Fuzzy rule-based demand forecasting for dynamic pricing of a maritime company

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In this study, the pricing problem of a transportation service provider company is considered. Our goal is to find optimal prices by using probabilistic dynamic programming. A fuzzy IF-THEN-rule based system is used to identify the demand levels under different prices and other characteristics of the journey. The results obtained by optimal price policies show that the revenue increases by applying dynamic pricing policy instead of fixed pricing. Thus, the diversification of pricing policies under different conditions is beneficial for the company.

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1. Introduction

Dynamic pricing, often referred to as individual-level price discrimination, has become much more common with the increased usage of internet sales [21]. Internet provides a media for companies to offer different pricing strategies in which prices vary over time, consumers, and/or circumstances. Companies try to maximize their profits using these strategies. This attempt is often called as revenue management.

The bulk of the literature deals with the revenue management and dynamic pricing problems of especially airlines, freight, and tourism markets. To the best of our knowledge, there has been no study conducted for maritime public transportation industry. In the transportation market, the demand fluctuates based on weather conditions, price, time of day and day of week. Based on these conditions, passengers decide on whether to travel or not. In other words, passengers’ demand/price elasticity may change depending on the weekdays or weekend, early or late hours of a day as well as the weather conditions. Considering this fact, the strength of fuzzy logic in capturing the ambiguity and exploiting imprecise data created a motivation for us to use fuzzy logic in demand forecasting. Fuzzy IF-THEN rules are generated in order to retrieve the price/demand elasticity of passengers. Using these findings, a dynamic pricing model is formulated in order to find the optimal policies under different conditions. Then the policies obtained by dynamic pricing are compared with the fixed pricing policy.

The paper includes seven sections. Literature review follows this section. In Section 3, problem description is given. Then dynamic pricing model is described in Section 4 while Section 5 presents the fuzzy rule based demand forecasting system. In Section 6, the case study is given and finally, conclusion and future research are discussed in Section 7.

2. Literature review

Revenue management literature related to the transportation industry is reviewed below in order to reveal the contribution of this study. For a more detailed review of pricing models used in revenue management, the interested readers can refer to Bitran and Caldentey [5] or Talluri and van Ryzin [40], and for a survey of the literature that considers both pricing and inventory decisions, see Elmaghraby and Keskinocak [11,18]. In the revenue management literature, dynamic pricing problems for a fixed stock of a single item sold in finite selling horizon have attracted considerable attention. Gallego and van Ryzin [15] formulate an intensity control model of the dynamic pricing problem and derive several structural properties. Zhao and Zheng [51] consider a similar problem with non-homogeneous demand and show that dynamic pricing policies may have a significant impact on revenue when demand is non-homogeneous. Zhang and Cooper [50] develop a Markov decision process formulation of a dynamic pricing problem for multiple substitutable flights between the same origin and destination, taking into account customer choice...

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among the flights. Kuyumcu and Garcia-Diaz [26] develop a new analytical procedure for joint pricing and seat allocation problem, which considers demand forecasts, number of fare classes, and aircraft capacities. They use polyhedral graph theoretical approach for optimization and show that it achieves significant computer time saving when compared to a general-purpose integer programming commercial software. A part of the existing literature considers the case of competition for the problem of dynamic pricing in transportation industry. Competition models consider that customers can choose the supplier, in this case, the supplier has to decide how to control the inventory level. However, in our study the market is monopolistic similar to the most of the research on dynamic pricing such as Gallego and van Ryzin [15], Chatwin [7], Feng and Xiao [13], Zhao and Zheng [51] and so on.

The previous literature on revenue management in the transport industry deals especially with the airline industry (some examples are: [26,50,28,19,18]). However, Maddah et al. [29] develop a discrete-time dynamic capacity control model for a cruise ship, being the first study developed for cruise ships. Another example is the study of Bhariil and Rangaraj [4]. They consider the case of Indian Railways with an application of the principles of revenue management. The strategy of overbooking is interpreted in terms of waitlist management by the railway company and cancellation action of customers; finally revenue management through differential pricing is suggested as a means to increase revenue on average. To the best of our knowledge, our study is the first attempt that models dynamic pricing for a maritime public transportation company.

Characterization of demand is required in order to model the dynamic pricing problem. Gamma, Normal shapes and Poisson function have been proposed to characterize demand distributions in the previous studies [26,47,28,50]. Some of the existing studies use various assumptions for modeling demand under different price levels based on previous data [47,14,29,28]. Some other studies use price elasticity functions derived from previous data [4,18]. Price elasticity, which characterizes consumer demand, has also been derived by market research using techniques such as conjoint analysis in order to design optimal pricing schemes [43]. Another approach in the literature is to forecast the demand using econometric models [22,16] and time series methods [22,49]. Recently, intelligent methods have been used to model the complexities of demand forecasting such as artificial neural networks [37,23].

In this study, we consider the problem of finding the optimal pricing policy of a maritime public transportation company. The market is monopolistic and the price has been fixed up to this time. Therefore there is no previous data, which shows the relation between different prices and demand levels. Due to the lack of previous data, it is not possible to employ most of the models used commonly in the literature. Moreover demand is also related to various criteria, such as weather, day of week, time of day as well as price. Due to this fact, the demand cannot be expected to be constant over time. Therefore, the fuzzy set theory proposed by Zadeh [48] and fuzzy-rule based system (FRBS) may be useful for incorporating the imprecision of the criteria (i.e. weather, etc.) in demand modeling. The contribution of this study is to estimate the demand via FRBS and to find optimal price policies for the maritime public transportation. Fuzzy sets enable modeling of imprecise and qualitative knowledge, as well as the transmission and handling of uncertainty at various stages [44]. Fuzzy logic allows us to emulate the human reasoning process and make decisions based on vague or imprecise data. Moreover linguistic terms represent the knowledge, experience, and subjective viewpoint of decision makers in a more intuitive way [20]. Fuzzy rule based systems have been one of the most popular methods to capture and represent fuzzy, vague and uncertain domain knowledge in the literature. The fuzzy rule based systems use fuzzy IF-THEN rules to generate a mapping from fuzzy sets in the input universe of discourse to fuzzy sets in the output universe of discourse based on fuzzy logic principles [44]. In recent years, much research has been proposed to develop FRBSs for many objectives such as modeling traffic flow behavior [35], analyzing stock prices [12], economic analysis of RFID investments [44], assessing supply chain performance [33], and predicting weather events [1].

3. Problem definition

This study deals with the dynamic pricing problem of a ferry line of the maritime public transportation company, which serves between Istanbul and Bandırma, Turkey. There exists only one service provider offering the ferry transportation in Istanbul. Thus, the market is monopolistic, which means that the formulated dynamic pricing model does not include competition. The service provider, which was a non-profit governmental establishment used to apply fixed pricing in order to serve for public welfare. However, the corporation has been privatized recently and considers applying dynamic pricing in order to increase the revenue. The realized demand in the past years show that the demand varies significantly based on the weather, days of the week and times of the day. Fuzzy IF-THEN rules are generated in order to retrieve the price/demand elasticity of passengers since there is no previous data about the relation between the demand and the price. Finally, using the outputs of the FRBS, a dynamic pricing model is built in order to see the optimal policies under different conditions. We employ a probabilistic dynamic programming method to find the optimal pricing policies. The diversification of optimal policies under different conditions results in revenue increase.

4. Dynamic pricing model

In this study, a dynamic pricing optimization problem faced by the maritime public transportation company is considered. The aim is to find the optimal prices of each journey that change by the day, time of trip and weather conditions. These are among the factors, which affect the level of demand. For instance, passengers usually make leisure-travels in the weekends or warm days. However, it is expected that the weather condition being warm does not influence the work-travels of the passengers. Furthermore, while price may be significant for leisure-travels, it may not be significant for work-travels. The important attributes for work-travels may be the time of the trip and seat availability.

The system state $S_t$ is defined with number of seats at time $t$. The actions $a_t$ are the prices and the optimal decision $q^*$ at time $t$ will be given according to these actions that will be evaluated at time $t$. In this case the possible price action set $A(S_t)$ of the $S_t$ state is as in (1) if there are $k$ prices observed.

$$A(S_t) = \{a_1, a_2, \ldots, a_k \}$$

Then, the policy $\pi$ is the set of decisions given for each period for each state $S_t, \pi_{S_t} = \{p_{r1}, p_{r2}, \ldots, p_{rn} \}$ where $p_{ri}$ is the optimal price action at time $t$ for state $S_t$. Demand is estimated under different prices together with some other criteria such as day, time of trip and weather condition through fuzzy rule based systems. In this model, demand is "lost" if not fulfilled in the same time period and does not have a cost. However, if there are empty seats, a cost, opportunity cost, is incurred. Therefore, the total demand for each journey at time $t$ is simply the number of seats sold. When demand $D_t$ is appeared, the system state $S_t$, which is the number of seats changes into $S_{t+1} = \max(0, S_t - D_t)$.

We use probabilistic Dynamic Programming (DP) to solve the problem. DP is useful in multistage decision processes where the optimal policy can be determined recursively. The aim of this...
stochastic optimization problem is to determine the optimal policy given in (2) [3,34].

\[
\max_{t} \mathbb{E} \left[ \sum_{t=1}^{T} V^*(S_t, pr_t^*) \right] 
\]  

(2)

where \( V \) is the value function for state \( S_t \). The optimality function is as follows which is known as Bellman Equation [3,34]:

\[
V_t(S_t) = \max_{a_t \in A(S_t)} \left( C_t(S_t, a_t) + \gamma \sum_{S_{t+1}} p(S_{t+1} | S_t, a_t) V_{t+1}(S_{t+1}) \right) 
\]

(3)

\( V_t(S_t) \) is the maximum expected discounted reward that can be earned during \( t \) periods if the state of the current period is \( S_t \). \( C_t(S_t, a_t) \) is the cost (or revenue) function of state \( S_t \) under decision \( a_t \) and \( p(\cdot) \) denotes the probability of next period's state \( S_{t+1} \) given the current state is \( S_t \). Moreover, \( \gamma \) is the discount factor between two periods. This general model qualifies to be a Markov decision programming algorithm tries to find optimal action for each state only the cost of empty seats. The value of being in state \( S_t \) at \( t \) is the profit obtained from the sales and the cost of empty seats.

The reward function \( C_t(S_t, a_t) \) is the profit obtained from the difference of the sales and the cost of empty seats.

The dynamic programming algorithm related to this dynamic pricing problem is shown in Fig. 1. Specifically, the inputs of the algorithm are the number of periods \( T \), beginning inventory level \( S_0 \), unit cost of empty seats, \( h \) and discount factor \( \gamma \) and the outputs of the algorithm are the optimal price of each state \( S_t \), \( pr_t \) and the values of each state \( S_t \), \( V_t(S_t) \). The optimal value is \( V_t(S_0) \) at period 1 when beginning inventory level is \( S_0 \).

5. Fuzzy rule-based demand forecasting system

A fuzzy rule-based system is a systematic reasoning methodology to model the complex behavior of a system by using fuzzy set theory. FRBS expresses the input–output relationship of the process by a collection of simple ‘if-then’ rules [38]. Fuzzy rule-based systems apply fuzzy methods to solve many types of “real world” problems, especially where a system is difficult to model, is controlled by a human operator or expert, or where ambiguity or vagueness is common such as health, education, etc. [45,31].

A typical fuzzy system consists of an inference system, membership functions, and rule-bases [27]. The structure of a fuzzy logic rule is expressed as follows [32]:

IF \( x_1 \) is \( A_1 \) AND \( x_2 \) is \( A_2 \) AND ... AND \( x_n \) is \( A_n \) THEN \( y \) is \( B \).

where \( x_i (i = 1, 2, ..., n) \) are input variables (antecedents) and \( y \) is the output variable (consequent). \( A_1, A_2, ..., A_n \) and \( B \) are the linguistic terms (e.g. low, medium, high) used for the fuzzy subsets (membership function distributions) of the corresponding input and output variables, respectively.

The two types of fuzzy logic inference systems implemented are the Sugeno type and Mamdani type. Both inference systems consider fuzzy inputs but Mamdani returns fuzzy continuous outputs while the output membership function of Sugeno is either linear or constant [30,39]. The Sugeno model is a data driven approach where membership functions and rules are developed using a training data set. The parameters for the membership functions and rules are subsequently optimized to reduce training error [24]. Sugeno model has been successfully applied in demand forecasting studies [36]. When training data is not available, Mamdani model, which relies on expert knowledge, is useful to approximate reasoning. Since Mamdani approach is not exclusively reliant on a data set, with sufficient expertise on the system involved, a generalized model for effective future predictions can be obtained [24].

Fuzzy inference process of Mamdani approach includes five steps: (i) fuzzification of the input variables, (ii) application of the fuzzy operator (AND or OR) in the antecedent, (iii) implication from the antecedent to the consequent, (iv) aggregation of the consequents across the rules, and (v) defuzzification. In the first step, the crisp input values are converted to the fuzzy values by the input membership functions. Using input membership functions, the degree to which input belong to each of appropriate fuzzy sets is determined.

Fig. 1. Dynamic programming algorithm.

Table 1. Dynamic programming algorithm.

<table>
<thead>
<tr>
<th>Step 01: Initialize:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 0a: Initialize number of periods ( T )</td>
</tr>
<tr>
<td>Step 0b: Initialize inventory level ( S_0 )</td>
</tr>
<tr>
<td>Step 0c: Initialize the terminal contribution ( V_T(S_T) ) for each state ( S_T \in S )</td>
</tr>
<tr>
<td>Step 0d: Set discount parameter ( \gamma )</td>
</tr>
<tr>
<td>Step 0e: Set ( t = T-1 )</td>
</tr>
<tr>
<td>Step 1: Calculate:</td>
</tr>
<tr>
<td>( V_t(S_t) = \max_{a_t \in A(S_t)} \left( C_t(S_t, a_t) + \gamma \sum_{S_{t+1}} p(S_{t+1}</td>
</tr>
<tr>
<td>for all ( S_t \in S ). Moreover, ( C_t(S_t, a_t) = (D_t \times a_t) - h \times (S_t - D_t) )</td>
</tr>
<tr>
<td>( D_t ) denotes the demand at time ( t ), ( h ) is the cost of empty seats and ( (S_t - D_t)^+ = \max(0, S_t - D_t) ) denotes the number of empty seats at time ( t ).</td>
</tr>
<tr>
<td>Step 2: If ( t &gt; 0 ), decrement ( t ) and return to step 1. Else, stop.</td>
</tr>
<tr>
<td>Step 3: Return ( pr_t^* ) for all ( S_t \in S ) at time ( t ).</td>
</tr>
</tbody>
</table>

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The second step is to take the fuzzified inputs (e.g., $\mu(x_1 = A_1) = 0.3$, $\mu(x_2 = A_2) = 0.7$) and apply them to the antecedents of the fuzzy rules. If the antecedent of a given rule has multiple parts, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation. The output is a single truth-value. This number (the truth-value) is then applied to the consequent membership function. The fuzzy operators AND and OR make use of the classical fuzzy operations intersection and union respectively, given in (5) and (6).

$$\mu_{A\&B}(x) = \min[\mu_{A(x)}, \mu_{B(x)}]$$  \hspace{1cm} (5)

$$\mu_{A\lor B}(x) = \max[\mu_{A(x)}, \mu_{B(x)}]$$  \hspace{1cm} (6)

In the implication step, the two widely used types of implication methods are clipping and scaling. Using these methods, the membership function of the consequent is reshaped associated with the antecedent value. Clipping implies a cut on the consequent membership function at the level of the antecedent truth, also named as (alpha-cut). Besides, implication by scaling is implemented by multiplying consequent's all membership degrees by the truth-value of the rule antecedent. The next step in the inference process is to unify the outputs of all rules. Thus, it is named as aggregation step. To do the unification, the list of clipped or scaled consequent membership functions are unified into one fuzzy set as an output variable.

Finally, defuzzification is applied to the aggregate output fuzzy set to generate a single output value. One of the most popular defuzzification methods is the centroid calculation, which returns the center of area under the curve of the membership function.

In this study, the Mamdani fuzzy inference system, which applies $\alpha$-cuts as implication and centroid as defuzzification method is expressed as follows in (7) [44]:

$$Y = \frac{\sum_{r=1}^{R} A_r^{\text{min}} C_{r}^{\text{out}}}{\sum_{r=1}^{R} A_r^{\text{out}}}$$  \hspace{1cm} (7)

where $A_r^{\text{min}}$ is the area of the output fuzzy subset covered by $r$ membership value using $r^{th}$ rule and $C_r^{\text{out}}$ is the center distance of the area $A_r^{\text{out}}$.

In order to identify the variables in the FRBS and to fuzzify the variables, we have used domain expert opinion. The fuzzy inference system offered by MATLAB 7.6.0 fuzzy logic toolbox is used in this study. Using the IF-THEN rules, the demand level for a scheduled ferry is estimated based on the input variables of weather condition, time as well as the ticket price. Ticket price is used as a variable, which might be controlled by the service provider, whereas weather and time are exogenous factors, which are not in control of the service provider. Effects of weather conditions on demand/ridership have been commonly modeled in the transportation forecasting literature either by directly classifying the temperature or indirectly using the months of the year [41,25]. Besides, time of day is treated as a significant factor that affects the level of demand for the transportation services and the traffic situation during different hours of the day [6,2,8,46]. Similarly, many studies have demonstrated the variation of traffic volume in different days of week and vacation times [42,46]. Ticket prices have also been considered as a variable affecting transit ridership [17,46].

In the fuzzification step, the membership functions of the input variables are defined using an expert opinion. The temperature of the trip day has been fuzzified into linguistic variables of very cold, cold, warm and high. Time of the trip has been represented by two variables, day of week and time of day. The crisp days of the week have been converted into fuzzy variables reflecting the closeness of the day to the weekend; the linguistic variables of early, mid and late are used. Early is standing for the beginning of the week and late for the end of the week. Demand is also related to the time of the day so the discrete hours of the day has been defined by five levels of linguistic variables namely very early, early, mid, late and very late. Finally, the price has been included in the inference model in order to reflect the price elasticity of the demand.

![Fig. 2. Membership functions of the input variables: (a) weather, (b) day of week, (c) time of day, (d) price and output variable: (e) occupancy percentage of the seats.](image-url)
numerical price values have been fuzzified with the terms of low, medium and high. Finally, the numerical value of “occupancy rate of the seats” has been used as the demand level, which is the output of the model. Demand levels are represented by the very low, low, medium, high and very high terms. The membership functions of the input variables and the output variable are illustrated in Fig. 1. The generated rules are presented in the Appendix.

Using the fuzzy if-then-rule based system, demand estimation is achieved and used in the dynamic pricing model presented in Section 4. As a result of this methodology, the optimal pricing scheme for the maritime public transportation service provider is identified with respect to the demand level in order to maximize the revenue. In the literature, triangular and trapezoidal fuzzy numbers are frequently utilized for fuzzy applications. In this study, both triangular and trapezoidal fuzzy numbers are used to consider the fuzziness of the decision elements. The membership functions of weather condition, day of week, time of day, price and percentage of demand are defined by the experts working at the maritime company and given in Fig. 2.

6. Application

6.1. Setting

In this study, we investigate the pricing policy of a maritime transportation service provider which operates on 19 lines, and serving 32 points with 10 fast ferries (vehicle-pass fast ferry), 25 sea buses (only passenger), and 17 conventional vehicle-passenger ferries. We analyzed one of the vehicle-passenger ferry lines namely Istanbul-Bandırma. Bandırma is a city, which is generally visited in summer due to being a tourism center for summer holidays. However, there is considerable travel demand to this city at all times because it connects Istanbul to southern cities.

6.2. Numerical results for demand estimation and pricing

6.2.1. Results of demand estimation

Our aim is to propose a framework to the practitioners in the maritime transportation companies. Hence, the dynamic pricing model is run based on an experimental design. In the designed experiment, three factors have been considered which are the temperature, day of week and time of day. For each factor, different levels are identified as in Table 1.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day of week</td>
<td>Early week</td>
<td>Monday, Tuesday, Wednesday</td>
</tr>
<tr>
<td></td>
<td>Late week</td>
<td>Thursday, Friday</td>
</tr>
<tr>
<td></td>
<td>Weekend</td>
<td>Saturday, Sunday</td>
</tr>
<tr>
<td>Time of day</td>
<td>Rush hour</td>
<td>07 am–10 am and 5–8 pm</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>Other hours of the day</td>
</tr>
<tr>
<td>Temperature</td>
<td>Cold</td>
<td>Less than 15 °C</td>
</tr>
<tr>
<td></td>
<td>Hot</td>
<td>Higher than 15 °C</td>
</tr>
</tbody>
</table>

The temperature factor is defined as two levels. The two levels are hot (weather temperatures higher than 15 °C) and cold (weather temperatures less than 15 °C).

Assuming that demand pattern in some days of the week are similar, the day of week factor are grouped into three levels. The early week level consists of Monday, Tuesday and Wednesday; late week consists of Thursday and Friday and weekend consists of Saturday and Sunday. The third factor, time of day has been split into two levels as the rush hour (07–10 am and 5–8 pm) and normal hours (the rest of the hours of the day). Based on the Design of Experiment (DoE) framework, we conducted scenarios given in Table 2. For each scenario, we run 3 cases to estimate the occupancy rate using the proposed FRBS as well as logit model. A case for each scenario is presented in Table 3.

The model results are expected to differ among scenarios since demand structures and price/demand elasticity of each case are different from each other. The results of demand estimation are obtained as occupancy rate by using the FRBS. The output value is the expected occupancy rate of the seats based on the weather, time and price of the trip, which are given in Table 3 for each scenario. In order to compare the results obtained using proposed FRBS, we refer to our previous study in which the demand estimation was conducted using a logit model with the same dataset [43].

The multinomial logit (MNL) models are utility-based models commonly used in the literature due to the simple probability formulation derivation. The alternative is selected depending on the highest utility (ui), which is formulated with the product attributes. In MNL models, the probability that an alternative j is chosen from a set S ⊆ N = {1, 2, ..., n} that contains j is given by (3) (Train, 2009):

\[ P_j(S) = \frac{e^{ui_j}}{\sum_{i=1}^{n} e^{ui_i}} \]  

(8)

The logit model was employed using crisp values so subjective data such as weather condition was not incorporated in the logit model. For comparison purposes, we include the effect of the temperature based on the average demand levels in the two periods: winter and summer. Then, we compare the estimation results of the logit model for winter with the FRBS estimation results of cold temperature and summer with the warm temperature.

Logit model yields the choice probabilities under specific states of the variables. Estimation using logit model is conducted such that; the demand at the present price level, 90 TL, is assumed to be similar to the past sales data of the specified time period, temperature and day of week. Then, using the choice probabilities obtained from logit model, we estimate the occupancy rate change with respect to the price changes. For example, if we decrease the ticket price we may observe the increase in the choice probabilities of the passengers. Depending on the seat occupancy rate under a specific ticket price and the increase in the choice probability with respect to a decrease in ticket price, we calculate the increase in the expected seat occupancy rate. The estimated occupancy rates of different prices, 70 TL and 110 TL for one case of each scenario are given in the last column of Table 3.

To evaluate the accuracy of the FRBS forecast, the past sales data given for price level 90 TL in Table 3 is used to calculate the mean squared error (MSE) and mean absolute percentage error (MAPE). The MSE value is calculated as 39.09 and the MAPE value as 8.69%. Since, MAPE value is under 10%, it can be concluded that
FRBS forecasts are highly competitive. Besides, we also calculate the MAPE in order to compare the results of FRBS and logit model. The obtained MAPE value is 11.35% which is still competitive.

### 6.2.2. Results of dynamic pricing

Table 4 shows the seven days dynamic pricing model results for a pre-determined demand path; 9, 11, 25, 38, 45, 57, and 65 seats. Comparing Case 1 and 4, it is seen that the temperature and the day of week are the same while time of day differs between them. Since the weather is warm and it is weekend, people tend to spend their holiday out of the city. On the other hand, in order to do this, they should leave the city early in the morning, that is why the revenue of Case 4 is much higher than Case 1. On the other hand, Case 3 also deals with Saturday mid-day case (similar to Case 1) but the temperature is higher, therefore the revenue is higher. Similarly, when Case 2 and 6 are compared, which only differ in temperature variable, it is seen that the revenue is higher for Case 2, since the temperature is higher. The main reason for these results is that Bandırma is a city, which is visited mostly in warm weathers for spending holidays. Moreover, Bandırma connects Istanbul to southern cities, where people usually travel for swimming in summer.

<table>
<thead>
<tr>
<th>Case No</th>
<th>Temperature</th>
<th>Day of week</th>
<th>Time of day</th>
<th>Price (TL)</th>
<th>Occupancy rate (FRBS%)</th>
<th>Occupancy rate (LM%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>26</td>
<td>Saturday</td>
<td>16:30</td>
<td>110</td>
<td>54.40</td>
<td>69.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>90</td>
<td>91.20</td>
<td>89.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70</td>
<td>92.50</td>
<td>100.00</td>
</tr>
<tr>
<td>Case 2</td>
<td>35</td>
<td>Friday</td>
<td>19:00</td>
<td>110</td>
<td>70.00</td>
<td>67.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>90</td>
<td>85.00</td>
<td>85.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70</td>
<td>93.40</td>
<td>100.00</td>
</tr>
<tr>
<td>Case 3</td>
<td>35</td>
<td>Saturday</td>
<td>13:00</td>
<td>110</td>
<td>70.00</td>
<td>71.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>90</td>
<td>92.80</td>
<td>92.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70</td>
<td>93.40</td>
<td>100.00</td>
</tr>
<tr>
<td>Case 4</td>
<td>26</td>
<td>Saturday</td>
<td>08:00</td>
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</table>

Table 4 shows the seven days dynamic pricing model results for a pre-determined demand path; 9, 11, 25, 38, 45, 57, and 65 seats.
Comparing Case 5 and 10, it is seen that the weather is hot and the time of day is the same, but the revenue is higher in Case 10 since it is the end of the week. Similarly Case 14 has lower revenue than Case 10, since the weather is colder. Case 8 and 9 can also be compared since they belong to the same day, there is not a big difference in temperature but the time of day variable is different. People go to work early in the morning, therefore the revenue is higher in Case 9. Similar discussion stands for Case 7 and Case 13, and the revenue is higher in Case 13.

There is a general conclusion that can be derived from the optimal dynamic policies, which states that the optimal decision for the last day of the week is setting the price at the highest level. This makes sense since the passengers are very willing to take that ferry. Thus, the highest price can be offered in order to use this opportunity. Moreover, most of the policies start selling the tickets with the highest price (7 days before the flight) but if the demand is not high enough, then the price is decreased and as it is stated before, on the last day the price is again increased to the highest level (110 TL) since the available number of seats is less and the demand is higher on the last days. To get more revenue, price level has increased in these last days.

Table 5 shows the diversification of revenues under optimal pricing policies based on FRBS, logit models and under fixed pricing policy. The results also show the superiority of using dynamic pricing over fixed pricing (the prices are fixed for each period).

Finally, we conduct a statistical analysis to present that dynamic pricing using FRBS forecasts (1st policy) will result in higher revenues compared to dynamic pricing using logit forecasts (2nd policy) and fixed pricing (3rd policy). We use the sample of revenues of all cases for the three policies. Then, we conduct a test for the difference in the means of the revenues of the 1st and 2nd policies as well as a test for the difference in the means of the revenues of the 1st and 3rd policies.

The null and alternative hypotheses of the first test are as follows:

\[ H_0: \text{mean revenue of dynamic pricing using FRBS forecasts is equal to mean revenue of dynamic pricing using Logit forecasts} \]
\[ H_1: \text{mean revenue of dynamic pricing using FRBS forecasts is larger than the mean revenue of dynamic pricing using Logit forecasts} \]

We find that null hypothesis is rejected at 10% significance level. Thus, we infer that the revenue of dynamic pricing using FRBS forecasts is larger than the revenue of dynamic pricing using logit forecasts at 10% significance level \( z_{\text{calculated}} = 1.32 > z_{0.10} = 1.28 \).

### Table 5

<table>
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<tr>
<th>Case Nr</th>
<th>Revenue of dynamic pricing (FRBS)</th>
<th>Revenue of dynamic pricing (Logit)</th>
<th>Revenue of fixed pricing</th>
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<td>Case 13</td>
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<td>Case 14</td>
<td>7,980</td>
<td>8,878</td>
<td>6,215</td>
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</table>

The null and alternative hypotheses of the second test are as follows:

\[ H_0: \text{mean revenue of dynamic pricing using FRBS forecasts is equal to mean revenue of fixed pricing} \] \( (\mu_{FRBS} = \mu_{fixed}) \).
\[ H_1: \text{mean revenue of dynamic pricing using FRBS forecasts is larger than the mean revenue of fixed pricing} \] \( (\mu_{FRBS} > \mu_{fixed}) \).

We find that null hypothesis is rejected at 5% significance level. Thus, we infer that revenue of dynamic pricing using FRBS forecasts is larger than the revenue of fixed pricing at 10% significance level \( z_{\text{calculated}} = 1.72 > z_{0.10} = 1.70 \).

### 7. Conclusion

In this study, we consider a dynamic pricing problem of a maritime public transportation company. There is no other service provider for ferry transportation in Istanbul, so the market is monopolistic. Since the service provider currently applies fixed pricing policy, we consider applying dynamic pricing due to its several benefits. The demand fluctuates based on the weather, days of the week and different times of the day.

Fuzzy IF-THEN rules are generated in order to forecast the demand because there is no previous data. The contribution of this study can be highlighted with the use of fuzzy logic. Fuzzy logic allows capturing the ambiguity and exploiting imprecise data in demand forecasting. We include weather conditions, which is a linguistic variable in the demand forecasting methodology different from the existing forecasting models in the dynamic pricing literature. Using these findings, a dynamic pricing model is formulated in order to see the optimal policies under different conditions. We compare the dynamic pricing results (based on FRBS and logit forecasts) with the fixed policies used currently by the company. Finally, we run hypothesis tests and show that dynamic pricing with FRBS forecasts yields higher revenue compared to the fixed pricing as well as dynamic pricing with logit forecasts at 5% and 10% significance levels respectively.

The dynamic pricing model is run based on the experimental design. The model results are expected to differ among the scenarios of the experiments since demand structures and price/demand elasticity of each scenario are different from each other. It has been seen that optimal prices and resulting revenues are higher for the scenarios which have higher demand.

The revenue increase gained by our dynamic pricing model is 28.8%, compared to the fixed pricing policy. On the other side, 17.1% increase is gained by the dynamic pricing policy using FRBS forecasts compared to logit model forecasts. This result is highly competitive when we compare it with the previous studies [10,9]. In the study of Di and Hualong [10], the pricing policy for container sea-rail intermodal transportation is investigated. Differently from our study, they try to allocate slots for freight transportation instead of passenger transportation. When they employ a dynamic pricing model, the total revenue is 396,901 yuan, however it is 387,100 yuan with the fixed pricing approach. The increase in the revenue is 2.53%. [9] compared their dynamic pricing model results with the fixed pricing policy under different scenarios. In their study, they consider dynamic congestion pricing in the presence of demand uncertainty and they find that the highest increase in the revenue is 7.71%.

In conclusion, we have used probabilistic dynamic programming to solve the optimization problem. The optimal policies give us an idea about the necessity of applying dynamic pricing policy instead of fixed pricing and show us the diversification of optimal policies under different conditions. For further research, competition with other maritime transport companies as well as
competition with different means of transport can be included to the model.

Appendix A

The rules of the proposed fuzzy if-then-rule based system:

1. If (Weather is warm) and (DayOfWeek is late) and (TimeOfDay is late) and (Price is low) then (percentage is veryHigh)
2. If (Weather is warm) and (DayOfWeek is late) and (TimeOfDay is early) and (Price is low) then (percentage is veryHigh)
3. If (Weather is warm) and (DayOfWeek is late) and (TimeOfDay is late) and (Price is high) then (percentage is medium)
4. If (Weather is veryCold) and (DayOfWeek is early) and (TimeOfDay is VeryLate) and (Price is low) then (percentage is low)
5. If (Weather is veryCold) and (DayOfWeek is early) and (TimeOfDay is VeryLate) and (Price is low) then (percentage is veryLow)
6. If (Weather is veryCold) and (DayOfWeek is early) and (TimeOfDay is VeryEarly) and (Price is low) then (percentage is veryLow)
7. If (Weather is hot) and (DayOfWeek is late) and (TimeOfDay is not VeryLate) and (Price is high) then (percentage is veryHigh)
8. If (Weather is hot) and (DayOfWeek is late) and (TimeOfDay is not VeryLate) and (Price is med) then (percentage is veryHigh)
9. If (Weather is hot) and (DayOfWeek is late) and (TimeOfDay is not VeryLate) and (Price is med) then (percentage is veryHigh)
10. If (Weather is not veryCold) and (DayOfWeek is mid) and (TimeOfDay is low) then (percentage is medium)
11. If (Weather is hot) and (DayOfWeek is mid) and (TimeOfDay is not mid) and (Price is not low) then (percentage is medium)
12. If (Weather is hot) and (DayOfWeek is mid) and (TimeOfDay is mid) and (Price is not low) then (percentage is medium)
13. If (Weather is veryCold) and (DayOfWeek is late) and (TimeOfDay is late) and (Price is high) then (percentage is medium)
14. If (Weather is veryCold) and (DayOfWeek is mid) and (TimeOfDay is mid) and (Price is high) then (percentage is low)
15. If (Weather is veryCold) and (DayOfWeek is mid) and (TimeOfDay is mid) and (Price is med) then (percentage is high)
16. If (Weather is veryCold) and (DayOfWeek is mid) and (TimeOfDay is mid) and (Price is low) then (percentage is high)
17. If (Weather is cold) and (DayOfWeek is early) and (TimeOfDay is early) and (Price is low) then (percentage is high)
18. If (Weather is cold) and (DayOfWeek is early) and (TimeOfDay is early) and (Price is high) then (percentage is high)
19. If (Weather is warm) and (DayOfWeek is early) and (TimeOfDay is mid) and (Price is med) then (percentage is medium)
20. If (Weather is warm) and (DayOfWeek is early) and (TimeOfDay is mid) and (Price is low) then (percentage is high)
21. If (Weather is warm) and (DayOfWeek is early) and (TimeOfDay is mid) and (Price is low) then (percentage is low)
22. If (Weather is cold) and (DayOfWeek is mid) and (TimeOfDay is low) and (Price is high) then (percentage is high)
23. If (Weather is cold) and (DayOfWeek is mid) and (TimeOfDay is low) and (Price is high) then (percentage is high)
24. If (Weather is cold) and (DayOfWeek is mid) and (TimeOfDay is late) and (Price is high) then (percentage is medium)
25. If (Weather is cold) and (DayOfWeek is mid) and (TimeOfDay is late) and (Price is low) then (percentage is high)
26. If (Weather is cold) and (DayOfWeek is mid) and (TimeOfDay is mid) and (Price is high) then (percentage is low)
27. If (Weather is cold) and (DayOfWeek is mid) and (TimeOfDay is mid) and (Price is low) then (percentage is medium)
28. If (Weather is hot) and (DayOfWeek is late) and (TimeOfDay is early) and (Price is low) then (percentage is high)
29. If (Weather is hot) and (DayOfWeek is late) and (TimeOfDay is early) and (Price is med) then (percentage is medium)
30. If (Weather is hot) and (DayOfWeek is late) and (TimeOfDay is early) and (Price is med) then (percentage is veryHigh)
31. If (Weather is hot) and (DayOfWeek is late) and (TimeOfDay is early) and (Price is high) then (percentage is high)

References


