Distributed Optimization for Utility-Energy Tradeoff in Wireless Sensor Networks

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Abstract—Wireless sensor networks (WSNs) are energy-constrained in nature, in this paper, we formulate the problem of data transport in sensor networks as a network utility maximization (NUM) problem, but we argue that each source utility not only depends on its source rate, but also on the consumed energy, this leads to a coupled utility model, where the utilities are functions of source rates and consumed energy. Differentiating from the classical NUM framework which usually takes the consumed energy as constraints, Our utility model regards consumed energy as one of the components of measure of the utility values, which indicates the tradeoff of source rates and consumed energy, it is a more accurate utility model for abstracting the energy characteristics for data gathering and transmission in WSNs.

Due to the coupled energy utility, our optimization problem is not separable. Despite the difficulty, we present a systematic approach to decouple our NUM problem with coupled utilities by introducing into the slack variables and using dual decomposition techniques, and obtain a distributed algorithm for solving our problem. The proposed algorithm can converge to the Pareto optimal tradeoff between rates and energy for all users.

I. INTRODUCTION

We consider a wireless sensor network of nodes distributed in a certain region, and assume that sensors are powered by small batteries that cannot be replaced. Under this energy constraint, sensor nodes can only transmit a finite number of bits in their lifetime. Consequently, reducing the energy consumption per bit for end-to-end transmissions is a key concern in WSNs. Since a significant portion of the communication in WSNs is due to data gathering, it is crucial to design energy-efficient communication strategies in implementing such an operation.

The severe energy constraints of the sensor nodes engender the need for designing efficient data gathering and transmitting algorithms based on mathematical modeling and optimization techniques. Over the past few years, since the publication of the seminal paper [1] by Kelly et al. in 1998, by allowing nonlinear, concave utility objective functions, the framework of Network Utility Maximization has found many applications in wireless as well as wireline networks [2], [3], [4]. Consider a communication network with $L$ links, each with a fixed capacity of $c_l$ bps, and $S$ sources with transmission rate of $x_s$ bps for $s \in S$. Each source $s$ uses a fixed set $L(s)$ links in its route path, and has a utility function $U_s(x_s)$. Each link $l$ is shared by a set $S(l)$ of sources. The basic version of NUM can be formulated as to maximize the total utility of the network

$$\sum_s U_s(x_s)$$

over the source rates $x_s$, subject to the constraints of link capacity

$$\sum_{s \in L(l)} x_s \leq c_l$$

for all links $l$ as following:

$$\begin{align*}
\text{maximize} & \quad \sum_s U_s(x_s) \\
\text{subject to} & \quad \sum_{s \in S(l)} x_s \leq c_l, \quad \forall l
\end{align*}$$

There are many nice properties of the above NUM model (1) due to the standard assumption on concavity of the utility functions and flow constraints. Moreover, problem (1) can be solved by using a dual-based distributed algorithm, in which the Lagrange dual variables can be interpreted as shadow prices for resource allocation. By regulating prices for resource allocation, each user and the network maximize their net utilities and net revenue, respectively.

In this paper, we formulate the problem of data transport in WSNs as a NUM problem whose objective function is to maximize the amount of information (utility) collected in each sensor node, subject to the wireless channel bandwidth and energy constraints. But differentiating from the current NUM framework which usually assumes that each user’s utility is only a function of local source rate, we believe energy is one of the most important components in WSNs, and think each user’s utility not only depends on its local source rate, but also on the total energy consumed in relaying its data packets to a sink. Moreover, a higher data rate results in greater sensing and transmitting energy costs, there is an inherent tradeoff between rates and energy consumed. Hence our utility functions indicate the tradeoff between the system utility and energy consumed, which are more accurate utility model for abstracting the energy characteristics for packet transmission in WSNs.

NUM based approaches have been explored in sensor networks recently. Byers et al. [5] consider the optimization problem of maximizing the overall utility of sensor networks during the system lifetime, in which its utility function maps the number of nodes participating in a sensory computation to a real value which quantitatively measures the utility. Sadagopan et al. [6] use NUM approach constructing a energy balance tree in sensor networks, where each sensor node’s utility depends on the selection of its parent node. Chen et al. [7] consider a NUM problem with utility function depending on source rates and the average end-to-end latency that a packet experiences from a sensor node to a sink. Nama et al. [8] consider utility-lifetime tradeoff problem, which model can...
be treated as special cases of our model. All of the work in [5]–[8] consider local utility function. In this paper we consider the forwarding energy costs, which leads to a globally coupled energy utility. Qiu et al. [9] incorporate the forwarding cost, which is similar to ours; but its utility only depend on source rates, its objective is maximize the net utility of the network, this is also a special case of our model. 

In the basic NUM model (1), convexity and separability properties of the optimization problem readily lead to a distributed algorithm that converges to the globally optimal solution. However, our new optimization formulations is not separable because of the globally coupled utility. This extended the coupled utilities model in [10] and [11], in which they consider the coupling between different source rates. Despite such difficulties, we develop simple and distributed algorithm based on pricing that converge to the optimal rate-energy tradeoff with readily verifiable sufficient conditions.

The rest of this paper is organized as follows. Section II presents our system model and problem. Section III describes a dual decomposition based distributed algorithm. Section IV provides the simulations results. Finally Section V concludes this paper.

II. System Model and Problem Formulation

We consider a static wireless sensor network with a set $N$ of sensor nodes and a set $L$ of logical wireless communication links. We assume that a link between node $i$ and $j$ exists if and only if node $i$ and $j$ are within the transmission range of each other, i.e., if and only if they can communicate directly. We assume that the destination nodes (also called sinks) are denoted by a set $D$. Let $x_s$ denote the non-negative data rate from sensor node $s \in N$ to a sink node, rate $x_s$ satisfies $m_s \leq x_s \leq M_s$, where $m_s$ and $M_s$ are the minimum and maximum transmission rates. For simplicity, we also assume that transmissions are perfectly scheduled, so that no one node's transmission interferes with another.

We define $S(n)$ as the set of sources that use node $n$ as a relay node (including source $n$ itself), and $R(n)$ as the set of nodes that relay packets for the source $n$ (including source $n$ itself). We assume that for each node $n$, the node capacity is $c_n$, which can be estimated as in [9]. For each node $n$, the aggregate relaying rate does not exceed the node capacity, thus we have

$$\sum_{s \in S(n)} x_s \leq c_n \quad \forall n \in N$$

(2)

A. Energy dissipation

In a sensor network, sensor nodes often have much tighter energy constrains than the sinks, hence we will focus only on the energy dissipated in sensor nodes. We assume that the initial energy of node $n$ denotes by $e_n$, and nodes are simply expected to continue collecting data until they exhaust their energy supply. We characterize the energy dissipation in each sensor node by the energy consumed in the sensing state per unit time $\varepsilon_s$, the energy consumed in transmitting and receiving one unit of data per unit time $\varepsilon_t$ and $\varepsilon_r$. We assume that the lifetime of a network is defined as the time at which the first node in the network drains out of energy [12], and the pre-specified, desired system lifetime denote by $T$. For each sensor node $s$, we denote the consumed energy in sending collected data to a sink node $d$ by $\sum_{n \in R(s)} p_n$, where $p_n$ denote the consumed energy of the node $n$ in relaying the data packets of the node $s$. We define the marginal energy of each node $s$ as the sum of the energy consumed by all other nodes for relaying its data packets. And for each sensor node $n$, the average consumed energy should satisfy

$$(\varepsilon_r + \varepsilon_t) \sum_{s \in S(n)} x_s - \varepsilon_r x_n + \varepsilon_s x_n)T \leq e_n \quad \forall n \in N$$

(3)

Combining equation (3) and the meaning of $\sum_{n \in R(s)} p_n$, we have

$$\sum_{n \in R(s)} p_n \leq \sum_{n \in R(s)} e_n$$

(4)

B. Problem Formulation

We formulate the problem of data transport in sensor networks as NUM problem, and use a coupled utility function, the problem can be formulated as:

$$\max_{m_s \leq x_s \leq M_s} \sum_{s \in N} U_s(x_s, \sum_{n \in R(s)} p_n)$$

subject to

$$\sum_{s \in S(n)} x_s \leq c_n \quad \forall n \in N$$

$$\sum_{n \in R(s)} p_n \leq \sum_{n \in R(s)} e_n$$

(5)

Where the utility function for node $s$ depends on source rate $x_s$ and consumed energy $\sum_{n \in R(s)} p_n$ for transmitting collected data from node $s$ to a destination $d$, which leads to an energy-coupled utility model. Solving the above problem (5) directly requires coordination among each source $s$ and all nodes which relay data for source $s$, thus is impractical in real network. In the next section, we will solve the dual problem and get a distributed algorithms, and interpret the resulting algorithm in the context of a joint transport layer problem and an energy problem.

III. Dual Decomposition Based Distributed Algorithm

In problem (5), though the source rates $x_s$ and consumed energy $\sum_{n \in R(s)} p_n$ are coupled in their constraints, respectively, but they are separable and the constraint set is still convex; in order to derive a distributed algorithm to solve problem (5) and to prove convergence to global optimum, the main difficulty is that each utility function is coupled by the marginal energy, because each source may not know the marginal energy consumed by other nodes for relaying its data packets. The key idea to tackle coupled utilities is to introduce slack variables and additional equality constrains, and to transfer the coupling in the objective function to coupling in the constraint set, which
can be then decoupled by dual decomposition and solved by introducing additional marginal price.

To solve problem (5), we introduce slack variables $p_{sn}$ for the coupled energy arguments in the utility functions, and add an equality constraints:

$$m_{s}x_{s} \leq x_{s} \leq M_{s}, p_{sn} \geq 0$$
subject to
$$\sum_{s \in N} U_{s}(x_{s}, \sum_{n \in R(s)} p_{sn})$$

$$\sum_{s \in (s)} x_{s} \leq c_{n} \quad \forall n \in N$$
$$\sum_{n \in R(s)} p_{n} \leq \sum_{n \in R(s)} e_{n}$$
$$p_{sn} = p_{n}, \quad \forall s, n \in R(s)$$

(6)

We use a dual decomposition approach to solve problem (6). By relaxing all the coupling constrains in problem (6), we write down the Lagrangian associated with problem (6) as follows, where $\lambda_{s}, \mu_{s}$ and $\gamma_{sn}$ are the Lagrange multipliers on node $s$ with their interpretation of capacity price, energy price and marginal price, respectively.

$$L(x_{s}, p_{n}, p_{sn}; \lambda_{s}, \mu_{s}, \gamma_{sn}) = \sum_{s} U_{s}(x_{s}, \sum_{n \in R(s)} p_{sn})$$
$$+ \sum_{s} \lambda_{s}(c_{s} - \sum_{n \in R(s)} x_{n})$$
$$+ \sum_{s} \mu_{s}(( - \sum_{n \in R(s)} p_{n})$$
$$+ \sum_{s} \gamma_{sn}'(p_{n} - p_{sn})$$

where $\gamma'$ means the transpose of $\gamma$. From above the expression, the Lagrangian can be separated into many subproblems, where each subproblem uses only local variables, i.e., the sth subproblem uses only variables with the first subscript index $s$. The dual function is given by

$$Q(\lambda_{s}, \mu_{s}, \gamma_{sn}) = \max_{m_{s} \leq x_{s} \leq M_{s}, p_{n} \geq 0, p_{sn} \geq 0} L(x_{s}, p_{n}, p_{sn}; \lambda_{s}, \mu_{s}, \gamma_{sn})$$

subject to

It is clear from the above expression of the Lagrangian that the dual function can be evaluated separately in each of the variable $\lambda_{s}, \mu_{s}$ and $\gamma_{sn}$. The dual problem is given by

$$\min_{\lambda_{s}, \mu_{s}, \gamma_{sn}} Q(\lambda_{s}, \mu_{s}, \gamma_{sn})$$
subject to

To solve the dual problem, we first consider problem (7). Since the Lagrangian is separable, this maximization of Lagrangian over $(x_{s}, p_{n}, p_{sn})$ can be conducted in parallel at each source $s$

$$\max_{m_{s} \leq x_{s} \leq M_{s}, p_{n} \geq 0, p_{sn} \geq 0} U_{s}(x_{s}, \sum_{n \in R(s)} p_{sn}) - x_{s} \sum_{n \in R(s)} \lambda_{n} - p_{s} \sum_{n \in S(s)} \mu_{n}$$
$$+ p_{s} \sum_{n \in S(n)} \gamma_{sn}'(p_{n} - p_{sn})$$
subject to

As a matter of fact, our utility functions depend on energy arguments, our primal problem (5) encompasses a wide variety of requirements and objectives for different applications, which have different design approaches to determine appropriate utility functions [13], [14]. Thus we can’t assure the objective function is strictly concave, the dual function might not be differentiable. Here we will solve the dual problem using subgradient projection method [15] as following distributed algorithm:

**Algorithm:** at each iteration $t$

- **Step 1:** The each node $s$ updates its capacity price as

$$\lambda_{s}(t + 1) = [\lambda_{s}(t) - \alpha(t)(e_{s} - \sum_{n \in S(s)} x_{n})]^{+}$$

and then broadcasts to the nodes who use node $s$ relaying their data packets. Note each node $s$ can update its $\lambda_{s}$ by its local information. Where $x_{n}$ is the solution of problem (12) for a given $(\lambda, \mu, \gamma)$, $\alpha(t)$ is a positive scalar stepsize, and $[a]^{+}$ denotes the projection of $a$ onto the set $R^{+}$ of non-negative real numbers.

- **Step 2:** The each node $s$ updates its energy price as

$$\mu_{s}(t + 1) = [\mu_{s}(t) - \beta(t)(e_{s} - \sum_{n \in S(s)} p_{n})]^{+}$$

and then broadcasts to the nodes who use node $s$ relaying their data packets. Note each node $s$ can update its $\mu_{s}$ by its local information. Where $p_{n}$ is the solution of problem (12) for a given $(\lambda, \mu, \gamma)$, $\beta(t)$ is a positive scalar stepsize.

- **Step 3:** The each node $s$ updates its marginal price as

$$\gamma_{sn}(t + 1) = [\gamma_{sn}(t) - \delta(t)(p_{sn} - p_{sn})]^{+}, \forall n \in R(s)$$

and then broadcasts to the nodes who relay data packets for node $s$. Note each node $s$ can update its $\gamma_{sn}$ by its local information. Where $p_{s}$ and $p_{sn}$ are the solution of problem (12) for a given $(\lambda, \mu, \gamma)$, $\delta(t)$ is a positive scalar stepsize.

- **Step 4:** For each node $s$ locally solves the problem (12).

$$\max_{m_{s} \leq x_{s} \leq M_{s}, p_{n} \geq 0, p_{sn} \geq 0} U_{s}(x_{s}, \sum_{n \in R(s)} p_{sn}) - x_{s} \sum_{n \in R(s)} \lambda_{n} - p_{s} \sum_{n \in S(s)} \mu_{n}$$
$$+ p_{s} \sum_{n \in S(n)} \gamma_{sn}'(p_{n} - p_{sn})$$
subject to

Note in problem (12), $\gamma_{sn}, \forall n \in R(s)$ are slack local variables for the node $s$, hence solving problem (12) can be conducted in distribution at each source $s$.

In each iteration $t$, by locally solving the problem (12), each source $s$ determines its data rate, calculate total energy consumed and the marginal energy for relaying the data packets of other nodes. In the current iteration the solution
maximizes the net utility of nodes on the capacity price, energy price and marginal price, in fact we can interpret the objective function (12) as that $U_s(x_s, \sum_{n \in R(s)} p_{sn})$ is the total utility, $x_s \sum_{n \in R(s)} \lambda_n$ is the capacity cost, $p_s \sum_{n \in S(s)} \mu_n$ is the energy cost, $p_s \sum_{n:s \in S(n)} \gamma_{sn} - \sum_{n \in S(s)} p_{sn} \gamma_{sn}$ is the marginal cost which is equal to minus of the relaying cost of other nodes for data packets of node $s$ and the relaying cost of node $s$ for the data packets of other nodes.

In our algorithm, to solve problem (12), source $s$ needs to know some information from nodes $n \in S(s)$, and source $s$ also needs to notify some information to the set of nodes $R(s)$. This can be obtained through the presence of acknowledgment packets in TCP [16].

After the above dual decomposition, the following result can be proved using standard techniques in distributed subgradient algorithm’s convergence analysis as in [17].

**Theorem 1:** By the above algorithm, dual variables ($\lambda(t)$, $\mu(t)$, $\gamma(t)$) converge to the optimal dual solutions ($\lambda^*$, $\mu^*$, $\gamma^*$), if the set sizes are chosen such that $\alpha(t) \to 0$, $\sum_{t=1}^{\infty} \alpha(t) = \infty$, $\beta(t) \to 0$, $\sum_{t=1}^{\infty} \beta(t) = \infty$ and $\delta(t) \to 0$, $\sum_{t=1}^{\infty} \delta(t) = \infty$.

By theorem 1, dual variables ($\lambda(t)$, $\mu(t)$, $\gamma(t)$) converge to the optimal dual solutions ($\lambda^*$, $\mu^*$, $\gamma^*$). Since primal problem (6) is a convex optimization problem, if Slater condition for strong duality [15] holds, the corresponding primal variables ($x^*$, $p^*$) are the globally optimal solutions of primal problem (6). In fact, the constraints in primal problem (6) are strictly feasible because that $c_n$ and $e_n$ are strictly positive. Thus, Slater condition for strong duality of primal problem (6) are holds. So, we can obtain the globally optimal solutions of primal problem (6) by the above distributed algorithm.

### IV. Simulations

In this section, we present numerical examples for the proposed algorithms by considering a sensor network, shown in Fig.1, with a linear topology consisting of three sensor nodes and a sink node. We assume that the interference is eliminated by using Time Division Multiple Access, and each active link has equal fractions of time. Every node transmits the collected data to sink node $d$. Only node 3 can directly communicate with sink node $d$, node 2 transfers the collected data to sink node $d$ by the forwarding of node 3, node 1 transfers the collected data to sink node $d$ by the forwarding of the node 2 and node 3.

![Network topology](image)

We assume that utility function for user $s$ is $U_s(x_s, \sum_{n \in R(s)} p_{sn})$, and with utility on rate and consumed energy with a given weight $a$ between rate and consumed energy

$$U_s(x_s, \sum_{n \in R(s)} p_{sn}) = a \log(x_s) - (1 - a) \sum_{n \in R(s)} p_{sn}$$

We assume that each sensor node has initial energies of 0.25 Joules. The parameters for the node energy dissipation model $\varepsilon_s$, $\varepsilon_r$ and $\varepsilon_t$ are chosen to be 50 nJ/bit/s, 135 nJ/bit/s, and 45 nJ/bit/s [8] respectively. We assume that the constant parameters have the following values: $m_s = 0.002$ (Mbps), $M_s = 0.02$ (Mbps) and the maximization node capacity is 0.02 (Mbps).

Fig.2 shows the network utility as the parameter $a$ is varied for 0.1 to 0.9. A larger value of $a$ implies that we put more weight on increasing network utility and less weight on decreasing energy consumption. The case of $a = 0$ corresponds to an energy minimization problem, hence, each node has a lower data rate. As the value of $a$ increases, the data rate of each user increases providing higher total system utility. This increase in network utility comes from a corresponding increase in energy consumption of the network. Fig. 3 and Fig. 4 show the node source rate and the consumed energy as the weight $a$ varies. As expected, as $a$ increases, node source rates increase, accordingly, the consumed energy of the node increase. When $a = 1$, our problem becomes an utility maximization problem by controlling the node source rate, hence each node consumes energy as much as possible to maximize the network utility.

![Network utility as a function a](image)

Fig.5 shows the network utility and the consumed energy trade-off curve in the pre-specified lifetime as the parameter $a$ varies. The curve is globally optimal, which clearly illustrates the inherent trade-off between the network utility and the energy costs in energy-limited wireless sensor networks. As expected, the total utility of the network almost linearly increase as the total consumed energy increase. Moreover, we can always find a better operating point on the curve than a operating point in the below of the curve. Hence, operating on the trade-off curve, i.e., the Pareto optimal trade-off curve, is the best.
V. CONCLUSION

Motivated by the application layer performance (utility) and the energy-limited characteristic in WSNs, we argue that the network utility not only depends on the source rates, but also on the consumed energy. And there is an inherent trade-off between them. We proposed a framework to trade off the source rates and the consumed energy. The basic network utility maximization is thus extended to concave maximization problem over nonlinear and coupled objective functions, which is much more difficult problem to be solved by distributed and globally optimal algorithms. In particular, the standard price-based distributed algorithm cannot be applied because of the coupled objective. We proposed a new price-based distributed algorithm. The key idea is that changing the coupled objective functions into the coupled constrains by introducing slack variables, which can be solved by the dual decomposition based method. Our algorithm presents three kinds of prices, i.e., capacity price, energy price and marginal price, which interact through the node capacity, node energy and forwarding energy.

REFERENCES