Abstract—This study considers all the possible actions a borrower may have, i.e., to default, to prepay, and to maintain the mortgage, during mortgage horizon. Then, we provide an effective and accurate pricing formula, which not only considers the effect that default might affect the mortgage value, but also more accurately explores the impact due to prepayment risk. In our model, we define prepayment value of the mortgage as the amount of outstanding principle. In contrast, previous literature defines prepayment value as a constant proportion of maintaining value of the mortgage. Finally, based on closed-form pricing formula, we analyze the yield, duration and convexity of risky mortgage loan, providing a better framework for risk management.

I. INTRODUCTION

For banks and investment managers, the unpredictable default or prepayment of a borrower is a serious risk. Furthermore, the fluctuation of the interest rate has a significant effect on the mortgage value. The change in interest rates not only directly influences the mortgage value, but also influences the behavior of the investor. Therefore, in order to show the connection between the mortgage value and the change in the interest rates, the calculation of duration and convexity is important for the risk management [1].

In our article, we aim at providing a closed-form formula for calculating the value of the fully-amortizing fix-rate mortgages (FRMs). This problem has been studied. Kau and Keenam [2] have collected the related articles of these pricing models. Since all the borrowers may default or prepay during the contract horizon, the contractual property is very similar to the American option.

Dunn and McConnell [3],[4] presented an option-based approach to model the valuation of mortgage. The option-based approach needs complicated numerical calculation to solve the partial differential equation, while these numerical methods need a great amount of calculations. Additionally, Collin-Dufresne and Harding[5] propose the closed-form solution based on the option-based approach to make the calculation more efficiently.

In this study, our evaluation model is presented based on the continuous-time intensity-form model. Intensity-form model has already been used in calculating the probability of termination risk of financial securities. Duffie and Wang [6] used the intensity-form model to precisely estimate the probability of default of a company. Kau et al. [7] presented a FRMs valuation model using the intensity-form model, and obtained the value based on the Monte Carlo simulation. Pliska [8],[9] and Longstaff [10] have developed a multi-stage decision model according to the intensity-based approach which considers the sequential refinancing. Gorovoy and Vadim Linetsky [11], proposed the closed-form solution, which modeled the exogenous and refinancing prepayment. Tsai et al. [12] derived a closed-form for calculating the mortgage value under continuous-time intensity-form model. In their paper, the intensity rates of prepayment and default were assumed to be linear functions of interest rate, and they also include jump process to model the occurrence of non-financial events that cause prepayment and default. They also assumed the loss rate of mortgage value to be a constant regardless of when the borrower prepays the mortgage.

In this article, to account for the termination risks, we derive a precise closed-form formula for valuing of FRMs. Then, we use backward recursion method to capture the possible behaviors of borrower. Following Duffie and Singleton [13], it is assumed that the intensity rates of prepayment and default are proportional to the interest rate (the state variable in our economy). To take into account the financial events (triggers), the jump structure is also included.

The major contribution of this paper is that we accurately explore the impact due to prepayment risk in the pricing model. Specially, mortgage value with prepayment risk is affected significantly by the difference between risk-free interest rate and coupon rate of the mortgage. Fig.1 shows one month Libor rate from September, 1991 to September, 2011. The Libor rate is 5.5 percentages in September, 1991 and 0.23 percentages in September, 2011. A FRM is a 30 years contract from September, 1990 to September, 2019. The value of the mortgage to the lender is different for the borrower to prepay at the two different points in time. Our article proposes an accurate pricing model which calculate mortgage precisely under such dynamics, and we also derive the closed-form formula to reduce the amount of computation.

Sensitivity analysis is also performed on the closed-form
The termination risks of a mortgage are prepayment risk and mortgage (FRM), taking termination risks into consideration. In this section, we present a model for pricing fixed-rate mortgage (FRM), taking termination risks into consideration. The rest of the paper is organized as follows: In section II, we present the pricing model in both discrete and continuous time settings. We then derive the closed-form pricing formula using the extended transform affine model in section III. In Section IV, sensitivity analysis of mortgage loan yield, duration and convexity based on our pricing formula is provided. Finally, section V presents conclusions.

In this section, we present a model for pricing fixed-rate mortgage (FRM), taking termination risks into consideration. The termination risks of a mortgage are prepayment risk and default risk. In this paper, we assume if the borrower prepay the mortgage, it means that he fully prepay this mortgage. Before deriving the model, we have to define some notations. We define $M_0$ as the initial mortgage principal, $c$ the fixed coupon rate, and $T$ the maturity of this mortgage. Because we assume the mortgage is a FRM, the payment is a constant for every payment date, and we assume the constant payment $Y$ per unit time, which can be derived as follows:

$$Y = M_0(c/(1 - \exp(-cT)))$$

(1)

For calculating the price of mortgage, we use a discrete time approximation to approximate the price of mortgage, and then derive the continuous formula of pricing mortgage. Assume the time interval of each payment date is $\Delta t$. The index $i$ means the point of valuation, where $i = 0, 1, ..., n$ and $n = T/\Delta t$. We assume there are two options for borrower to deal with the mortgage at each time point $i$. That is to prepay or maintain the mortgage. When the borrower prepay the mortgage, lender will get the outstanding principal at time $i$, denoted as $M_i$. The probability of the prepayment at time $i$ is $P^p_{i+1}$. When the borrower maintains this mortgage, there are two conditions. The borrower may default the mortgage with the probability $P^d_{i+1}$, or may pay on time with the probability of $1 - P^p_{i+1} - P^d_{i+1}$. If the borrower pays on time, the value of mortgage will be $Y\Delta t + V_{i+1}$. And if the borrower defaults the mortgage, there will be some loss to the mortgage. We denote the loss rate of default as $\eta_{i+1}$, $0 < \eta_{i+1} \leq 1$. The rate is a random variable showing the partial loss of the mortgage value during the default. Then, the value of the mortgage obtained after default will be denoted as $(1 - \eta_{i+1})(Y\Delta t + V_{i+1})$.

$$Q_i = \exp(-r_1\Delta t)[(1 - P^p_n - P^d_n)\exp(-r_n\Delta t) + Y\Delta t(1 - \eta_n)P^d_n\exp(-r_n\Delta t) + M_{n-1}P^p_n]$$

(2)

Let $Q_i = \exp(-r_1\Delta t)[(1 - P^p_n - P^d_n) + (1 - \eta_n)P^d_n]$, and approximate $\exp(x)$ by $(1 + x)$ on small value of $x$ using the notion of Taylor series expansion [14], we can rewrite $Q_i$ as:

$$Q_i \approx 1 - [r_1\Delta t + P^p_n - r_1\Delta tP^p_n + \eta_nP^d_n(1 - r_1\Delta t)]$$

$$\approx \exp[-(r_1\Delta t + P^p_n - r_1\Delta tP^p_n + \eta_nP^d_n(1 - r_1\Delta t))]$$

(3)

then we can rewrite equation (2) as:
\[ V_{n-1} = E_{n-1}[Y \Delta t Q_n + M_{n-1}P^P_n] \]  
(4)

The value of the mortgage at time point \( i = n - 2 \) is:

\[ V_{n-2} = E_{n-2}[(Y \Delta t + V_{n-1})Q_{n-1} + M_{n-2}P^P_{n-1}] \]  
(5)

Substitute \( V_{n-1} \) from equation (4) into equation (5), and use property of expectation \( E_i[E_{i+1}[\cdot]] = E_i[\cdot] \), we obtain a new equation (6):

\[ V_{n-2} = E_{n-2}[Y \Delta t Q_{n-1} + Y \Delta t Q_n Q_{n-1} + M_{n-1}P^P_{n-1} + M_{n-2}P^P_{n-1}] \]  
(6)

Iterate to initial point, we can obtain the initial mortgage value \( V_0 \) as:

\[ V_0 = E_0[Y \Delta t \sum_{i=1}^n \prod_{j=1}^i Q_j] + E_0 \sum_{i=1}^n M_{i-1}P^P_i(\prod_{j=1}^i Q_j/Q_i) \]  
(7)

where \( Q_0 = 1 \), and \( \prod_{i=1}^1 Q_j/Q_i = Q_0(\prod_{j=1}^1 Q_j) = \prod_{j=1}^1 Q_j-1 \). Substitute \( Q_i \) from equation (3), and \( M_i = M_0(1 - \exp[-c(T - i \Delta t)])/(1 - \exp(-cT)) \), then we can obtain \( V_0 \) as:

\[ V_0 = Y E_0[Y \Delta t \sum_{i=1}^n \exp(- \sum_{j=1}^i (r_j \Delta t + P^P_j - r_j \Delta t P^P_j) + \eta_j P^D_j(1 - r_j \Delta t))] + M_0/(1 - \exp(-cT)) \]
\[ = E_0 \sum_{i=1}^n (1 - \exp(-c(T - (i - 1) \Delta t)))P^P_i \exp( - \sum_{j=1}^{i-1} (r_j \Delta t + P^P_j - r_j \Delta t P^P_j + \eta_j P^D_j(1 - r_j \Delta t))] + (r_j \Delta t + P^P_j - r_j \Delta t P^P_j + \eta_j P^D_j(1 - r_j \Delta t))] \]

Now, we want to derive the continuous form of the initial mortgage value \( V_0 \). We model the default and prepayment probabilities as Poisson Processes with time-varying intensities. Specially, the conditional probabilities of default and prepayment in \([t, t + dt]\) is proportional to \( dt \) with the intensities of default and prepayment as coefficients. We define the intensities of default and prepayment as \( \lambda^D_i \) and \( \lambda^P_i \), where \( 0 \leq t \leq T \). Then we can represent the conditional probabilities of default and prepayment as \( P^D_t = \lambda^D_t dt \) and \( P^P_t = \lambda^P_t dt \). The continuous form of the initial mortgage value \( V_0 \) can be expressed as follows:

\[ \lim_{\Delta t \to 0} V_0 = Y \int_0^T E_0[\exp(- \int_0^t (r_u + \lambda^P_u + \eta \lambda^D_u)du)]dt + M_0/(1 - \exp(-cT)) \int_0^T E_0[(1 - \exp(-c(T - t)))] \lambda^D_t \exp(- \int_0^t (r_u + \lambda^P_u + \eta \lambda^D_u)du)dt \]  
(10)

where \( \lambda_t \) is in the integral of equation (9), we can apply 1 to approximate \( \lambda_t(t) \). According to the definition above, \( \eta_t \) is the random variable denoting the loss given default. Jokivuolle and Peura [15] shows that there is no significant difference on pricing mortgages both when the loss given default is a random variable or a constant. In that case, we can have \( \eta_t = \eta \). Otherwise, \( \lambda^D_t r_u(du)^2 \) and \( \lambda^D_t \eta_t r_u(du)^2 \) are very small in real world, so we can neglect these terms. From the above, we can rewrite equation (9) as:

\[ V_0 = Y \int_0^T E_0[\exp(- \int_0^t (r_u + \lambda^P_u + \eta \lambda^D_u)du)]dt + M_0/(1 - \exp(-cT)) \int_0^T E_0[(1 - \exp(-c(T - t)))] \lambda^D_t \exp(- \int_0^t (r_u + \lambda^P_u + \eta \lambda^D_u)du)dt \]  
(10)

III. THE CLOSED-FORM PRICING FORMULA

The value of the mortgage equals the expectation of the future cash flow under the risk-neutral measure. To obtain the closed-form pricing formula, we set up a state variable describing the variation in the economy, which is fruitful in developing the tractable and conventional solution from Duffie and Singleton [13]. The affine jump-diffusion (AJD) is specified, which is a jump diffusion process for which the drift vector, covariance matrix and jump intensity all have affine dependence on the state variable related to the interest rate.

In this section, we introduce the interest rate process as the state variable in the economy. To characterize the financial events (triggers), Kau [16] proposed the interest rate
process follows a double exponential jump diffusion process as follows:
\[ dr_t = \kappa(\tau - r_t)dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t} v_i\right) \]  
(11)

where \( \kappa \) denotes the speed of adjustment to revert to long-run mean, and is a positive constant. \( \sigma \) is the volatility of short term interest rate. \( \tau \) denotes the long-run mean of the interest rate. Kou [16] defined \( N_t \) as a Poisson process with intensity \( \lambda \). \( v_i \) denotes magnitude of a jump which is used to model unanticipated attack in macroeconomic environment.

The behavior of \( v \) is captured by an asymmetric double exponential distribution with the density
\[ f_v(\nu) = p\eta_1 \exp(-\eta_1 \nu)1_{\nu\geq0} + q\eta_2 \exp(-\eta_2 \nu)1_{\nu\leq0} \]  
(12)

where \( p, q \geq 0, p+q = 1 \). \( W_t \) is a standard Brownian motion independent of \( N_t \) and \( f_v(\nu) \). \( 1_{\cdot} \) is an indicator function. Finally, \( \sigma \) is the volatility term, which is also constant.

Following Duffie and Singleton [13], this study further represents the intensity rates of prepayment and default, \( \lambda^P_t \) and \( \lambda^D_t \), respectively, as given by:
\[ d\lambda^P_t = \lambda^P_t dr_t \]  
(13)

\[ d\lambda^D_t = \lambda^D_t dr_t \]  
(14)

From above discussion, we begin to take the first line of expectation for the mortgage value by substituting equation (15) and equation (16) into equation (10) as follows:
\[ E_0[\exp(\int_0^t - (r_u + \lambda^P_u + \eta \lambda^D_u)du)] = \exp(-(\lambda^P_0 + \eta \lambda^D_0))E_0[\exp(-(1 + \lambda^P_t + \eta \lambda^D_t)\int_0^t r_udu)] \]
\[ = -\exp(\lambda^0_P + \eta \lambda^0_D - (1 + \lambda^P_t + \eta \lambda^D_t)\mu_t + (1 + \lambda^P_t + \eta \lambda^D_t)^2\sum_{i}v_i/2 + \lambda(\theta(-(1 + \lambda^P_t + \eta \lambda^D_t)) - 1)) \]  
(17)

where \( \theta(.) \) is the moment generation function of double exponential jump structure:
\[ \theta(c) = p\eta_1/(\eta_1 - c) + q\eta_2/(\eta_2 - c) \]  
(18)

Next, the extended transform (Duffie and Singleton) [13] is applied for the second line of equation (10) for the closed-form mortgage pricing formula. This is regarded as the following calculation:
\[ E_0[\lambda^P_t \exp(\int_0^T r_u + \lambda^P_u + \eta \lambda^D_u du)] = \exp(\alpha(t) + \beta(t)r_t)(A(t) + B(t)r_t) \]  
(19)

where \( A(t) \) and \( B(t) \) satisfy the following ordinary differential equations (ODEs) [13]:
\[ \begin{cases} -\dot{B}(t) = -\kappa B(t) \\ -\dot{A}(t) = \kappa f B(t) + \beta(t)\sigma^2 B'(t) + \lambda \nabla \theta(\beta(t))B(t) \end{cases} \]  
(20)

with boundary conditions: \( B(T) = \lambda^P_T, A(T) = 0 \). Before solving the above ODEs system, we need to find the solution for \( \beta(t) \). Particularly, \( \alpha(t) \) and \( \beta(t) \) also need to satisfy the following ODEs [13]:
\[ \begin{cases} \dot{\beta}(t) = (1 + \lambda^P_t + \eta \lambda^D_t) + \lambda B(t) \\ \dot{\alpha}(t) = (\lambda^P_t + \eta \lambda^D_t) - \kappa f \beta(t) + \beta(t)\sigma^2/2 - \lambda(\theta(\beta(t)) - 1) \end{cases} \]  
(21)

with boundary conditions: \( \alpha(T) = 0, \beta(T) = 0 \). We lay out the formulations for pricing mortgage value:
\[ \beta(t) = (1 + \lambda^P_t + \eta \lambda^D_t)(\exp(-\kappa(T - t)) - 1) \]  
(22)
\[ \alpha(t) = (\lambda^0_P + \eta \lambda^0_D)T - \dot{\tau}(1 + \lambda^P_t + \eta \lambda^D_t)(\exp(-\kappa(T - t)))/\kappa - t - (\alpha^2(1 + \lambda^P_t + \eta \lambda^D_t)/(2\kappa^2))\exp(-2\kappa(T - t))/(2\kappa) - 2\exp(-\kappa(T - t))/\kappa + t - M(t) + c_1 \]  
(23)

where
\[ c_1 = M(T) - (\lambda^0_P + \eta \lambda^0_D)T + \dot{\tau}(1 + \lambda^P_t + \eta \lambda^D_t)(1/\kappa - T) + (\alpha^2(1 + \lambda^P_t + \eta \lambda^D_t)/(2\kappa^2))(-3/(2\kappa) + T) \]  
(24)
Thus, we have the closed-form pricing formula for mortgage loan with prepayment and default risk

\[ M(t) = \int_0^t (p_1 u/(\eta_1 - \beta(u)) + p_2 u/(\eta_2 - \beta(u)))du \]  

(25)

Take \( \beta(t) \) into equation (20), we further obtain the following results:

\[ B(t) = \lambda_1^P \exp(-\kappa(T - t)) \]  

(26)

\[ A(t) = -\bar{\nu} \lambda_1^P \exp(-\kappa(T - t)) - \lambda_1^P \sigma^2(1 + \lambda_1^P + \eta \lambda_1^D) \]  

( exp(-3\kappa(T - t))/3\kappa - \exp(-2\kappa(T - t))/\kappa - \exp(-\kappa(T - t))/\kappa \]  

- N(t) + c_2 \]  

(27)

\[ c_2 = \bar{\nu} \lambda_1^P - \lambda_1^P \sigma^2(1 + \lambda_1^P + \eta \lambda_1^D)^2/(3\kappa^3) + N(T) \]  

(28)

\[ N(t) = \int_0^t (p_1 u/(\eta_1 - \beta(u))^2 + p_2 u/(\eta_2 - \beta(u))^2) \]  

\[ \exp(-\kappa(T - u))du \]  

(29)

Thus, we have the closed-form pricing formula for mortgage loan:

\[ V_0 = Y \int_0^T (\exp(-\lambda_0^0 P + \eta \lambda_0^D) t - (1 + \lambda_1^P + \eta \lambda_1^D) \bar{\nu}_t + (1 + \lambda_1^P + \eta \lambda_1^D)^2 \bar{\nu}_t + \lambda_1 t(\exp(-(1 + \lambda_1^P + \eta \lambda_1^D)) - 1))dt + M_0/(1 - \exp(-cT)) \int_0^T ((1 - \exp(-(T - t))) \]  

\[ \exp(\alpha(t) + \beta(t) r_0 - (\lambda_0^P + \eta \lambda_0^D) t)(A(t) + B(t) r_0)du \]  

(30)

Therefore, we obtain the relationship between YTM and mortgage loan fair value such that

\[ Y \exp(-R t)dt = \]  

\[ Y \exp(-(\lambda_0^0 P + \eta \lambda_0^D) t - (1 + \lambda_1^P + \eta \lambda_1^D) \bar{\nu}_t + (1 + \lambda_1^P + \eta \lambda_1^D)^2 \bar{\nu}_t + \lambda_1 t(\exp(-(1 + \lambda_1^P + \eta \lambda_1^D)) - 1)) + M_0/(1 - \exp(-cT))(1 - \exp(-(T - t)) \]  

\[ \exp(\alpha(t) + \beta(t) r_0 - (\lambda_0^P + \eta \lambda_0^D) t)(A(t) + B(t) r_0)/t \]  

(32)

The yield of a risky mortgage is

\[ R = -\log(\exp(-(\lambda_0^0 P + \eta \lambda_0^D) t - (1 + \lambda_1^P + \eta \lambda_1^D) \bar{\nu}_t + (1 + \lambda_1^P + \eta \lambda_1^D)^2 \bar{\nu}_t + \lambda_1 t(\exp(-(1 + \lambda_1^P + \eta \lambda_1^D)) - 1)) + M_0/(Y(1 - \exp(-cT))(1 - \exp(-(T - t)) \]  

\[ \exp(\alpha(t) + \beta(t) r_0 - (\lambda_0^P + \eta \lambda_0^D) t)(A(t) + B(t) r_0))/t \]  

(33)

The above formula describes the effect of the significant parameters of our model on the YTM. Particularly, the sensitivity analysis could be performed on this foundation, which also convey some information to participator in the risk management field.

The duration of the mortgage loan with prepayment and default risks is explored in the following way. First, the risk-adjusted duration is defined as

\[ D = -(1/V_0)(\partial V_0/\partial R), \]  

(34)

where \( R \) denotes the YTM. The duration can be also derived as follows:

\[ D = -(1/V_0)Y \int_0^T t \exp(-R t)dt = \int_0^T t \omega_1 dt \]  

(35)

where \( \omega_1 = \exp(-(R t))/V_0 = \exp(-(R t))/ \int_0^T \exp(-(R t))dt \), which represents the weight of cash flows at time \( t \). Next, the definition of the convexity for a risky mortgage is

\[ C = (1/V_0)(\partial^2 V_0/\partial R^2) \]  

(36)

where \( C = (1/V_0) \int_0^T t^2 \exp(-R t)dt = \int_0^T t^2 \omega_1 dt \).

B. Sensitivity Analysis

This subsection numerically illustrates the influence of the parameters on the mortgage yield, duration, and convexity for the closed-pricing formula. For sensitivity analysis, we have to set up some parameters. We assume the mortgage is a 20 years contract with \( M_0 = 1 \) million. Let \( \lambda_0^0 = 0.001, \lambda_0^D = 0.001, \lambda_1^P = 0.5, \lambda_1^D = 0.5, \kappa = 0.5, \eta = 0.7, \sigma = 0.1, \bar{\nu} = 0.02, r_0 = 0.02, \eta_1 = 0.01, \eta_2 = 0.01, \lambda = 0.001, c = 0.03, p = 0.4, \) and \( q = 0.6 \). Under these settings, we can obtain \( V_0 = 906,060, R = 0.041, D = 8.641, \) and
\( C = 106.915 \). Then, we want to analyze how these four values change with four parameters, \( \lambda^D, \lambda^L, \kappa, \) and \( \sigma \).

The key task for portfolio management is to explore the implication of the duration and convexity of their mortgage holding. Additionally, the main determinant of the investment decision is the yield, which is also considered in the following way.

Fig. 3. Yield to maturity for risky mortgage loan with respect to different levels of the influence of interest rate on the intensity of default rate and prepayment rate.

Fig. 3 illustrates the influences of the interest rate (state variable) on the yield for the holding of risky mortgage loan. This figure shows that a higher yield will be required when the effect of interest rate on the default and prepayment intensities (the value of \( \lambda^D \) and \( \lambda^L \)) increases. It also means that the influence of interest rate on mortgage value is negative.

Fig. 4. Yield to maturity for risky mortgage loan with respect to different levels of the volatility of interest rate (\( \sigma \)) and the adjust speed of revert to long-term mean (\( \kappa \)).

The left-hand side of Fig. 4 illustrates the yield as a function of the level of volatility of interest rate. A larger volatility implies that high required rate is requested when there is a great degree of change in the state variable, reducing risky mortgage loan value. Additionally, the right-hand side of Fig. 4 shows that an effect related to how adjustment speed to revert to long-term mean influence the yield of risky mortgage loan. Larger speed to revert to long-term mean of the state variable reduces the required rate for holding the mortgage loan. Remarkably, the yield converges to a constant as \( \kappa \to \infty \).

To provide the implication of the duration for risk management, the results are presented in Fig. 5. It presents a view related to how the influences of the interest rate (state variable) on the yield influences the duration of mortgage loan. Increasing \( \lambda^D \) and \( \lambda^L \) decreases the mortgage duration. It is worthy to note that our finding is similar to Chance

Fig. 5. Duration for risky mortgage loan with respect to different levels of the influence of interest rate on the intensity of default rate and prepayment rate.

[19] and Derosa et al. [20], the influences of interest rate on default and prepayment rates will lead to smaller mortgage duration.

Fig. 6. Duration for risky mortgage loan with respect to different levels of the volatility of interest rate (\( \sigma \)) and the adjust speed of revert to long-term mean (\( \kappa \)).

The left-hand side of Fig. 6 explains the duration as a function of the level of volatility of interest rate. The mortgage duration is inversely proportional to the volatility of the interest rate. The right-hand side of Fig. 6 shows that there is a positive relationship between the duration and the speed to revert to long-term mean. Similarly, the mortgage duration converges to a constant as \( \kappa \to \infty \).

To provide the implication of the duration for risk management, the results are presented in Fig. 5. It presents a view related to how the influences of the interest rate (state variable) on the yield influences the duration of mortgage loan. Increasing \( \lambda^D \) and \( \lambda^L \) decreases the mortgage duration. It is worthy to note that our finding is similar to Chance

Fig. 7. Convexity for risky mortgage loan with respect to different levels of the influence of interest rate on the intensity of default rate and prepayment rate.

Furthermore, we find that the decreasing curve of the mortgage convexity with respect to the positive influence of the interest rate on the intensity of default rate and prepayment rate, based on our closed-form pricing formula, from Fig. 7. Noticeably, Fig. 8 is similar to Fig. 6.

Several points are worth noting. According to sensitivity analysis, this study claims that there are positive relationship between the yield and the effects of volatility of the interest
rate when the jump structure is included in state variable dynamics. Otherwise, there are negative relationship between mortgage duration, or convexity to the influence of the interest rate on the intensity of default rate and prepayment rate. A great degree of change in the state variable reduces the magnitude of risky mortgage duration and convexity. Finally, the speed of adjustment of interest rate increases mortgage duration and convexity.

VI. CONCLUDING REMARKS

This article provides an accurate and efficient pricing model for the valuation of FRM. It is complicated and time-consuming to measure the risky mortgage yield, duration, and convexity since the uncertainty of borrowers’ behaviors (default and prepayment), which lead to the uncertainty of future cash flow and the life of the contract. Our closed-form formula provides an accurate and efficient computation of those risk measures for financial institutions. The major difference between our pricing model and previous studies is that we consider the impact of prepayment in the valuation of mortgage. The mortgage loan dynamics resulting from prepayment should be time-varied with the changing in interest rate. Hence, we use a backward recursion method to value mortgage loan and consider the impacts of prepayment risk directly. And in order to account for the financial events, our formula includes jump structure in the state variable process, also providing the solution to the problem of mortgage hedging. The duration and the convexity of fixed-income securities are very important hedging tools for risk management. Particularly, the duration of the interest-rate-sensitive securities determines the hedging position for offsetting the loss due to the uncertainty of interest rate. In other words, we propose the practical implementation of pricing risky mortgage loan to relax the assumption that the prepayment behavior of debtor is constant. Finally, on the economic side, we provide the implication in the relationship between the risk measures (e.g. duration and convexity) and the changing of interest rate for banks and investment managers.

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