A Simple Method to Enhance the Detection of Second Order Cyclostationarity

Miao Shi †, Yeheskel Bar-Ness ‡, and Wei Su ∗

∗Center for Wireless Communication and Signal Processing Research
ECE Department, New Jersey Institute of Technology, Newark, NJ 07102 USA

†U.S. Army RDECOM CERDEC, Fort Monmouth, NJ 07703 USA

Abstract—Cyclostationarity is a significant feature of numerous physical and man-made processes. Specially in wireless communications, it has been explored and applied for many processing, like timing and frequency synchronization, channel estimation, and modulation classification. In this paper, we notice that the second order time varying auto-correlation of the received communication signal contains a cyclostationary real and a non-cyclostationary complex part. We propose a simple method to combine the positive and negative part of the cyclo-spectrum (the Fourier transform of the time varying auto-correlation) to enhance the performance of detecting the second order cyclostationarity of these signals. The idea redis related to the concept of cyclic Wiener filter put forth by Gardner in 1993 [4]. Monte Carlo simulations are given to verify the results.

Index Terms—2nd order Cyclostationarity, communication signal.

I. INTRODUCTION

Cyclostationarity is an important feature of the wireless communication signals and many man made processes. Exploration of cyclostationarity has started in the 1950s, when Bennett apply it in synchronization algorithms for communications systems [1]. The detection of cyclostationarity of communication signals has been studied by Gardner [2], [3]. Giannakis and Dandawate proposed statistical tests to show the existence of cyclostationarity [5]. Cyclostationarity was also used for estimating frequency offset, timing recovery and channel equalization [6]–[7]. More papers on the application of cyclostationarity have been listed by E. Serpedin and el. [8]. Hence improving capability of detecting cyclostationarity will ease performing these processes.

In this paper, we consider a scheme to enhance the performance of detecting (probability of detections) the second order cyclostationarity of the received communication signal. By noticing that the second order time varying auto-correlation of the received communication signal contains a cyclostationary real part and a non-cyclostationary complex part, we propose to combine the positive part and the negative part of the cyclo-spectrum. (Note that we use the term "cyclo-spectrum" to denote the Fourier transform of the time varying correlation function.) We note that the DFT at the positive and the negative cyclic frequencies of the real cyclostationary part is conjugate to each other, while the DFT of the non-cyclostationary part (noise part), which is complex, does not have such feature. This property is similar to having cyclostationary part from two different bases. Therefore, it presents extra degree of freedom that can enhance detection performance of the second order cyclostationarity. Throughout the paper, we assumed for simplicity that frequency offset is zero, and w.l.o.g. we take we discard time offset.

The paper is organized as follows. Section II introduces the system model and the second order cyclostationarity definition. In Section III, we propose the prewhitening method [5] as well as our simple direct combining method to enhance the cyclostationarity detection. In Section IV, the cases with added pulse shaping filtering are considered. Performance of these two aforementioned methods are analyzed thereafter in Section V. In Section VI, Monte Carlo simulations are conducted to test the improvement in probability of detecting cyclostationarity using the new method under different scenarios. Last in Section VII, we conclude this paper.

II. SIGNAL MODEL AND SECOND ORDER CYCLOSTATIONARITY

A signal \( x(t) \) is termed second-order cyclostationary if its time-varying correlation function \(^1\)

\[
R(t, \tau) = E[x(t)x^*(t + \tau)]
\]

(1)

is periodic in \( t \) for a given delay \( \tau \). Denote \( M \) as the length of the data samples. The cyclo-spectrum at cyclic frequency \( \alpha \), \( C(\alpha, \tau) \), is the Fourier transform of \( R(t, \tau) \) with respect to time \( t \) [5], given by

\[
C(\alpha, \tau) = \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{t=0}^{M-1} R(t, \tau) \exp(-j\alpha t)
\]

(2)

Let \( x(t) \) be the received signal given by

\[
x(t) = \sum_{m} s(m)h(t - mT_b) + w(t)
\]

(3)

\(^{1}\)In literature, \( E[x(t)x(t + \tau)] \), ie. without conjugation is also considered.

978-1-4244-2324-8/08/$25.00 © 2008 IEEE.
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2008 proceedings.
where \( s(m) \) is the transmitted symbol which can be any complex modulated signal (QAM, MPSK, OFDM) with variance \( \sigma_w^2 \). \( T_s \) is the signal duration of one symbol in time domain. \( h(t) \) is the channel response including effect of the transmitting and receiving pulse shaping filters. \( w(t) \) is additive white Gaussian noise (AWGN) with variance \( \sigma_w^2 \).

The received sampled sequence \( x(n) \) is obtained by oversampling the received signal with sampling frequency \( 1/T_s' \). When the oversampling factor \( q = \frac{T_s}{T_s'} \) is larger than 1, the *cyclo-spectrum*, \( C(\alpha, \tau) \), will have a peak corresponding to the symbol rate 1/\( T_s \), i.e. \( \hat{\alpha} = 2\pi/q \).

Dandawaté and Giannakis [5] proposed a generalized maximum likelihood method to estimate the cyclic frequency \( \hat{\alpha} \).

Define \( \hat{C}(\alpha, \tau_i) \) as the estimate of \( C(\alpha, \tau_i) \), the *cyclo-spectrum* at cyclic frequency \( \alpha \) with delay \( \tau_i \), which is given by

\[
\hat{C}(\alpha, \tau_i) = \frac{1}{M} \sum_{i=0}^{M-1} x(t) x^*(t + \tau_i) \exp(-j\alpha t) \tag{4}
\]

and \( \hat{c}(\alpha, \vec{\tau}) \) by the vector of dimension \( 2p \),

\[
\hat{c}(\alpha, \vec{\tau}) = \left[ \begin{array}{c}
\text{Re} \left( \hat{C}(\alpha, 1) \right) \\
\text{Im} \left( \hat{C}(\alpha, 1) \right) \\
\vdots \\
\text{Re} \left( \hat{C}(\alpha, p) \right) \\
\text{Im} \left( \hat{C}(\alpha, p) \right)
\end{array} \right] \tag{5}
\]

where \( \vec{\tau} \) is a vector of \( \tau \). Since the *cyclo-spectrum* for different \( \tau \) have the same cyclic frequency \( \hat{\alpha} \) [5], the estimator (6) prewhitens the *cyclo-spectrums* \( C(\alpha, \tau_i) \), \( i = 1, \ldots, p \), before combining them together. The cyclic frequency estimator for delay \( \vec{\tau} \) is given by

\[
\hat{\alpha} = \arg \max_{\alpha} \hat{c}(\alpha, \vec{\tau}) \hat{Y}^{-1} \hat{c}(\alpha, \vec{\tau})^H
\]

where \( \hat{Y} \) is of size \( 2p \times 2p \) is the asymptotic covariance matrix of the estimator \( \hat{c}(\alpha, \vec{\tau}) \), which can be computed by using the equations (16) and (17) given in [5].

As noticed from (6), the scheme proposed by Dandawaté and Giannakis [5] combines the *cyclo-spectrums* at different delay \( \tau \) to enhance the detection probability of the cyclostationarity test. In the following, we present a new method for enhancing the cyclostationarity of the received signal by combining the information contained in the positive and negative frequency part of the *cyclo-spectrum*.

III. NEW METHODS TO ENHANCE THE PERFORMANCE OF DETECTING THE SECOND ORDER CYCLOSTATIONARY

A. direct combining scheme

To simplify the derivation, we assume first that *no pulse shaping filtering* is used in the transmitter, hence the received signal \( x(t) \) oversampled with \( q > 1 \) is given by

\[
x(n) = \sum_{m} s(m) g(n - mq) + w(n) \tag{7}
\]

where we normalize the sample duration at the receiver, \( T'_b \) to 1. \( g(l) \) is the rectangular pulse given by

\[
g(l) = \begin{cases} 1, & 0 \leq l < q, \ l \text{ is an integer} \\ 0, & \text{otherwise} \end{cases}
\]

The discrete time-varying correlation function \( R(n, \tau) \) at \( \tau = 1^2 \) will be periodic with period \( q \) (when \( q \) is an integer) or almost-periodic \( ^3 \) (when \( q \) is a fractional number) [5]. That is,

\[
R(n + lq, 1) = R(n, 1) \tag{9}
\]

when \( l \) is any integer. Hence the choice of \( \tau = 1 \) does not restrict generality. Note that since the received signal is now changed to discrete sequence, we will use the discrete number to indicate the time delay without specific mentioning.

For example, when \( q \) is an integer, and since the signal \( s(m) \) is independent of the noise, \( R(n, 1) \) will be a rectangular pulse train given by

\[
R(n, 1) = \begin{cases} \sigma_w^2, & 0 \leq [(n - \nu) \mod q] < q - 1 \\ 0, & [(n - \nu) \mod q] = q - 1 \end{cases} \tag{10}
\]

where \( \nu \) is the timing offset, \( \nu = 0, \ldots, q - 1 \), though we will assume \( \nu = 0 \) in our derivation. Obviously, the above function is a periodic function with period \( q \).

In practice, the estimate of the time-varying correlation function, \( \hat{R}(n, 1) \) is given by

\[
\hat{R}(n, 1) = x(n)x^*(n + 1)
\]

\[
= [x(n) - w(n)] [x(n + 1) - w(n + 1)]^* + w(n)w^*(n + 1) + [x(n) - w(n)]w^*(n + 1) + w(n)[x(n + 1) - w(n + 1)]^*
\]

where \( n = 0, \ldots, M - 1 \), with \( M \) is the length of the observed data samples. Note that the estimate of the time-varying correlation function contains four terms. The first results from the signal, which we denote as \( R_s(n, 1) \). We split \( R_s(n, 1) \) into two complementary sequences: \( \hat{R}_{ps}(n, 1) \) and \( \hat{R}_{np}(n, 1) \). The first shown to be *cyclostationary sequence*, which we call the cyclostationary signal part, is defined as

\[
\hat{R}_{ps}(n, 1) = \begin{cases} [x(n) - w(n)]^*, & 0 \leq [(n - \nu) \mod q] < q - 1 \\ [x(n + 1) - w(n + 1)]^*, & [(n - \nu) \mod q] = q - 1 \end{cases} \tag{12}
\]

Clearly, when the received signal is oversampled, the expectation of the \( \hat{R}_{ps}(n, 1) \) is periodic and is given by (10).

The other part of \( \hat{R}_s(n, 1) \) is non-cyclostationary, which we call the *non-cyclostationary signal part*, is defined as

\[
\hat{R}_{np}(n, 1) = \begin{cases} 0, & 0 \leq [(n - \nu) \mod q] < q - 1 \\ [x(n) - w(n)]^*, & [(n - \nu) \mod q] = q - 1 \end{cases}
\]

where \( \nu \) is taken as integer after sampling in the receiver.

\(^3\)To simplify the derivation, we only consider the periodic case.
The expectation of $\hat{R}_{nps}(n,1)$ is zero for all $n$ due to the fact that the signal $s(m)$ and $s(l)$, $m \neq l$, are independent from each other.

To conduct cyclostationarity test, we use at the front end of the receivers a wide band filter, causing the noise samples be independent and thus contributes to the non-cyclostationary part of $\hat{R}(n,1)$. The non-cyclostationary sequence noise part (the other three terms of (11)) is given by

$$\hat{R}_{nps}(n,1) = w(n)w^*(n+1) + [x(n) - w(n)]w^*(n+1) + w(n)[x(n+1) - w(n+1)]^*$$

(14)

The expectation of $\hat{R}_{nps}(n,1)$ is zero for all $n$.

As an example of the sequence of (12), (13) and (14), for OFDM systems, when the oversampling factor $q = 3$, timing synchronized ($\nu = 0$), the sequence of the oversampled signal is given by $(s(0), s(0), s(0), s(1), s(1), ...)$, then the received signal $x(n)$, $n = 0, ..., M$ is given by

$$x(n) = \begin{bmatrix} s(0) + w(0), s(0) + w(1), s(0) + w(2), \\ s(1) + w(3), s(1) + w(4), ... \end{bmatrix}$$

(15)

The cyclostationary signal part of the time-varying correlation, $\hat{R}_{ps}(n,1)$, $n = 0, ..., M$ is given by

$$\hat{R}_{ps}(n,1) = [s(0)^2, |s(0)|^2, 0, |s(1)|^2, |s(1)|^2, ...]$$

(16)

while the sequence of the non-cyclostationary signal part $R_{nps}(n,1)$, $n = 0, ..., M$ is given by

$$\hat{R}_{nps}(n,1) = [0,0,0,s(0)s^*(1),0,0,...]$$

(17)

The non-cyclostationary noise part $\hat{R}_{nps}(n,1)$, $n = 0, ..., M$ is given by

$$\hat{R}_{nps}(n,1) = \begin{bmatrix} w(0)w^*(1) + s(0)s^*(1), \\ s^*(0)w(0), w(1)w^*(2), \\ s(0)w^*(2) + s^*(0)w(1), \\ w(2)w^*(3) + s(0)w^*(3) + s^*(1)w(2), ... \end{bmatrix}$$

(18)

We note from (16) that the cyclostationary sequence, $\hat{R}_{ps}(n,1)$ is real while from (17) and (18) the non-cyclostationary part, $\hat{R}_{nps}(n,1)$ and $\hat{R}_{nps}(n,1)$ are complex. According to the property of DFT, the Fourier transform of a real periodic sequence has symmetric feature, that is, $F(-\alpha) = F^*(\alpha)$, where $F(\alpha)$ denotes the DFT at frequency $\alpha$. Hence, the DFT of $\hat{R}_{ps}(n,1)$ will have the aforementioned symmetry; while the non-cyclostationary parts, $\hat{R}_{nps}(n,1)$ and $\hat{R}_{nps}(n,1)$ are complex and aperiodic which does not have the above symmetries. Note that these conclusions can also be applied to the case where delay $\tau > 1$.

Straightforwardly going to frequency domain, the detection of cyclostationarity will be enhanced by combining the positive frequency part with the negative frequency part of the cyclo-spectrum. This is a simple method which we call the direct combining scheme. In fact, with this approach, the cyclic frequency estimator is given by

$$\hat{\alpha} = \arg \max_{\alpha} C^*(\alpha, \tau) = \arg \max_{\alpha} \left[ \frac{1}{q} \left( \frac{C^*(\alpha, \tau) + C^*(-\alpha, \tau)}{2} \right) \right]$$

(19)

where $C^*(\alpha, \tau)$ is half of the cyclo-spectrum of (4) with $\alpha$ positive, and $C^*(-\alpha, \tau)$ is the other half corresponds to $\alpha$ negative, $\alpha \in [0, \pi]$. $\kappa_1(\alpha)$ is the cost function for this scheme. Note that $\hat{\alpha}$ is a function of $\tau$.

B. prewhiten combining scheme

In [5], Dandawaté and Giannakis proposed a generalized maximum likelihood scheme to estimate the cyclic frequency by combining the cyclo-spectrums for $p$ different delay $\tau$ (6), which can be regarded as prewhiten before combining. In the following, we apply this approach to combine the corresponding negative frequency part and the positive frequency part of the cyclo-spectrum together i.e with $p = 1$. Note that the asymptotic covariance matrix $\hat{Y}$ in (6) is estimated by using the cyclo-spectrum vector $\hat{c}(\alpha)$ $^4$. We propose two ways to obtain the covariance matrix $\hat{Y}$. First, we present a method which use the exact covariance matrix of the cyclo-spectrum $\hat{C}^*(\alpha)$. Then we use the asymptotic estimates of $\hat{Y}$ similar to the method proposed by Dandawaté and Giannakis. To distinguish these two methods from the method proposed in previous, we call in the following these two methods as prewhitening combining method.

Assuming that the FFT which generates the cyclo-spectrum has size $M$, which is the same as the length of the observed sequence, when $(M \text{mod} q) = 0$ or the period $q$ fits the measurement $M$, the cyclo-spectrum of $\hat{R}_{ps}(n,1)$ has peaks only at indices $\alpha = lM/q$ and zeros elsewhere. $l$ is an integer. This is because the Fourier transform of a periodic signal is an impulse train in the frequency domain with the impulse values proportional to the discrete Fourier series. To simplify the derivation, we can approximate the cyclo-spectrum of $\hat{R}_{ps}(n,1)$ will generate peaks only at indices $\alpha = 2\pi/q$ and zeros elsewhere when the observed sequence is long enough (more periods of data is taken in the measurement.) The cyclo-spectrum of $\hat{R}_{ps}(n,1)$ can be expressed by using the Fourier transform of one period of the cyclostationary sequence, which is given by

$$\hat{C}_ps(\alpha) = \sum_{l=0}^{q-1} Z_l \delta \left( \alpha - \frac{2\pi l}{q} \right)$$

(20)

where $\delta(\cdot)$ is a delta function. $Z_l$, $l = 0, \cdots, q-1$, is the $l$th sample of the DFT of one period of the periodic sequence $\frac{q-1}{\sigma_1^2, \cdots, \sigma_q^2}$, $0, \sigma_1^2, \cdots, \sigma_q^2$. To simplify the derivation, in the following, we assume the signal is timing synchronized, or $\nu = 0$. When $\nu \neq 0$, the above sequence will be rotated accordingly, and thus change the value of $Z_l$.

$^4$Note that we dropped the dependence of $\tau$ as we took it to equal to one.
The noise in the cyclo-spectrum is due to the non-cyclostationary part which is the overall result of \( \hat{R}_{nps}(n, 1) \) and \( \hat{R}_{npn}(n, 1) \). Since the signal is independent of the noise, it is possible to find directly the cyclo-spectrum noise introduced by \( \hat{R}_{nps}(n, 1) \) and \( \hat{R}_{npn}(n, 1) \) separately. When \( \nu = 0 \), from (13), the DFT of \( \hat{R}_{nps}(n, 1) \) is given by

\[
\hat{C}_{nps} (\alpha) = \frac{1}{M} \sum_{k=0}^{[M/q]-1} s(k)s^*(k+1) \cdot \exp(-jqa_k) \exp[-j\alpha(q-1)]
\]

(21)

Note that when \( \nu \neq 0 \), according the property of the DFT, \( \hat{C}_{nps}(\alpha) \) will have an additional exponential term on the right corresponding to the non-zero timing offset \( \nu \). Since both \( \hat{C}_{nps}(\alpha) \) and \( \hat{C}_{nps}(-\alpha) \) are complex, and \( \text{Re}[\hat{C}_{nps}(-\alpha)] = \text{Re}[\hat{C}_{nps}(\alpha)] \), \( \text{Im}[\hat{C}_{nps}(\alpha)] = -\text{Im}[\hat{C}_{nps}(\alpha)] \), the covariance matrix of \( \hat{C}_{nps}(\alpha) \) and \( \hat{C}_{nps}(-\alpha) \), \( \text{cov}_{nps} \) is given by

\[
\text{cov}_{nps}(\alpha) = \begin{bmatrix}
\hat{c}_{r+}^2 & \hat{c}_{r+}\hat{c}_{r-} & \hat{c}_{r+}\hat{c}_{i+} & \hat{c}_{r+}\hat{c}_{i-} \\
\hat{c}_{r-}\hat{c}_{r+} & \hat{c}_{r-}\hat{c}_{r-} & \hat{c}_{r-}\hat{c}_{i+} & \hat{c}_{r-}\hat{c}_{i-} \\
\hat{c}_{i+}\hat{c}_{r+} & \hat{c}_{i+}\hat{c}_{r-} & \hat{c}_{i+}\hat{c}_{i+} & \hat{c}_{i+}\hat{c}_{i-} \\
\hat{c}_{i-}\hat{c}_{r+} & \hat{c}_{i-}\hat{c}_{r-} & \hat{c}_{i-}\hat{c}_{i+} & \hat{c}_{i-}\hat{c}_{i-} \\
\end{bmatrix}
\]

(22)

where

\[
\hat{c}_{r+} = \text{Re}[\hat{C}_{nps}(\alpha)] = \frac{1}{M} \sum_{k=0}^{[M/q]-1} \left\{ z_r(k) \cos[q\alpha k + \alpha(q-1)] + z_i(k) \sin[q\alpha k + \alpha(q-1)] \right\}
\]

(23)

\[
\hat{c}_{r-} = \text{Re}[\hat{C}_{nps}(-\alpha)] = \frac{1}{M} \sum_{k=0}^{[M/q]-1} \left\{ z_r(k) \cos[q\alpha k + \alpha(q-1)] - z_i(k) \sin[q\alpha k + \alpha(q-1)] \right\}
\]

(24)

\[
\hat{c}_{i+} = \text{Im}[\hat{C}_{nps}(\alpha)] = \frac{1}{M} \sum_{k=0}^{[M/q]-1} \left\{ -z_r(k) \sin[q\alpha k + \alpha(q-1)] + z_i(k) \cos[q\alpha k + \alpha(q-1)] \right\}
\]

(25)

\[
\hat{c}_{i-} = -\text{Im}[\hat{C}_{nps}(-\alpha)] = \frac{1}{M} \sum_{k=0}^{[M/q]-1} \left\{ -z_r(k) \sin[q\alpha k + \alpha(q-1)] - z_i(k) \cos[q\alpha k + \alpha(q-1)] \right\}
\]

(26)

Note that \( z_r(k) = \text{Re}[s(k)s^*(k+1)] \) and \( z_i(k) = \text{Im}[s(k)s^*(k+1)] \). Since the signal is i.i.d.,

\[
\mathbb{E}[z_r(l)z_r(m)] = \mathbb{E}[z_i(l)z_i(m)] = 0, \quad l \neq m,
\]

the real and imaginary part of the signal \( s(k) \) is also i.i.d. Through mathematical manipulation, we have

\[
\text{cov}_{nps}(\alpha) = \frac{\sigma_r^4}{2Mq} \begin{bmatrix}
\hat{c}_{r+}^2 & \hat{c}_{r+}\hat{c}_{r-} & \hat{c}_{r+}\hat{c}_{i+} & \hat{c}_{r+}\hat{c}_{i-} \\
\hat{c}_{r-}\hat{c}_{r+} & \hat{c}_{r-}\hat{c}_{r-} & \hat{c}_{r-}\hat{c}_{i+} & \hat{c}_{r-}\hat{c}_{i-} \\
\hat{c}_{i+}\hat{c}_{r+} & \hat{c}_{i+}\hat{c}_{r-} & \hat{c}_{i+}\hat{c}_{i+} & \hat{c}_{i+}\hat{c}_{i-} \\
\hat{c}_{i-}\hat{c}_{r+} & \hat{c}_{i-}\hat{c}_{r-} & \hat{c}_{i-}\hat{c}_{i+} & \hat{c}_{i-}\hat{c}_{i-} \\
\end{bmatrix}
\]

(27)

Similarly, we can get

\[
\text{cov}_{npn}(\alpha) = \frac{\sigma_n^4}{2Mq} \begin{bmatrix}
\hat{c}_{n+}^2 & \hat{c}_{n+}\hat{c}_{n-} & \hat{c}_{n+}\hat{c}_{n+} & \hat{c}_{n+}\hat{c}_{n-} \\
\hat{c}_{n-}\hat{c}_{n+} & \hat{c}_{n-}\hat{c}_{n-} & \hat{c}_{n-}\hat{c}_{n+} & \hat{c}_{n-}\hat{c}_{n-} \\
\hat{c}_{n+}\hat{c}_{n+} & \hat{c}_{n+}\hat{c}_{n-} & \hat{c}_{n+}\hat{c}_{n+} & \hat{c}_{n+}\hat{c}_{n-} \\
\hat{c}_{n-}\hat{c}_{n+} & \hat{c}_{n-}\hat{c}_{n-} & \hat{c}_{n-}\hat{c}_{n+} & \hat{c}_{n-}\hat{c}_{n-} \\
\end{bmatrix}
\]

(28)

Note that except at the cyclic frequency \( l\pi/q \), the covariance matrix \( \text{cov}_{nps} \) is approximately zeros, that is, the cyclo-spectrum noise generated by the signal is almost white. Similarly from (14), we can get \( \hat{C}_{npn}(\alpha) \), the DFT of \( \hat{R}_{npn}(n, 1) \), and the covariance matrix of \( \hat{C}_{npn}(\alpha) \) and \( \hat{C}_{npn}(-\alpha) \), \( \text{cov}_{npn} \). Except the frequency at 0, \( \pm \pi \), \( \text{cov}_{npn}(\alpha) \) is given by

\[
\text{cov}_{npn}(\alpha) = \frac{\sigma_r^4}{2M} \mathbf{I}_4 + \text{cov}_{npn2}(\alpha)
\]

(32)

where \( \mathbf{I}_4 \) is the unit matrix of size 4, and \( \text{cov}_{npn2}(\alpha) \) is the covariance matrix generated by the cross term of the signal and noise, which is a \( 4 \times 4 \) matrix given by

\[
\text{cov}_{npn2}(\alpha) = \begin{bmatrix}
\varsigma_{11}(\alpha) & \varsigma_{12}(\alpha) & \varsigma_{13}(\alpha) & \varsigma_{14}(\alpha) \\
\varsigma_{21}(\alpha) & \varsigma_{22}(\alpha) & \varsigma_{23}(\alpha) & \varsigma_{24}(\alpha) \\
\varsigma_{31}(\alpha) & \varsigma_{32}(\alpha) & \varsigma_{33}(\alpha) & \varsigma_{34}(\alpha) \\
\varsigma_{41}(\alpha) & \varsigma_{42}(\alpha) & \varsigma_{43}(\alpha) & \varsigma_{44}(\alpha) \\
\end{bmatrix}
\]

(33)

where

\[
\varsigma_{11}(\alpha) = \varsigma_{22}(\alpha) = \frac{\sigma_r^4 \sigma_n^2}{Mq} \sum_{k=0}^{[M/q]-1} \cos[(2i+1)\alpha + 2qk\alpha]
\]

(34)
\[
\varsigma_{13}(\alpha) = \varsigma_{14}(\alpha) \\
= -\frac{\sigma_x^2\sigma_w^2}{Mq} \left\{ q - \sum_{k=0}^{[M/q]-1} \sum_{i=0}^{q-2} \cos[(2i+1)\alpha + 2qk\alpha] \right\} \\
\varsigma_{12}(\alpha) = \varsigma_{34}(\alpha) = -\frac{\sigma_x^2\sigma_w^2}{Mq} \cos \alpha \\
\varsigma_{13}(\alpha) = \varsigma_{24}(\alpha) = -\frac{\sigma_x^2\sigma_w^2}{Mq} \sum_{k=0}^{[M/q]-1} \sum_{i=0}^{q-2} \sin[(2i+1)\alpha + 2qk\alpha]
\]

and
\[
\varsigma_{14}(\alpha) = \varsigma_{23}(\alpha) = 0
\]

The proof is not shown due to the page limit.

In summary, the first term on the RHS of (32) is diagonal, which means that the cyclo-spectrum noise from the additive noise \( w(n) \) is white (except the frequency at 0, \( \pm \pi \)). However, the second term on the RHS of (32) is not diagonal, meaning that the cross term of the signal and noise will bring correlated cyclo-spectrum noise. Similar to (6), the cyclic frequency estimator using prewhiten combining scheme is given by
\[
\hat{\alpha} = \arg \max_{\alpha} \left[ \varsigma_{12}(\alpha) \right] \\
= \arg \max_{\alpha} \left[ \varsigma'(\alpha) \left[ \text{cov}_{wps}(\alpha) + \text{cov}_{dps}(\alpha) \right]^{-1} \varsigma'(\alpha) H \right]
\]

where as in (5)
\[
\varsigma'(\alpha) = \begin{bmatrix} \text{Re} \left( \hat{C}'(\alpha) \right) & \text{Re} \left( \hat{C}'(-\alpha) \right) \\ \text{Im} \left( \hat{C}'(\alpha) \right) & -\text{Im} \left( \hat{C}'(-\alpha) \right) \end{bmatrix}
\]

except \( \hat{C}'(\alpha) \) is only half of the cyclo-spectrum. (The prime identifies this fact).

In the above prewhiten scheme, the covariance matrix is derived theoretically. According to (22) and (32), to calculate the covariance matrix, we need the prior knowledge of oversampling factor \( q \), timing offset \( \nu \), signal power \( \sigma_x^2 \) and noise power \( \sigma_w^2 \). Dandawaté and Giannakis avoid this difficulty by evaluating the covariance matrix experimentally without resorting to the particular information of other parameters except the cyclo-spectrum itself. Following their approach, we will present a modified and more practical prewhiten combining scheme next.

We directly apply Dandawaté and Giannakis’ scheme to get the covariance matrix for the negative and positive half of the cyclo-spectrum. Note that \( x(n)x^*(n+\tau) \) and \( x(n+\tau)x^*(n) \) are conjugate to each other. If the positive part of the Fourier transform of \( x(n)x^*(n+\tau) \) is \( \hat{C}'(\alpha) \), then according to the properties of Fourier transform, the positive part of the Fourier transform of \( x(n+\tau)x^*(n) \) will be \( \hat{C}'(-\alpha) \). Therefore, by calculating the covariance matrix of the cyclo-spectrums of \( x(n)x^*(n+\tau) \) and \( x(n+\tau)x^*(n) \), we can get the covariance matrix of \( \hat{C}'(\alpha) \) and \( \hat{C}'(-\alpha) \) without exact calculation using the prior knowledge of \( q, \nu, \sigma_x^2 \) and \( \sigma_w^2 \). Later in simulation, to distinguish from the previously proposed prewhiten combining scheme, we will call this scheme as the experimental prewhiten combining scheme, while the previously proposed one as the theoretical prewhiten combining scheme.

### IV. Cases with Pulse Shaping

In practice, pulse shaping filtering is applied for spectral management. For the second order cyclostationarity test, the pulse shaping filter will introduce correlations between successive samples and thus degrade its performance.

To simplify the derivation, we assume the cyclostationarity test is conducted before the pulse shaping match filtering in the receiver. Similar to the no pulse shaping filter case, we can get the mean value and the covariance matrix of the cyclic spectrum noise due to the periodic and aperiodic signal and noise. The exact derivation will not be shown here due to the page limit. It was also found that for the pulse shaping case, the theoretical derivation of the prewhiten scheme is very complicated. Therefore, in simulation with pulse shaping, we only test the performance of the direct combining and the experimental prewhiten combining scheme which use the estimated asymptotic covariance matrix.

### V. Simulation

Monte Carlo simulations are conducted to test the performance of the above methods (5000 trails for each point). In the first scenario, no pulse shaping is used in the transmitter. The received signal is oversampled with \( q = 3 \) (in the half cyclo-spectrum, there will be only one peak. When \( q > 3 \), there will be multiple peaks in the cyclo-spectrum, and hence need an approximate range to detect the peak corresponding to the oversampling rate.) The methods, direct combining (direct) and prewhiten combining schemes are applied respectively. Note that we apply two methods for the prewhiten combining scheme; one is the theoretical prewhiten combining scheme which uses the exact covariance we derived (prewhiten 1) and the other is experimental prewhiten combining scheme which uses the asymptotic covariance matrix (prewhiten 2) [5]. We also applied the original algorithm in [5] (ori) without using our schemes but for fairness, with the search range limited to half of the cyclo-spectrum. QPSK and OFDM modulation are used. The number of the received symbols for cyclostationarity test was 1000. The detection is considered successful when the absolute cyclic frequency \( \hat{\alpha} \) is detected within \( 2\pi(3/M) \) error. As it is shown in Fig.1, for both OFDM and QPSK, all the schemes outperform the original scheme. The direct combining scheme has almost similar detection performance as the theoretical prewhiten (prewhiten 2) combining schemes.

*The oversampling factor \( q \neq 2 \) will insure that cyclic frequency is not equal to \( \pm \pi \).

*We only consider one cyclo-spectrum with negative and positive parts to combine, which corresponds to \( p = 1 \) for (5) in [5]. When \( p > 1 \), for each cyclo-spectrum, it can be divided into two parts, negative and positive, to combine as in (41).
In this paper, a new method to enhance the detection performance of the second order cyclostationarity in communication system is proposed. This due to the fact that this new method introduce extra degrees of freedom in cyclostationarity test, which enables us to combine the positive and negative half of the cyclo-spectrum together. Two schemes were proposed. One is the direct combining scheme, where we simply sum up the two half cyclo-spectrum. The other scheme is the prewhiten combining scheme, where the two half cyclo-spectrums are prewhitened before summing up together. We used both theoretical derivation of the covariance matrix and the asymptotic covariance matrix for the prewhiten combining scheme. We show that the direct combining scheme at certain scenarios can detect cyclostationarity as good as the prewhiten combining scheme. Both combing schemes will outperform the original method without combining the cyclo-spectrum, by approximately 3dB detection gain, when measure at 50% detection probability.

VI. CONCLUSION

In this paper, a new method to enhance the detection performance of the second order cyclostationarity in communication system is proposed. This due to the fact that this new method introduce extra degrees of freedom in cyclostationarity test, which enables us to combine the positive and negative half of the cyclo-spectrum together. Two schemes were proposed. One is the direct combining scheme, where we simply sum up the two half cyclo-spectrum. The other scheme is the prewhiten combining scheme, where the two half cyclo-spectrums are prewhitened before summing up together. We used both theoretical derivation of the covariance matrix and the asymptotic covariance matrix for the prewhiten combining scheme. We show that the direct combining scheme at certain scenarios can detect cyclostationarity as good as the prewhiten combining scheme. Both combing schemes will outperform the original method without combining the cyclo-spectrum, by approximately 3dB detection gain, when measure at 50% detection probability.

REFERENCES