Combinatorial Constructions of Optimal Optical Orthogonal Codes with Weight 4

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Abstract

A \((v, k, \lambda)\) optical orthogonal code \(C\) is a family of \((0, 1)\) sequences of length \(v\) and weight \(k\) satisfying the following two correlation properties:

\begin{align*}
(1) \quad &\sum_{0 \leq t \leq v-1} x_t x_{t+i} \leq \lambda \quad \text{for any } x = (x_0, x_1, \ldots, x_{v-1}) \in C \text{ and any integer } i \neq 0 \pmod{v}; \\
(2) \quad &\sum_{0 \leq t \leq v-1} x_t y_{t+i} \leq \lambda \quad \text{for any } x = (x_0, x_1, \ldots, x_{v-1}) \in C, y = (y_0, y_1, \ldots, y_{v-1}) \in C \text{ with } x \neq y, \text{ and any integer } i,
\end{align*}

where the subscripts are taken modulo \(v\). A \((v, k, \lambda)\) optical orthogonal code with

\[\left\lfloor \frac{1}{k} \left\lfloor \frac{v-1}{k-1} \left\lfloor \frac{v-2}{k-2} \left[ \cdots \left\lfloor \frac{v-\lambda}{k-\lambda} \right\rfloor \cdots \right] \right\rfloor \right\rfloor \]

codewords is said to be optimal. Optical orthogonal codes are essential for success of fiber-optic code division multiple access communication systems. The use of an optimal optical orthogonal code enables the largest possible number of asynchronous users to transmit information efficiently and reliably. In this paper, various combinatorial constructions for optimal \((v, 4, 1)\) optical orthogonal codes, such as those via skew starters and Weil’s theorem on character sums, are given for \(v \equiv 0 \pmod{12}\). These improve the known existence results on optimal optical orthogonal codes. In particular, it is shown that an optimal \((v, 4, 1)\) optical orthogonal code exists for any positive integer \(v \equiv 0 \pmod{24}\).