamma_{n}}{\beta}\right)
\]

(4)

So the effective SINR \( \gamma_{eff} \) can be obtained as
\[ \gamma_{\text{eff}} = -\beta \ln \left( \frac{1}{N} \sum_{n=1}^{N} \exp\left(-\frac{\gamma_n}{\beta}\right) \right) \]  
\[ (5) \]

where \( \beta \) is a linear scaling factor which can be tuned to optimize the BLER prediction. In practice, different optimal \( \beta \) should be obtained numerically for different modulation and coding schemes. Since only one scaling factor is used in (5), the modulation schemes for all tones are the same. When different modulation schemes are used, multiple scaling factors should be used. However, to jointly optimize the multiple factors numerically, the computational complexity increases exponentially with the number of different factors.

III. GENERALIZED BIT-WISE E-ESM

According to [9], the exponential average operation used by E-ESM was sparked by the Chernoff bound of the error probability of channel coding. However, the exponential average operation in E-ESM was imposed on the effective SINR of modulated symbol rather than the bit SINR in original derivation in Chernoff bound. Although the linear scaling factor \( \beta \) tries to translate the symbol SINR to bit SINR, the relationship between the symbol SINR and bit SINR is in fact not linear. Therefore, E-ESM cannot achieve good performance even though \( \beta \) is optimized numerically.

To solve this problem, a generalized BE-ESM is proposed, where the symbol SINR is accurately mapped to an equivalent bit SINR, then the exponential average operation can be directly imposed on the equivalent bit SINR to get the bit-wise effective SINR. Thus, the key problem is how to evaluate the equivalent SINR contained in the soft information of each individual bit after soft demapping at the receive side. For BPSK and QPSK, the soft information of each individual bit can be treated as noisy BPSK symbol. A complicated multi-Gaussian distribution approximation method was applied by MMIB-ESM in their derivation [12]. This paper presents a very simple equivalent BPSK channel concept as illustrated in the following sub-section.

A. Mapping symbol SINR to bit SINR

Assume that \( \gamma \) is the symbol-level SINR of a modulated QAM symbol with constellation size \( M \), and \( m = \log_2 M \) is the number of bits carried by one symbol. Then this QAM symbol channel can be decoupled into \( m \) parallel BPSK channels each with an equivalent bit SINR \( \gamma_b \) (\( b = 1 : m \)) as shown in Fig. 2, where \( \gamma_b \) is given by

\[ P_{e,b}^{m}(\gamma) = P_{e,b}^{m}(\gamma) \]
\[ \Rightarrow \gamma_b = P_{e,b}^{m}\left[ P_{e,b}^{m}(\gamma) \right] = \Psi_b^{m}(\gamma) \]  
\[ (6) \]
\[
P_{c3}(\lambda) = P_{c3}(\lambda) = Q\left(\frac{1}{\sqrt{21}}\lambda + \frac{3}{4} \sum_{i=2}^{3} (-1)^{i-2} Q\left(\frac{(2i-1)^2}{21}\lambda\right) - \frac{2}{4} \sum_{i=4}^{5} (-1)^{i-4} Q\left(\frac{(2i-1)^2}{21}\lambda\right) + \frac{1}{4} \sum_{i=6}^{7} (-1)^{i-6} Q\left(\frac{(2i-1)^2}{21}\lambda\right)\right)
\]

where *Q*(·) is the Q-function. Note that precise BERs of different bits of the QAM symbols are derived here.

**B. BIT-WISE EFFECTIVE SINR**

After decoupling, the exponential average operation can be directly imposed on the equivalent bit SINR. Thus, the Bit-wise effective SINR can be given by

\[
\gamma_{b,\text{eff}} = -\ln \left\{ \frac{1}{mN} \sum_{n=1}^{N} \sum_{b=1}^{m} e^{-\gamma_{b,n}} \right\}
\]

It can be seen from (7) that BE-ESM uses an accurate symbol to bit SINR mapping function \(\Psi_{b,m}(\cdot)\) to replace the linear scaling factor \(\beta\) in E-ESM. With \(\Psi_{b,m}(\cdot)\), it is easy for BE-ESM to support various modulation schemes within one data block. In specific, a generalized version of equation (7) can be written as

\[
\gamma_{b,\text{eff}} = -\ln \left\{ \frac{1}{mN} \sum_{n=1}^{N} \sum_{b=1}^{m} e^{-\Psi_{b,n}(\gamma_{n})} \right\}
\]

where \(m_{n}\) denotes the modulation order of symbol on the \(n\)-th tone.

Moreover, it is also convenient to extend this method to MIMO system with linear spatial detection method where the post detection SINR of the system can be used to calculate the bit-wise effective SINR. For MIMO system with MLD, the equivalent bit SINR can also be obtained directly from the derived BER of each individual bit (discussed in another paper), which is also simpler compared with MMIB-ESM.

**IV. EQUIVALENCY BETWEEN BE-ESM AND MMIB-ESM**

It has been shown in [12] that MMIB-ESM outperforms other methods like C-ESM, MI-ESM, and E-ESM. In the following context, it will be shown that the proposed BE-ESM can be equivalent to MMIB-ESM. First of all, equation (7) can be rewritten as

\[
\gamma_{b,\text{eff}} = -\ln \left\{ 1 - \frac{1}{mN} \sum_{n=1}^{N} \sum_{b=1}^{m} (1 - e^{-\Psi_{b,n}(\gamma_{n})}) \right\}
\]

Defining a new function, \(B_{m}(\gamma)\), as

\[
B_{m}(\gamma) = \frac{1}{m} \sum_{b=1}^{m} (1 - e^{-\Psi_{b,n}(\gamma)})
\]

it can be seen that \(B_{m}(\gamma)\) in BE-ESM corresponds to the Mean Mutual Information per-Bit function \(I_{b}(\gamma)\) in MMIB–ESM [12]. In order to show the similarity between these two functions, the function curves are presented in Fig. 3a) for different modulation schemes (different \(m\)). It is shown that they are almost same to each other except a SINR offset between each pair of them. This offset can be easily removed by introducing a extra scaling factor \(\alpha\) as follows

\[
B_{m}(\gamma) = \frac{1}{m} \sum_{b=1}^{m} (1 - e^{-\Psi_{b,n}(\gamma)\alpha})
\]

When \(\alpha = 1.36\), another set of curves are plot in Fig. 3b), where the SINR offset becomes negligible.

**Figure 3. Comparison between MMIB-ESM and BE-ESM**

It should be noted that both MMIB-ESM and BE-ESM are derived based on heuristic ideas. MMIB-ESM is based on the constraint channel capacity, while BE-ESM is based on the Chernoff bound. Theoretically, it is difficult to prove which method could perform better than the other. Fortunately, they are quite similar to each other, however, with entirely different mechanism and therefore different feasibility for further enhancement. Before the presentation of this paper, it is widely
accepted by some wireless standard bodies such as 802.16m that MMIB-ESM is the most promising solution for link abstraction. This paper points out something they just missed.

V. NUMERICAL RESULTS

Performance comparison between BE-ESM and MMIB-ESM is carried out using the method stated in Fig. 1. First of all, consider an OFDM system with 16QAM, 1/2 convolutional coding and 1000 bytes block length. 20 instantaneous channel realizations with different transmitted powers are generated. The simulated BLERs are shown in Fig. 4 as a function of the effective SNR calculated with BE-ESM or MMIB-ESM. It can be seen that these curves are quite close to each other in both cases. Specifically, the square roots of the variances of effective-SINR for the same BLER are within 0.1dB for both of them. Therefore, the proposed BE-ESM can provide an accurate estimation for the system performance as well as MMIB-ESM. By curve fitting over these simulated curves, the curve of \( BLER = f_1(\gamma_{eff}) \) can be obtained.

Moreover, the performance of BE-ESM is investigated with various MCSs. The simulated system is a MIMO-OFDM system with 2 transmit and 2 receive antennas. As shown in Table I, various combinations of different coding rate, different modulation scheme, different number of spatial streams are considered. Different modulation schemes over different spatial streams are also considered (MCS 16-21). For single spatial stream MCSs, STBC is used. For two spatial streams MCSs, SDM with MMSE detector is considered. Table 1. list of evaluated MCSs

<table>
<thead>
<tr>
<th>MCS Label</th>
<th>Code rate</th>
<th>Nr. of Streams</th>
<th>Modulation of Streams 1</th>
<th>Modulation of Streams 2</th>
<th>MUX Label</th>
<th>Code rate</th>
<th>Nr. of Streams</th>
<th>Modulation of Streams 1</th>
<th>Modulation of Streams 2</th>
</tr>
</thead>
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<td>1</td>
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<td>QPSK</td>
<td></td>
<td>1</td>
<td>1</td>
<td>BPSK</td>
<td>QPSK</td>
</tr>
<tr>
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<tr>
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<td>1</td>
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<tr>
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<tr>
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<tr>
<td>6</td>
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</table>

Fig. 5 only shows the curves obtained through curve-fitting (i.e. \( f_2(\gamma_{eff}) \)) for different MCs. It is seen that for MCs with the same coding rate but different modulations, the BLER vs. effective SINR curves are quite close to each other. This is because BE-ESM calculates the bit-wise effective SINR. Thus, the impact of different modulations is removed. Moreover, since only the curves of different coding rates are needed to be stored, the realization of BE-ESM in practice is simplified.

VI. CONCLUSIONS

Effective SINR mapping has been widely adopted in link abstraction. The accuracy of the link abstraction is determined by the utilized ESM function. This paper presents a new bit-wise exponential ESM function, BE-ESM. It has been shown that BE-ESM can perform as well as MMIB-ESM but with a very simple structure. Furthermore, it has also been analytically shown that the BE-ESM and MMIB-ESM are equivalent to each other in certain extents.

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