A General Model of a Kind of Parallel Manipulator for Active Control Based on KANE’s Dynamics

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Abstract—Since many applications in precision engineering will strongly require a careful isolation of the process from the vibration environment to provide a sufficiently quiescent gravity environment, the active isolation system which can achieve a very low remaining vibration level has become more and more important in recent years. In this paper, a general dynamic model of a kind of parallel platform for vibration control applications based on KANE’s method is presented. The analysis result is a state-space, analytical set of linearized equations of motion. At the end of this paper the control system architecture is introduced for the kind of parallel platform as a multiple-input/multiple-output (MIMO) problem.

I. INTRODUCTION

A careful isolation of the process from the vibration environment will be strongly required in many applications of precision engineering. These disturbances in the form of vibrations will degrade the performance of the sensitive instruments which are essential for precision positioning. To counteract the vibration on precision instruments, active isolation systems are proposed, since these units achieve a very low remaining vibration level, especially for low frequency disturbances without the resonance behavior of a passive isolation system. In recent years, many researchers devote to use parallel manipulators for active vibration control since the parallel manipulators offer the advantages of high stiffness, low inertia, and high speed capability at the expense of smaller workspace, more complex mechanical design, difficult direct kinematics and control algorithm. There have been intensively evaluated by industry and institutions. Some designers adopt the flexure hinges instead of conventional mechanism joints since the backlash and friction in the conventional joints influence the performances of parallel mechanisms remarkably.

In this paper, a kind of parallel robot with flexure hinges will be analyzed to achieve active vibration control for precision instruments or the micro-gravity payloads. In order to provide effective model-based isolation, the task of controller design requires a prior development of an adequate dynamic model of the isolation system. A general dynamic model of the parallel manipulator is presented in a state-space framework intended to facilitate the design of an optimal controller. The chosen approach is the method of “KANE’s Dynamics”.

II. SYSTEM DESCRIPTION

As shown in Fig. 1, the kind of parallel platform using flexure hinges at all joints consists of a moving platform, a fixed base, and several limbs with identical kinematic structure. Each limb connects the mobile platform to the fixed base by two flexure hinges, where the prismatic actuator is fixed at the limb. In Fig. 1, the flexure hinge is a slender shaft configuration which can be replaced by any other kind of flexure hinge as shown in Fig. 2. This kind of parallel manipulators can achieve a 3-DOF movement such as 3-UPU, 6-DOF such as 6-SPS or 6-UPS, and even can be a redundant mechanism with more than six limbs. It is very important to select actuators that offer the merits involving smooth motion, high accuracy, and fast response, etc., which make them much suitable for active vibration isolation for precision engineering applications.

III. DYNAMIC MODEL

Let \( \mathbf{d} \) be the vector of the prismatic actuators variables and the vector \( \mathbf{o} o' = [x \ y \ z]^T \) of the reference point \( o' \) be the position.
of the moving platform. In addition, let \( \mathbf{b}_i \) be the vector \( oB_i \) and \( \mathbf{m}_i \) be the vector \( o'M_i \). \( \mathbf{m}_i^2 \) is the vector \( o'M_i \) in the moving platform fixed coordinate system. The mass of moving platform is \( M \). The mass of each limb is \( m_u \).

**A. Coordinate Systems**

As shown in Fig. 3, let a reference frame \( \hat{s}_j \) attach to the fixed platform at the center \( O \) with the \( \hat{s}_1 \) towards the point \( B_1 \) and \( \hat{s}_3 \) vertical to the fixed platform. Locate a coordinate system \( \hat{f}_j \) to the moving platform at the center \( o' \) with \( \hat{f}_1 \) toward the point \( M_1 \) and \( \hat{f}_3 \) vertical to the moving platform. \( \mathcal{R} \) is the transformation matrix from \( \hat{f}_j \) coordinate system to \( \hat{s}_j \). Let each limb be fixed a right-handed coordinate system with origin \( o_i \) (\( i=1, \cdots, n \)) located at the center of mass for the \( i \)th limb, with axis directions determined by an orthonormal set of unit vectors \( \hat{c}_i^j \) (\( j=1, \cdots, 3 \)). The rotation matrix is \( \mathcal{R} \). The rotation matrix is \( \mathcal{R}_c. \) If the DOF of selecting flexure hinge is less than 3, the corresponding angle can be set to 0. Similarly, let the orientation of the \( \hat{s}_j \) coordinate system, relative to the \( \hat{c}_j^i \), be described by consecutive positive rotations \( Q_i^j \) about the \( \hat{c}_1^i, Q_2^j \) about the moved two-axis and \( Q_3^j \) about the moved three-axis when the flexure hinge is 3-DOF joint. The rotation matrix is \( \mathcal{R}_{c}\). If the DOF of selecting flexure hinge is less than 3, the corresponding angle can be set to 0. Similarly, let the orientation of the \( \hat{s}_j \) coordinate system, relative to the \( \hat{c}_j^i \), be described by consecutive positive rotations \( Q_i^j \) about the \( \hat{c}_1^i, Q_2^j \) about the moved two-axis and \( Q_3^j \) about the moved three-axis. The rotation matrix is \( \mathcal{R}_{cesi}. \)

Let \( c_k \) and \( s_k \) represent the cosines and sines of the respective angles \( Q_k \) (\( k=1, \cdots, 6 \)). It is assumed that the small-angle approximations hold for angles \( Q_k \). Then the rotation matrices among the several coordinate systems for the \( i \)th limb are as follows:

\[
\mathcal{R}_{cfi} = \begin{bmatrix}
    c_2^i & c_3^i & c_2^i & s_2^i & c_3^i & s_2^i \\
    -s_2^i & s_2^i & c_3^i & s_2^i & c_2^i & s_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
\end{bmatrix}
\]

\[
\mathcal{R}_{cesi} = \begin{bmatrix}
    c_2^i & c_3^i & c_2^i & s_2^i & c_3^i & s_2^i \\
    -s_2^i & s_2^i & c_3^i & s_2^i & c_2^i & s_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
    -s_3^i & s_3^i & c_2^i & s_3^i & c_2^i & c_3^i \\
\end{bmatrix}
\]

**B. Generalized Speeds for the System**

In order to determine the angles of flexure hinges directly, a coordinate system is fixed on origin \( o_i \) with axis directions determined by an orthonormal set of unit vectors \( \hat{c}^i_0 \). The axis directions are located coincident with the corresponding fixed base coordinate system \( \hat{s}^i_0 \) when the moving platform is in the initial position. Let \( q^i_k \) (\( k=1, \cdots, 6 \)) be the angles at the upper and lower flexure hinges. For the \( i \)th limb, the associated generalized coordinates are the angles at the flexure hinges \( q^i_k \).

Define the system generalized speeds \( \dot{u} \) as the time change rate of the generalized coordinates in the inertial reference frame:

\[
\dot{u}^i = [\dot{q}^i_1 \cdots \dot{q}^i_6] \tag{2}
\]

Two intermediate reference frames are introduced previously to permit describing the angular velocity of \( \mathbf{\hat{c}}^i_0 \) to the \( \mathbf{\hat{c}}^i_{e0} \). Designate the unit vectors of these intermediate reference frames by \( \mathbf{\hat{m}}^i_0 \) and \( \mathbf{\hat{n}}^i_0 \). Similarly, other two intermediate reference frames are introduced previously to permit describing the angular velocity of \( \mathbf{\hat{s}}^i_j \) to the \( \mathbf{\hat{c}}^i_{e0} \). Designate the unit vectors of these intermediate reference frames by \( \mathbf{\hat{v}}^i_j \) and \( \mathbf{\hat{w}}^i_j \). The expressions for the angular velocities of each limb are:

\[
\omega^i = -u^i_1 \hat{c}^i_0 - u^i_2 \hat{v}^i_2 - u^i_3 \hat{w}^i_3 \tag{3}
\]

The expression for the angular velocities of moving platform is:

\[
\omega^i_0 = u^i_1 \hat{c}^i_0 + u^i_2 \hat{m}^i_0 + u^i_3 \hat{c}^i_0 - u^i_2 \hat{v}^i_2 - u^i_3 \hat{w}^i_3 \tag{4}
\]

The position vectors are:

\[
\mathbf{r}_o = d_i \mathbf{\hat{c}}^i_0 + \mathbf{b}_i - \mathbf{m}_i = d_i \mathbf{\hat{c}}^i_0 + \mathbf{b}_i + \mathbf{R} \mathbf{m}_i \tag{5}
\]

Differentiating Eq. (5) with respect to time:

\[
\mathbf{v}_o = d_i \omega \times \mathbf{\hat{c}}^i_0 + \mathbf{d}_i \omega \times \mathbf{\hat{w}}^i_3 - \omega \times \mathbf{R} \mathbf{m}_i
\]
C. Linearized Partial Velocities

The partial velocities and partial angular velocities are formed by inspection of the relevant velocity vectors. These partial velocities are then linearized by neglecting higher order terms.

\[ v_{o_1, u_2} = \frac{d_i}{2} \omega_{o_1} \times \hat{c}_3 + d_3 \hat{c}_3 \]  

(6)

D. Generalized Active Forces

Let \( F^{m_i} \) and \( M^{m_i} \) represent the force and moment exerted by the \( i \)th limb on the moving platform at the \( i \)th flexure hinge. Since \( F^{m_i} \) is a noncontributing force, it can be ignored in the analysis. The total moment \( M^m \) due to the three flexure hinges connecting the moving platform can be calculated as

\[ M^m = \sum_{i=1}^{n} M^{m_i} = \sum_{i=1}^{n} (k_1^i q_1^i \hat{c}_1 + k_2^i q_2^i \hat{m}_2 + k_3^i q_3^i \hat{n}_3) \]  

(8)

where \( k_1^i, k_2^i \) and \( k_3^i \) is pertinent upper flexure hinge stiffness. Let \( F^D \) and \( M^D \) represent the disturbance force and moment acting on the mass center of moving platform.

In terms of the above, the moving platform’s contribution to the set of generalized active forces, for the \( j \)th generalized speed, is:

\[ Q_{m, u_j} = v_{o_1, u_j} \cdot (Mg + F^D) + \omega_{o_1, u_j} \cdot (M^m + M^D) \]  

(9)

Let \( F^{a_i} \) be the force exerted by the \( i \)th actuator on each limb at the mass center. The forces and moments acting on the \( i \)th limb are caused by the moving platform (through the \( i \)th upper flexure hinge), gravity of the upper arm, and the fixed base (through the lower flexure hinge). The contributing loads of the each limb are as follows:

\[ Q_{o_1, u_j} = v_{o_1, u_j} \cdot (mu \hat{g} + F^{a_i}) + \omega_{o_1, u_j} \cdot (M^{a_i} - M^{m_i}) \]  

\[ M^{a_i} = k_3^i q_3^i \hat{c}_1 - k_1^i q_1^i \hat{c}_2 - k_2^i q_2^i \hat{m}_2 \]  

(10)

E. Generalized Inertia Forces

The contribution \( Q_{m, u_j}^* \) to the generalized inertia forces for the \( j \)th generalized speed is:

\[ Q_{m, u_j}^* = v_{o_1, u_j} \cdot (-Ma_{o_1}) + \omega_{o_1, u_j} \cdot (-I_o \omega_{o_1} - \omega_{o_1} \times I_o \omega_{o_1}) \]

The contribution \( Q_{o_1, u_j}^* \) to the generalized inertia forces due to the \( i \)th upper arm for the \( j \)th generalized speed is:

\[ Q_{o_1, u_j}^* = v_{o_1, u_j} \cdot (-m_o a_{o_1}^{(r)}) + \omega_{o_1, u_j} \cdot (-I_{o_i} \omega_{o_1} - \omega_{o_1} \times I_{o_i} \omega_{o_1}) \]

F. Constraint Equations

Since the velocity and angular velocity of the moving platform center of mass \( o \) is irrespective of the actuator path chosen for describing its position, a set of constraint equations can be written in vector using the following:

\[ v_{o_1} = d_i \hat{c}_3 + b_i = d_i \hat{c}_3 + b_j - m_j \quad (i = 1, 2, \ldots, n) \]

\[ \omega_{o_1} = u_1^i \hat{c}_3 + u_2^i \hat{m}_2 + u_3^i \hat{n}_3 - u_1^j \hat{c}_3 - u_2^j \hat{m}_2 - u_3^j \hat{n}_3 \]

(11)

Expand the equations and rearrange to the form:

\[ u_j^i = \sum_{s=1}^{6} A_{rs} u_j^s + D_r (i = 2, \ldots, n; j = 1, \ldots, 6; r = 7, \ldots, 6n) \]  

(12)

G. Dynamical Equations

The holonomic generalized active force for the \( r \)th \( (r = 1, \cdots, 6) \) generalized speed is:

\[ F_r = Q_{m, u_r} + \sum_{i=1}^{6} Q_{o_1, u_r} \]  

(13)

Likewise, the contribution to the set of holonomic generalized inertia force is:

\[ F_r^* = Q_{m, u_r}^* + \sum_{i=1}^{6} Q_{o_1, u_r}^* \]  

(14)

The nonholonomic and holonomic generalized active forces are related to each other as follows:

\[ \Phi_r = F_r + \sum_{s=7}^{6n} A_{rs} F_s (r = 1, \cdots, 6) \]  

(15)

Similarly, the nonholonomic and holonomic generalized inertia forces are related to each other as follows:

\[ \Phi_r^* = F_r^* + \sum_{s=7}^{6n} A_{rs} F_s^* (r = 1, \cdots, 6) \]  

(16)

Kane’s dynamical equations are:

\[ \Phi_r + \Phi_r^* = 0 (r = 1, \cdots, 6) \]  

(17)

The final dynamics equation of this parallel multi-body system can be written as:

\[ M \cdot \dot{u} + K \cdot q = Q \]  

(18)
IV. STATE-SPACE EQUATIONS

According to the Eq. (2) and (18), the state vector consists of \(q\) with the 3 coordinates and the three independent generalized speeds \(u\):

\[
\begin{bmatrix}
1 & 0 \\
0 & M
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{u}
\end{bmatrix}
=
\begin{bmatrix}
0 & N \\
K & C
\end{bmatrix}
\begin{bmatrix}
q \\
u
\end{bmatrix}
+
\begin{bmatrix}
0 \\
E
\end{bmatrix} = \(x\) \tag{19}\]

where \(M, K\) and \(C\) are system mass, stiffness, and damping matrices. \(I\) is identity matrix. Vector \(\omega\) is the disturbance vector and \(E\) is the input matrix which is time-varying matrix functions of the coordinates which accounts for the unknown direct disturbance force \(F^D\) and \(M^D\). \(N\) is a constant matrix.

V. CONTROL STRATEGY

Modern control methods provide the potential for both high-performance and robust stability in the presence of parametric uncertainties. \(H_2\) and \(H_\infty\) methods have gained widespread recognitions and are used in controller synthesis for multi-degree of freedom active vibration isolation problems which are always be multiple-input/multiple-output (MIMO) system. Mixed \(H_2/H_\infty\) controllers provide a means for maximizing robust stability for a given level of mean-square nominal performance while directly optimizing for controller order constraints.

A schematic of the active vibration isolation system is shown in Fig. 4. The generalized plant of a standard control problem is given by:

\[
\begin{align*}
\dot{X} &= A \cdot X + B_1 \cdot \omega + B_2 \cdot u \\
\dot{Z} &= C_1 \cdot X + D_{12} \cdot u \\
\dot{Y} &= C_2 \cdot X + D_{21} \cdot \omega + D_{22} \cdot u
\end{align*}
\tag{20}\]

where \(X\) is the state vector, \(\omega\) is the disturbance vector, \(\dot{u}\) is the control vector, \(Z\) is the performance vector and \(Y\) is the measurement vector. A general compensator for this system is:

\[
\dot{X}_c = A_c \cdot X_c + B_c \cdot Y, \quad \dot{u} = C_c \cdot X_c \tag{21}\]

where \(X_c\) is the state vector of the controller.

Let

\[
\dot{X} = \begin{bmatrix}
X \\
X_c
\end{bmatrix}
\]

The closed loop system dynamics is given by:

\[
\dot{X} = \tilde{A} \cdot X + \tilde{B} \cdot \omega, \quad Z = \tilde{C} \cdot X \tag{22}\]

where

\[
\tilde{A} = \begin{bmatrix}
A & -B_c C_c \\
B_c C_2 & A_c - B_c D_{22} C_c
\end{bmatrix}, \quad \tilde{B} = \begin{bmatrix}
B_1 \\
B_c D_{21}
\end{bmatrix}, \quad \tilde{C} = \begin{bmatrix}
C_1 & -D_{12} C_c
\end{bmatrix} \tag{23}\]

For an \(H_2\) problem, the objective is to minimize the \(H_2\)-norm of the closed loop transfer function from disturbance inputs to performance outputs:

\[
T_{zz} = \tilde{C}(sI - \tilde{A})^{-1}\tilde{B} \tag{24}\]

For the \(H_\infty\) problem, the objective is to minimize the \(H_\infty\)-norm of the transfer function from disturbance inputs to performance outputs. The performance index for the mixed \(H_2/H_\infty\) problem is a weighted combination of the Lagrangian for the \(H_2\) problem and the Lagrangian for the \(H_\infty\) problem \[4],[5].

VI. CONCLUSION

This paper is concerned with the development of a vibration isolation system for highly sensitive equipment applications, which attenuates the vibration transmission above some corner frequencies so as to protect the payload from the jitter induced by the various disturbance sources. In this paper, a kind of parallel robot with flexure hinges will be analyzed to achieve active vibration control for precision instruments or micro-gravity science payloads. A dynamics model of the parallel manipulator is presented in a state-space framework intended to facilitate the design of an optimal controller, which is based on KANE’s method with concise features. The investigations of this paper will be helpful to active vibration isolation devices based on parallel manipulators.

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1833