Congestion Game Models for Capacity and Bandwidth Relation in Dynamic Spectrum Access

Y. B. Reddy, Heather Smith, and Michael Terrell
Grambling State University, USA

Abstract—Dynamic spectrum access is a technique to utilize the spectrum resources efficiently in a cognitive radio environment. To use spectrum efficiently various models were designed by researchers. Game theoretical models are few of the efficient techniques recently introduced in wireless communications. Game models are very useful for opportunistic selection of spectrum and help to utilize available spectrum efficiently. We introduced the spectrum underlay model and discussed the current state of allocation algorithms. We then introduced the congestion game model for opportunistic spectrum access with minimum interference. The theoretical results conclude that the congestion game model helps to use the underlay spectrum efficiently with minimum interference.

Keywords—bandwidth; capacity; congestion games; overlay; underlay;

I. INTRODUCTION

It is a known fact that spectrum is a scarce resource and it must be used efficiently and effectively. This means the unused (overlay) and under used (underlay) spectrum must be used efficiently and at the same time it must generate more finance. These are two different approaches (overlay and underlay) targeted together for better use of existing spectrum. The concept of cognitive radio (CR) enters automatically with overlay and underlay (uses ultra wide band) because they are unused and underused spectrum layers by primary user. The game models helps for the opportunistic access of unused and under user spectrum by the secondary users.

The spectrum underlay functions are below the noise level of primary users and similar to spread spectrum techniques (SST). The underlay technique is efficient use of underutilized spectrum. In this approach (underlay spectrum use) the secondary users can achieve high data rating, low power with extremely low transmission power. Overlay technique is opportunistic spectrum access or use of spectrum holes (identify and instantaneous use of spectrum). The concepts and more explanations are found in [1-3, 19, 20].

Dynamic spectrum access (DSA) is the sharing of existing spectrum by unlicensed users (secondary or cognitive users) with licensed users (primary users) without interference to the licensed users. Sharing of spectrum requires following certain rules and policies. Spectrum sharing rules and protocols that allow the bandwidth to share is discussed in [4]. The important point is the amount of information that the secondary systems or cognitive systems need to know to use the unused spectrum. The information includes the current state of other cognitive users accessing same resource, unutilized and underutilized spectrum.

There are many models developed including efficient channel utilization, channel modeling, and allocation of resources for underlay techniques [9-11]. Recently, the concentration increased to design better and efficient models for overlay and underlay techniques. The developments include game models for spectrum sharing which produces encouraging results and moves new direction in research [8-11]. The researchers introduced the role of games and game models for efficient use of spectrum. Non-cooperative and congestion game models are more suitable to the overlay and underlay spectrum usage.

Some of the game models presently trying to apply for overlay and underlay spectrum are zero sum games, non-zero sum games, potential games, cooperative games, non-cooperative games, and congestion games. The behavior and stability of game models is measured through Nash equilibrium. The fundamental is that the players must reach Nash equilibrium to provide stable condition. Some of the models are discussed below:

• The zero sum game is played between two players. The net result of two players that equals zero means that the gain of one player equates the loss of another player. Zero sum game is not useful in the current problem because we have to utilize available spectrum efficiently. There is no loss involved.

• The non-zero sum game model can be used in overlay spectrum utilization because if one or more of the players (cognitive users) cooperate then some or all of the players will be benefited. The cooperation of primary user with secondary users may not be possible but the cooperation of secondary users will help to benefit most of the secondary users to use spectrum efficiently.

• The potential function (game model) is a useful tool to analyze equilibrium properties of games since the
incentive of all players are mapped into single function. In finite potential games [12, 13], the change in any player’s payoffs from a unilateral deviation is exactly matched by a change in potential which concludes that the Nash equilibrium is a local maxima. In ordinal potential games only the signs of the differences have to be the same. If we treat the total incentive as utilizing unused spectrum, we may use the potential games to find the unused spectrum and make it available to the cognitive users.

- In cooperative games the players stay close together for the overall benefit of all players. In non-cooperative games each player makes independent decisions and any cooperation is self enforcing. The cooperative game may be useful in the current problem of efficient utilization of spectrum.

Congestion games [15, 16] are a class of non-cooperative games where players share a common set of strategies. The utility of a player from using a certain resource depends on the total number of players that are using the same resource. That is the resulting payoff is a function of the number of active users (congestion). In this paper we use the congestion game model for better utilization of the unutilized spectrum by cognitive users.

The remaining sections of the paper discusses the recent developments related to spectrum overlay and underlay, problem formulation for maximum utilization of spectrum resource (capacity), congestion game model for opportunistic spectrum access and conclusions.

II. RECENT DEVELOPMENTS

The detailed analysis of “evaluation of overlay/underlay waveform via soft decision spectrally modulated, spectrally encoded (SD-SMSE) framework for enhancing spectrum efficiency” is available in [14]. The thesis discusses the techniques for overlay/underlay and efficient use of spectrum. In [7] sensor net with CR is used to use the spectrum in underlay mode. The Hidden Markov Model is used to predict the interference at primary user so that the secondary user can benefit better. A similar resource allocation in underlay mode is discussed in [6]. The authors in [6] proposed the admission control algorithm for underlay in code division multiple accesses (CDMA) for better quality of service with minimum interference.

Xie [17] discussed the overlay cognitive radio market model. In this model the primary user imposes the spectral mask so that the secondary user gets better spectrum use opportunities. Furthermore, for efficient usage of unused spectrum, the primary user uses the economics competition for purchase of power allocation on each channel. The model uses the market equilibrium while controlling the interferences.

Wu et al [18] discussed the auction-based dynamic spectrum allocation to lease the spectrum by secondary users. They proposed a mechanism called multi-winner spectrum auction with collusion resistant pricing strategy to allocate the spectrum optimally. The greedy algorithm helps to reduce the complexity in multi-band auctions.

The game models for spectrum sharing and controlled interference is studied by Nie[5], Ji [8], Halldorsson [9], and Liu [10]. Nie et al [5] formulated the channel allocation as potential game and showed the improvement of overall network performance. Ji [8] discussed the dynamic spectrum sharing through game theory approach and discussed the analysis of network, user’s behavior, and optimal analysis of the spectrum unused allocation. Halldorsson [9] viewed channel assignment as a game and provide the price anarchy depending upon the assumptions on the underlying network. Finally, Liu [10] used congestion game model for spectrum sharing and base station channel adaptation. In this paper, we study the congestion game model for spectrum sharing without interfering with primary users.

III. PROBLEM FORMULATION

To transmit the information on a data channel the information rate R must be less than or equal to the channel capacity C (R ≤ C) [21]. If R > C, the errors cannot be avoided. In a brand limited channel operating in the presence of additive white Gaussian noise (AWGN), the Shannon’s channel capacity is given by

\[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \]  

(1)

Where C is the capacity in bits per second, B is the bandwidth of the channel in Hertz, and S/N is the signal-to-noise ratio. As the bandwidth increases the noise power also increases and N can be written as: \( N = \eta B \). where noise power spectral density is \( \eta/2 \). The equation (1) becomes

\[ C = B \log_2 \left( 1 + \frac{S}{\eta B} \right) \]  

(2)

\[ C = \frac{S}{\eta} \log_2 \left( 1 + \frac{S}{\eta B} \right) \]  

But

\[ \lim_{\frac{S}{\eta B} \to \infty} \left( 1 + \frac{S}{\eta B} \right)^{\frac{S}{\eta B}} = e \]

\( \{ e \} \) is usually defined by: \( e = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n \)

The equation (3) becomes

\[ C = \frac{S}{\eta} e \]  

(4)
In equation (3), if we represent the spectral density in terms of noise temperature then C will be independent of bandwidth as in equation (4) and maximum transmission is possible. This information is subject to the condition and is related to any channel, but we need to find the unused and under used bandwidth utilization.

To use the total capacity of the band (not unlimited use), we must include primary users, cognitive users, and users that use ultra wide band. Therefore, the total capacity of the transmission occupied by primary users, cognitive users, and underlay spectrum users is

\[ C = B \log_2 \left(1 + \frac{\varphi_{pu} B_{pu} + \varphi_{su} B_{su} + \varphi_{ud} B_{ud}}{N_0 B + \phi_d B_{sd}} \right) \]  

(5)

where \( \varphi_{pu} \) is the spectral density of primary user and \( B_{pu} \) is the corresponding band width

\( \varphi_{su} \) is the spectral density of cognitive (secondary) user (overlay) and \( B_{su} \) is the corresponding band width

\( \varphi_{ud} \) is the spectral density of underlay user and \( B_{ud} \) is the corresponding band width

\( N_0 \) is the noise for power spectrum density AWGN (Additive White Gaussian Noise)

\( \phi_d \) is the average spectrum density and \( B_{sd} \) is the corresponding band width

Since we are considering the effect of interference temperature on primary signals, we ignore the overlay part in the equation (5). Therefore, equation (5) can be written as:

\[ C = B \log_2 \left(1 + \frac{\varphi_{pu} B_{pu} + \varphi_{ud} B_{ud}}{N_0 B + \phi_d B_{sd}} \right) \]  

(6)

We calculate the total channel capacity \( C_T \) of the primary users without any significant interference from underlay users (assuming the underlay users are active) as:

\[ C_T = B \log_2 \left(1 + \frac{\sum_{i=1}^{N} \varphi_{pu_i} B_{pu_i} + \sum_{i=1}^{M} \varphi_{ud_i} B_{ud_i}}{N_0 B + \sum_{i=1}^{M} \phi_d B_{sd_i}} \right) \]  

(7)

\[ 2^{C_T/B} - 1 = \frac{\sum_{i=1}^{N} \varphi_{pu_i} B_{pu_i} + \sum_{i=1}^{M} \varphi_{ud_i} B_{ud_i}}{N_0 B + \sum_{i=1}^{M} \phi_d B_{sd_i}} \]

\[ (2^{C_T/B} - 1)(N_0 B + \sum_{i=1}^{M} \phi_d B_{sd_i}) = \sum_{i=1}^{N} \varphi_{pu_i} B_{pu_i} + \sum_{i=1}^{M} \varphi_{ud_i} B_{ud_i} \]

Therefore the maximum bandwidth utilized by underlay user without disturbing the primary users is

\[ (2^{C_T/B} - 1)(N_0 B + \sum_{i=1}^{M} \phi_d B_{sd_i}) (\text{occupy by secondary user}) \]  

(10)

But we calculate the interference temperature by multiplying the average spectrum density and corresponding bandwidth [22, 23]

\[ T_i(f, B) = \frac{P_i(f, B)}{kB} \]  

(11)

where \( k \) is 1.38*10^-23, Joules per Kelvin degree. \( P_i \) is the average interference power in Watts centered at \( f \).

Using equations (10) and (11) the maximum bandwidth occupied by the secondary user is

\[ B = (2^{C_T/B} - 1)(N_0 B + \sum_{i=1}^{M} \phi_d B_{sd_i}) \]  

(12)

The power required for underlay user to occupy maximum bandwidth without interference to a primary user is

\[ \frac{1}{(2^{C_T/B} - 1)} - N_0 \times kB^2 = P_i(f, B) \]  

(13)

But the maximum transit power required by the secondary user at each bandwidth is [22]

\[ \frac{1}{(2^{C_T/B} - 1)} - N_0 \times kB = \frac{Bk}{\Theta}(T_i(f, B) - T_1(f, B)) \]  

(14)

\[ \frac{1}{(2^{C_T/B} - 1)} - N_0 \Theta = T_L(f, B) - T_1(f, B) \]  

(15)

\( \Theta \) is the loss factor, \( T_1 \) is the interference temperature, and \( T_L \) is the maximum tolerable interference. Therefore

\[ \frac{1}{(2^{C_T/B} - 1)} - N_0 \Theta > 0, \]
since \( \Theta \) greater than 0, then

\[
\frac{1}{(2^{\frac{C_T}{B}} - 1) - N_0} = \frac{\lambda}{\Theta} + N_0
\]

where \( \lambda \) is a constant and \( >0 \)

Therefore,

\[
\frac{1}{(2^{\frac{C_T}{B}} - 1)} = N_0 + \frac{\lambda}{\Theta} > 0 \tag{16}
\]

Equation (16) concludes that

\[
C_T = B \frac{\log e}{N_0 + \frac{\lambda}{\Theta}} = B \beta
\]

That is, we must maintain the value \( \beta >1 \), then the spectrum capacity will be under control and \( \beta \) is

\[
\beta = \frac{\log e}{N_0 + \frac{\lambda}{\Theta}}
\]

The above spectrum sharing problem is to determine power allocations (spectrum density multiply by corresponding band width) to secondary users which satisfy the power constraint utility function. The \( \beta \) value depends upon the \( \lambda \) \((<1)\), \( \Theta \) \((>1 \text{ and } <2)\) and \( N_0 \) \((10E-10)\) the noise. The figure 1 shows that the capacity or data transfer is low at low noise. High data transfer rate is possible by increasing the noise ans band. Figure 2 shows the capacity calculation with very high noise. From Figure 1 and Figure 2 we observe that after adjusting the noise to certain level, capacity or transfer of data reaches uniform level. If the band width is zero then there is no data transfer irrespective of noise level.

Figure 1: Capacity as function of noise

Figure 2: Capacity as function of noise with very high noise

IV. GAME MODELS

The current problem is an efficient use of unused spectrum by the secondary users (with selfish nature) that results the potential utilization of the scarce resource spectrum. Since the efficient utilization of unused spectrum by each selfish user effects global utilization, we can design this problem as a congestion game and potential utilization as the utility function. Any change in the local utilization of the resource reflects the global activity (utility function).

The congestion game is a non-cooperative game where players (cognitive users) share a common set of strategies (unused spectrum utilization). The payoff of a particular player depends upon the number of players (cognitive users) playing with the same strategy. The spectrum will be utilized opportunistically by the cognitive (secondary) users (CU) at different times. Our main aim is to maximize the utilization of the spectrum by the cognitive users without interfering with primary users.

Let \( N \) be the CUs and \( R \) be the resource or unused spectrum available to CUs. Each user uses a strategy \( S \) to get a resource \( R \) resulting in a payoff \( U \) (the utility function). The game \( G \) is represented as

\[
G = (N, R, S, U)
\]

where, \( N = n_1, n_2, \ldots, n_n \) the set of users

\( R = r_1, r_2, \ldots, r_n \) the set of resources

\( S = s_1, s_2, \ldots, s_n \) the set of strategies

Used by each user to get a resource and resulting to payoff \( U \).

\[
U = u_1, u_2, \ldots, u_n \text{ the set of utilities}
\]
The payoff is a resulting value of all users N using resources R. The total payoff of user i using strategy $s_i$ is given by

$$u_i(s_i) = \sum_{r \in s_i} U(N_r(s_i))$$  \hspace{1cm} (18)$$

where $N_r(.)$ is the total number of users using resource r under the strategy s.

If the players share the same set of strategies then the utility function is given by

$$U(s) = \sum_{k=1}^{r} \sum_{m=1}^{n} S_k(m)$$  \hspace{1cm} (19)$$

If any $i^{th}$ player shifts to new strategy $j$ then the potential is changed by

$$\Delta U = S_j(n_j + 1) - S_{s_i}(n_{s_i})$$  \hspace{1cm} (20)$$

where $S_{s_i}(n_{s_i}) \geq S_j(n_j + 1)$ and $S_{s_i}$ is the strategy tuple of a user selected that particular strategy.

This is the gain or loss of $i^{th}$ player. If we have reached local maxima and any strategy tuple of changing one coordinate will not change the potential function $U$ which corresponds to Nash Equilibrium. Let equation (15) the difference between average temperature and Interference temperature by secondary users is

$$T_c(f_c) - T_c(f_c, B) = \left(\frac{1}{2^{C_r/B}} - 1\right) - N_0$$  \hspace{1cm} (21)$$

Let $\Delta U$ be the change in interference by any $i^{th}$ player shifts to new strategy, then using equations (20) and (21) we obtain

$$\Delta U = \left(\frac{1}{2^{C_r/B}} - 1\right) - N_0$$  \hspace{1cm} (22)$$

$$\frac{\Delta U}{\Theta} + N_0 = \left(\frac{1}{2^{C_r/B}} - 1\right)$$

$$\frac{1}{2^{C_r/B}} = 1 + \frac{\Delta U}{\Theta} + N_0$$

$$2^{C_r/B} = 1 + \frac{\Delta U}{\Theta} + N_0$$

$$C_r = B \log(1 + \frac{\Delta U}{\Theta} + N_0)$$

$$C_r = \frac{B}{\Theta} \log e$$  \hspace{1cm} (23)$$

From equations (17) and (23) we get

$$\log e B = \frac{B}{N_0 + \frac{\Delta U}{\Theta} + N_0} \log e$$  \hspace{1cm} (24)$$

That is

$$N_0 + \frac{\Delta U}{\Theta} = N_0 + \frac{\Delta U}{\Theta}$$  \hspace{1cm} (25)$$

It results to $\lambda = \Delta U$  \hspace{1cm} (26)$$

The equation (26) shows that shift in new strategy is a small value and $\lambda > 0$ and will not produce any significant interference to the primary user while cognitive user is in operation above the above constraints. Further, it shows the game model works equally good for shifts in new strategy with small change from current strategy. The Figure 3 is drawn for the equation (23) to show the capacity variation for game model. Capacity is uniform after certain noise level but achieves better capacity with the same noise level. So, game model achieves the capacity with same level of noise as non-game model. The $\Delta U$ value is small and its increment is also small (0.05).

Compare the Figure 3 with Figure 1, the game model performs better and higher data rate is possible with same bandwidth.

![Figure 3: Game model: Capacity as a function of Noise](image-url)

**V. CONCLUSIONS**

The model for underlay cognitive users (unused spectrum used by cognitive users) without having interference with primary users is discussed in this paper. In the current research the relation between capacity, noise, and bandwidth was developed using the temperature interference formula. The capacity and bandwidth model was designed using the congestion model and was compared with the temperature...
interference model. The results show that a shift in strategy using congestion model will not disturb the primary user (do not produce significant amount of interference) in the presence of a cognitive user and improves the performance.

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VII. References