A Novel Piezoactuated XY Stage with Parallel, Decoupled and Stacked Flexure Structure for Micro/Nano Positioning

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Abstract—This paper presents the design and manufacturing processes of a new piezoactuated XY stage with integrated parallel, decoupled, and stacked kinematics structure for micro/nanopositioning application. The flexure-based XY stage is composed of two decoupled prismatic-prismatic (PP) limbs which are constructed by compound parallelogram flexures and compound bridge-type displacement amplifiers. The two limbs are assembled in a parallel and stacked manner to achieve a compact stage with the merits of parallel kinematics. Analytical models for the mechanical performance assessment of the stage in terms of kinematics, statics, stiffness, load capacity, and dynamics are derived and verified with finite element analysis (FEA). A prototype of the XY stage is then fabricated and its decoupling property is tested. Moreover, Bouc-Wen hysteresis model of the system is identified by resorting to particle swarm optimization (PSO), and a control scheme combining the inverse hysteresis model-based feedforward with feedback control is employed to compensate for the plant nonlinearity and uncertainty. Experimental results reveal that a submicron accuracy single-axis motion tracking and biaxial contouring can be achieved by the proposed mechanism and controller designs as well.

Index Terms—Micro-/nanopositioning, flexure mechanisms, mechanism design, hysteresis modeling, motion control.

I. INTRODUCTION

IN THE field of micro-/nanomanipulation such as biological manipulation [1], [2] and scanning probe microscopy [3], flexure-based compliant stages [4], [5] are popular devices to achieve ultrahigh precision positioning with (sub)micrometer level accuracy. The reason lies in that the compliant mechanisms deliver motions by making use of elastic deformations of the material instead of conventional mechanical joints [6], [7]. Hence, they provide the merits including free of backlash, free of friction, repeatable motion, and vacuum compatibility, etc. In addition, most of the mechanisms employ piezoelectric actuators (PZTs) for actuation since PZT is capable of linear positioning with (sub)nanometer level resolution, large blocking force, high stiffness, and rapid response characteristics. A great number of piezoactuated flexure stages have been developed in research laboratories and even commercialized on the markets (e.g., the piezo stages manufactured by Physik Instrumente GmbH & Co.). This research is focused on the investigation of XY stages due to their promising applications in micro-/nanofields.

In the perspective of kinematics structures, the XY stages can be classified into two categories in terms of serial and parallel ones. In serial stages, two one-degree-of-freedom (1-DOF) linear stages are jointed consecutively as a stacking or nesting architecture in the majority of cases [8]–[10], where the moving platform (output platform) of the stage is directly supported by the last stage solely. The serial stage has the advantages of simple structure and simple control strategy because the two stages can be operated independently. However, it exhibits certain disadvantages in terms of high inertia, low natural frequency, asymmetric dynamics of X and Y axes, and cumulative errors. In contrast, parallel stages are based on parallel-kinematics, whose moving platforms are directly supported by all the connecting limbs simultaneously [11]–[16]. By this way, the performances of low inertia, high load-carrying capacity, identical dynamic behaviors of X and Y axes, and high accuracy can be relatively easily achieved. Therefore, parallel stages are more suitable for micro-/nanopositioning from the above points of view.

In accordance with the requirements of specific applications, a parallel XY stage can be implemented in micro-, meso- or macro-scale [17]. This research is concerned with macro-scale stages with the size of tens to hundreds of millimeters. Within this scale, the existing parallel XY stages [11]–[16] are all designed as monolithic planar structures which are free of assembly so that they can be easily manufactured by such processes as wire electrical discharge machining (EDM). As a consequence, the monolithic stage potentially occupies a large planar area in dimension since it is fabricated from a piece of material. Nevertheless, under such a situation, it may not be suitable for the applications where the micro-/nanomanipulation inside a limited space (e.g., inside a scanning electron microscope) is required.

To overcome such limitations of monolithic XY parallel stage, a concept of parallel stage with stacked structure is proposed in this research to obtain a compact size while maintaining the inherent advantages of parallel kinematics. Moreover, under the circumstances where the stage is underactuated or sensory feedback of the positions of moving platform is not permitted, a decoupled XY stage with proper
Section VIII concludes this research.

In the current research, the hysteresis is modeled using Bouc-Wen model which has fewer parameters and is easier to integrate with the rest of the system model. A hybrid control strategy employing the inverse hysteresis model-based feedforward combined with a proportional-integral-derivative (PID) feedback control is implemented on the XY stage to achieve a submicron accuracy positioning.

In the rest of the paper, the design and assembly procedures of the new XY stage are addressed in Section II. Analytical models for the prediction of kinematics, statics, stiffness, stress, load capacity, and dynamics properties of the stage are established in details in Sections III and IV, respectively. The models are then validated by finite element analysis (FEA) carried out in Section V, where the mechanical properties of the stage are characterized as well. Afterwards, a prototype of stage is fabricated and the open-loop performance is tested in Section VI. Then, Section VII presents the hysteresis model identification and hybrid controller design procedures, where extensive experiments are conducted to demonstrate the positioning capability of the developed XY stage. Finally, Section VIII concludes this research.

II. DESIGN AND ASSEMBLY OF THE XY STAGE

To design a decoupled XY stage with parallel structure, a 2-PP (P stands for prismatic joint) mechanism is adopted due to its simple structure. In what follows, the design procedure of a PP limb with decoupled translations is presented.

It is known that the compound parallelogram flexure as shown in Fig. 1(a) is capable of 1-DOF ideal translation \((d_x)\) if a force \(f_x\) is applied on its output stage. At the same time, the compound bridge-type displacement amplifier [see Fig. 1(b)] can guide a linear output motion in the vertical direction \((y\text{-axis})\) once driven by a linear actuator. In addition to the roles of linear motion guidance and amplification, this type of amplifier has much larger input stiffness and larger lateral stiffness than the conventional bridge-type amplifier [29]. The large input stiffness calls for an actuator with large blocking force and stiffness. Hence, PZT is the most suitable actuator for drives of the amplifier. On the other hand, the large lateral stiffness indicates that the output end of the amplifier can tolerate a large lateral load. This merit provides protection to the inner PZT which can only tolerate small magnitude of lateral load. Therefore, combining the above 1-DOF stage with the amplifier, a PP limb with decoupled translations along the \(x\) - and \(y\) -axes is obtained as shown in Fig. 1(c). This PP limb can be employed as a basic modular to design an XY parallel stage with decoupled structure.

For instance, with different arrangement schemes of two limbs, three XY stages are constructed in Fig. 2. As planar mechanisms, the XY stages are commonly designed as monolithic structures. Furthermore, to generate a double-symmetric architecture, four PP limbs can be adopted. For example, a 4-PP stage corresponding to the 2-PP one as shown in Fig. 2(a) is developed in [16]. As monolithic structures, the XY stages are easy to fabricate. However, they exhibit relatively large dimensions in plane. Hence, they are not suitable for the applications where the micro-/nanomanipulation inside a limited space is desirable. Although the stages in Figs. 2(b) and (c) have a more compact design than the one in Fig. 2(a), they still occupy a large area if long-travel PZT actuators are employed to obtain a large workspace.

To overcome this drawback, an XY stage with two-layer stacked structure is proposed as described in Fig. 3(a). It can be observed that, with an orthogonal assembly of the two PP limbs, the stage still produces decoupled output motion.
In contrast, comparing to the above monolithic XY stages, it has a more compact size. Its limitation lies in that the two limbs and output platform are required to be assembled together to construct such an XY stage. Considering that fixing holes can be easily machined with a tolerance of ±5 µm, precise assembly of the components is not impossible nowadays. Moreover, four PP limbs can be employed to design a double-symmetric XY stage as illustrated in Fig. 3(b) to reduce parasitic motions and thermal gradient effects. Even so, in the current research, the two-limb version as shown in Fig. 3(a) is adopted to demonstrate the conceptual design of a compact XY stage with integrated parallel, decoupled and stacked structure.

The CAD assembly processes of such an XY stage are graphically shown in Fig. 4. The components of the stage can be assembled together through four steps [Figs. 4(a)–4(h)]. In the additional step shown in Figs. 4(i)–4(j), two displacement sensors are fixed at the base for the measurement of the stage output displacements. Besides, the top and side views of the assembled model are depicted in Figs. 4(k) and 4(l), respectively. Concerning practical implementation of the XY stage shown in Fig. 3(b), the output platform and limbs can be fabricated separately by the wire-EDM process. Then, the components can be assembled together via a procedure similar to the one depicted in Fig. 4. It is noticeable that although the right circular shape flexure hinge as described in Fig. 5(a) is adopted in this research for illustrations, other types of hinges can also be employed as well. In the following sections, analytical models for the kinematics, statics, stiffness, and dynamics of XY stage are derived in sequence.

### III. Kinematics and Statics Modeling

In view of the decoupling design procedure as addressed in preceding discussions, it is assumed that the XY stage owns a totally decoupled property for the convenience of analytical modeling. Actually, this assumption is justified by the finite element analysis results obtained in Section V later. Based on the assumption, it is easy to deduce that the properties in the two working axes of the XY stage are identical. Thus, given the input displacements ($q_1$ and $q_2$) of the two PZT actuators, the stage output motion ($d_x$ and $d_y$) and actuator input forces ($F_{in1}$ and $F_{in2}$) can be calculated by the following kinematics and statics equations:

$$
\begin{bmatrix}
  d_x \\
  d_y \\
\end{bmatrix} =
\begin{bmatrix}
  A_s & 0 \\
  0 & A_s \\
\end{bmatrix}
\begin{bmatrix}
  q_1 \\
  q_2 \\
\end{bmatrix}
$$

(1)

$$
\begin{bmatrix}
  F_{in1} \\
  F_{in2} \\
\end{bmatrix} =
\begin{bmatrix}
  K_{in} & 0 \\
  0 & K_{in} \\
\end{bmatrix}
\begin{bmatrix}
  q_1 \\
  q_2 \\
\end{bmatrix}
$$

(2)

where $A_s$ is the amplification ratio of the displacement amplifier and $K_{in}$ is the input or actuation stiffness of the stage.
Through the above relations, the kinematics and statics problems are converted into the calculation of amplification ratio and input stiffness of the XY stage, respectively. In the current research, a pseudo-rigid body (PRB) model of the XY stage is established by considering each flexure hinge as a 1-DOF revolute joint combined with a torsional spring with stiffness $K_r$. The stiffness equation of $K_r$ with the best accuracy as compared in [30] is adopted for calculation. Due to the double symmetry of the compound bridge-type displacement amplifier, one quarter of the amplifier 2 [lower-left part in Fig. 3(a)] is picked out as shown in Fig. 5(b) for the purpose of analysis. Assign $F_x$, $\Delta x$ and $\Delta y$ be the input force, input and output displacements of one quarter of the amplifier 2, respectively. Then, the input force, input and output displacements of the whole amplifier can be expressed as $2F_x$, $2\Delta x$ and $2\Delta y$, respectively.

A. PRB Model

First, only the compliances of the flexure hinges are considered in the PRB model. That is, it is assumed that each flexure hinge has 1-DOF rotational compliance arising from the rotational deformation, and other elements are all considered as rigid bodies.

The free-body diagram of one amplifier leg is shown in Fig. 6(a). Under the equilibrium status, the equation of moments at point $A_1$ can be derived as follows.

$$F_y l_x/2 + 2M_r = F_x l_y/2$$

with the moment

$$M_r = K_r \Delta \alpha$$

where $K_r$ and $\Delta \alpha$ denote the rotational stiffness and deformation of a notch hinge, respectively.

Differentiating both sides of the displacement relation $l_y = l_a \sin \alpha$ (where $l_a$ and $\alpha$ are variable during the operation) with respect to time, allows the generation of:

$$\Delta y = l_a \cos \alpha \Delta \alpha = l_x \Delta \alpha.$$  (5)

Concerning the force $F_y$ in (3), it is an internal force applied on the output end of the amplifier 2, which arises from the deformation of the compound parallelogram in limb 1 [see Fig. 3(a)]. The translational stiffness $K_{\text{limb}}$ of the compound parallelogram is contributed by the rotational stiffness of the eight notch hinges, which can be calculated below based on the potential energy analysis:

$$K_{\text{limb}} = 2K_r/l^2$$  (6)

where $l$ denotes the length of the limb leg [see Fig. 1(a)]. Thus, the force $F_y$ can be expressed as:

$$F_y = K_{\text{limb}} \Delta y.$$  (7)

Substituting (4), (5), and (7) into (3), gives a relation between the variables $F_x$ and $\Delta \alpha$ as follows:

$$K_{\text{limb}} l_x^2 \Delta \alpha/2 + 2K_r \Delta \alpha = F_x l_y/2$$

which allows the expression of $\Delta \alpha$ in terms of $F_x$:

$$\Delta \alpha = \frac{l_y F_x}{4K_r + l_x^2 K_{\text{limb}}} = \frac{l_y F_x}{4K_r + 2K_r l_x^2/l^2}.$$  (9)

Moreover, in view of the virtual work principle, an equation can be obtained for one quarter of the amplifier:

$$F_x \Delta x - F_y \Delta y = 4M_r \Delta \alpha$$

which implies that the work done by external forces is equal to that done by internal forces.

Inserting (4), (5), (7), and (9) into (10) leads to a relation of $F_x$ and $\Delta x$ solely, which further gives

$$\Delta x = \frac{l_y F_x}{4K_r + l_x^2 K_{\text{limb}}} = \frac{l_y F_x}{4K_r + 2K_r l_x^2/l^2}.$$  (11)

Dividing (5) by (11) along with the consideration of (9), yields

$$A_{s1} = \frac{\Delta y}{\Delta x} = \frac{l_y}{l_x}$$

which represents the amplification ratio of the stage derived based on the PRB model. In addition, the input stiffness can be calculated by (11):  

$$K_{\text{in1}} = \frac{F_x}{\Delta x} = \frac{2K_r (2 + l_x^2/l^2)}{l_y^2}.$$  (13)

It is noticeable that $l_x$ and $l_y$ in (12) and (13) [and subsequent (24)–(25) and Table I] denote the initial values of these two parameters because parameters $l_x$ and $l_y$ are changing during the operation of the amplifier. Similarly, the initial value of the variable parameter $\alpha$ is used to express the amplification ratio of the amplifier in the literature [29], [31].

B. PRB Model with Beam Bending

However, the FEA simulation conducted in the following Section V reveals that the above theoretical result (12) overestimates the application ratio with a large deviation, which stems from the assumption that other components are all rigid bodies with infinite stiffness. On the contrary, the compliance of the input rod [$OA_{10}$, see Fig. 6(b)] of the amplifier has a great influence on the amplification ratio as discovered by FEA. Therefore, in addition to the rotational deformations of notch hinges, the bending deformations of the input rods also deserve a particular consideration in order to generate a more accurate model for the prediction of amplification ratio.
In this research, the compliant rod is treated as a Euler-Bernoulli beam. From the beam theory, the bending equation of a flexure element can be written as:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI(x)}$$  \hspace{1cm} (14)

where $y$ denotes the perpendicular distance to the neutral axis $x$, $M(x)$ is the moment exerted on a smaller length of $dx$ at position $x$ about the neutral axis, $I(x)$ is the area moment of inertia of the corresponding cross-section about the neutral axis, and $E$ is the Young’s modulus of the material. It follows that the slope of the beam, i.e., the angular deflection with respect to the neutral axis, can be derived by:

$$\theta = \frac{dy}{dx} = \int \frac{d^2y}{dx^2} dx = \int \frac{M(x)}{EI(x)} dx.$$  \hspace{1cm} (15)

The free-body diagram of the input rod $OA_{10}$ is shown in Fig. 6(b). Conventionally, the forces exerted at points $A_{10}$ and $A_{20}$ should be considered independently to derive the deformations of these two points, respectively. In this research, for the convenience of analysis, the forces exerted at points $A_{10}$ and $A_{20}$ are transferred to their midpoint $A$, and the deflection at point $A$ is assumed to be the bending deformation of the input rod. This assumption will be confirmed by the FEA results obtained in the following Section V.

Considering the force equilibrium at the input point $O$ under static force, the force at point $A$ can be derived by $2F_{Ax} = F_x$. In addition, the equation of moments at the point $O$ can be written as

$$M_O = F_x(l_1 + l_2)/2 + 2M_r.$$  \hspace{1cm} (16)

Referring to the coordinate system as assigned in Fig. 6(b), the moment at the position $x$ can be generated as:

$$M(x) = F_x(x - M_O) = F_x(2x - l_1 - l_2)/2 - l_yF_x/(2 + l_y^2/l_z^2)$$  \hspace{1cm} (17)

which is derived with the consideration of (16), (4), and (9).

Then, according to (15), the angular deformation can be obtained as follows:

$$\theta(x) = \int \frac{M(x)}{EI(x)} dx + C$$  \hspace{1cm} (18)

where $C$ is a constant. In view of the zero angular deflection at the position $O$, one has $\theta(0) = 0$. Then, substituting this boundary condition into (18) allows the calculation of the constant $C = 0$ and

$$\theta(x) = \frac{6F_x[x^2 - (l_1 + l_2)x - 2l_yx/(2 + l_y^2/l_z^2)]}{Ewh_1^3}.$$  \hspace{1cm} (19)

Afterwards, the equation of linear deformation can be expressed as

$$y(x) = \int \theta(x) dx + D$$  \hspace{1cm} (20)

where $D$ is also a constant. Inserting (19) into (20) and considering the boundary condition $y(0) = 0$, gives $D = 0$ and

$$y(x) = \frac{F_x[2x^3 - 3(l_1 + l_2)x^2 - 6l_yx^2/(2 + l_y^2/l_z^2)]}{Ewh_1^3}.$$  \hspace{1cm} (21)

Hence, by substituting the coordinate $(l_1 + l_2)/2$ of point $A$ into (21), the linear deformation of the input rod $OA_{10}$ at position $A$ can be obtained:

$$\Delta x_2 = \frac{-F_x(l_1 + l_2)^2[l_1 + l_2 + 3l_y/(2 + l_y^2/l_z^2)]}{2Ewh_1^3}$$  \hspace{1cm} (22)

which is a function of the single variable $F_x$ and represents one part of the input displacement in addition to the PRB model result (11). Thus, the overall input displacement can be calculated as the summation of (11) and (22), i.e.,

$$\Delta x_{all} = \Delta x + |\Delta x_2|$$  \hspace{1cm} (23)

which further gives an expression for the input stiffness of the stage:

$$K_{in2} = \frac{F_x}{\Delta x_{all}} = \left\{ \frac{l_y^2}{2K_r(2 + l_y^2/l_z^2)} \right\}$$
In addition, combining (5) and (23) allows the derivation of the amplification ratio:

$$A_{x2} = \frac{\Delta y}{\Delta x_{\text{in}}} = \frac{\frac{l_x l_y}{2K_r(2 + l_x^2/2)}}{\frac{l_y^2}{2K_r(2 + l_y^2/2)}} + \frac{(l_1 + l_2)^2(l_1 l_2 + 3l_y(2 + l_y^2/2))}{2Ewh_1^3} \right)^{-1} \tag{24}$$

For an XY stage with parameters described in Table I, by varying the parameters $l_x$ and $l_y$ while keeping other parameters constant, the variation tendency of the stage amplification ratio obtained by (25) is illustrated in Fig. 7. It is observed that the amplification ratio increases as the increasing of $l_x$ and decreasing of $l_y$. That is, the stage amplification ratio $A_s$ is a direct proportion function of the parameter ratio $l_x/l_y$. This is coincident with the variation tendency predicted by the PRB model (12).

IV. STIFFNESS, STRESS, AND DYNAMICS ANALYSIS

A. Stiffness and Compliance Analysis

The in-plane stiffness ($K_x$ and $K_y$) of the XY stage relates the in-plane external forces ($F_x$ and $F_y$) applied on the output platform to the induced deflections ($t_x$ and $t_y$). On the contrary, as the inverse of stiffness, the compliances ($C_x = 1/K_x$ and $C_y = 1/K_y$) reflect the linear deflections under external loads as follows:

$$\begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} C_x & 0 \\ 0 & C_y \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}. \tag{26}$$

These compliance factors characterize the output compliance (or output stiffness) of the XY stage in case of suffering from external loads.

Due to the symmetry of the $x$- and $y$-axes of the stage structure, it can be deduced that $C_x = C_y$ in theory. In what follows, the output compliance $C_x$ of the stage is derived with an $x$-axis external force $F_x$ applied on the output platform.

Referring to Fig. 3(a), one observes that the load $F_x$ causes a linear deformation $t_x$ of the stage output platform, which is then transferred to both limb 1 and limb 2 as a consequence. In limb 1 (bottom layer), this deflection is transmitted to the displacement amplifier, which induces the rotation of the 16 flexure hinges with the identical angle ($\theta_1$):

$$\theta_1 = t_x/2l_o. \tag{27}$$

In addition, thanks to the large lateral stiffness of the mechanical amplifier, the deformation of limb 2 (top layer) is reflected as the linear motion of the limb compound parallelogram flexure. It causes the eight flexure hinges to rotate with the same angle $\theta_2$, i.e.,

$$\theta_2 = t_x/2l. \tag{28}$$

The potential energies of the XY stage arise from the elastic deformations of the material and are stored in the two limbs, which can be expressed in two different ways:

$$\frac{1}{2}K_x t_x^2 = 16 \times \frac{1}{2}K_1 \theta_1^2 + 8 \times \frac{1}{2}K_2 \theta_2^2 \tag{29}$$

where $K_x$ is the output stiffness of the stage in $x$-axis direction, and $K_r$ represents the rotational stiffness of each flexure hinge around the working axis.

Inserting (27) and (28) into (29) gives an expression for the output stiffness:

$$K_x = \frac{4K_r}{t_x^2} + \frac{2K_r}{t^2} \tag{30}$$

which implies that the output compliance is

$$C_x = \frac{(t_0^2 + 2l^2)}{4K_r}. \tag{31}$$

B. Stress and Load Capacity Analysis

The in-plane load capacity $F_x^{\max}$ (or $F_y^{\max}$) means the maximum external force that can be applied at the XY stage in the $x$-$y$ plane without causing the failure of the material.

For the in-plane operation, only the bending stress is taken into account to derive the load limit, because the axial tensile or compressive stress of the flexure hinge is far less than the maximum bending stress. For a flexure hinge bearing a bending moment around its rotation axis, the maximum angular displacement $\theta_{\max}$ occurs when the maximum stress $\sigma_{\max}$, which occurs at the outermost surface of the thinnest portion of the hinge, reaches to the yield stress $\sigma_y$.

The relationship between the maximum bending stress and maximum rotational deformation of the flexure hinge has been derived in [32]:

$$\sigma_{\max} \frac{E(1+\eta^{9/20})}{\eta^{2}f(\eta)} \theta_{\max} \tag{32}$$

where $\eta = t/2r$ is a dimensionless geometry factor, and $f(\eta)$ is a dimensionless compliance factor defined as:

$$f(\eta) = \frac{1}{2\eta + \eta^2} \left[ \frac{3 + 4\eta + 2\eta^2}{(1+\eta)(2\eta + \eta^2)} \right] + \frac{6(1+\eta)}{(2\eta + \eta^2)^{3/2}} \tan^{-1}\left(\frac{2 + \eta}{\eta}\right)^{1/2} \tag{33}$$

Assume the maximum external force $F_x^{\max}$ is applied on the output platform of the XY stage, which results in a linear deflection $t_0^{\max}$ of the platform along the $x$-axis direction due to the maximum rotational deformation $\theta_{\max}$ of the hinges. According to the geometry of the stage, the maximum angular deflection may occur on the hinge belong to either amplifier 1 or compound parallelogram flexure 2.

Concerning the amplifier, the 16 flexure hinges rotate with the same angle:

$$\theta_1^{\max} = t_0^{\max}/2l_o \tag{34}$$

<table>
<thead>
<tr>
<th>TABLE I ARCHITECTURAL PARAMETERS OF AN XY STAGE</th>
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<tr>
<td>Parameter</td>
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where $t_{x1}^{max}$ is the corresponding maximum linear deformation of the stage. Under such case, the maximum stress calculated by (32) should satisfy:

$$\sigma_1^{max} = \frac{E(1+\eta)^{9/20}}{2\mu_1^2 f(\eta)} t_{x1}^{max} \leq \sigma_y$$

(35)

which allows the derivation of

$$t_{x1}^{max} \leq \frac{2\mu_1^2 f(\eta)\sigma_y}{E(1+\eta)^{9/20}}.$$  

(36)

Concerning the compound parallelogram flexure, the eight hinges suffer from the identical angular deformation:

$$\theta_2^{max} = \frac{t_{x2}^{max}}{2l}$$

(37)

where $t_{x2}^{max}$ is the corresponding maximum linear deformation of the XY stage. Similarly, a relation for this maximum deformation can be obtained as:

$$t_{x2}^{max} \leq \frac{2\mu_2^2 f(\eta)\sigma_y}{E(1+\eta)^{9/20}}.$$  

(38)

In order to make the maximum stress in both the amplifier and compound parallelogram flexure stay within the yield stress, the maximum linear deformation of the stage is obtained as:

$$t_{x}^{max} = \min\left\{t_{x1}^{max}, t_{x2}^{max}\right\} = \min\left\{\frac{2\mu_1^2 f(\eta)\sigma_y}{E(1+\eta)^{9/20}}, \frac{2\mu_2^2 f(\eta)\sigma_y}{E(1+\eta)^{9/20}}\right\}.$$  

(39)

Once the maximum deflection is generated above, the maximum in-plane load that can be supported by the XY stage is given by:

$$F_x^{max} = K_{x}t_{x}^{max}$$

(40)

where the in-plane stiffness $K_x (= K_y)$ is expressed in (30).

C. Dynamics and Resonance Frequency Calculation

For the XY stage, considering the kinematics relation in (1), the coordinate vector of $q = [q_1 \ q_2]^T$ is adopted to describe its in-plane motion. Then, the kinetic and potential energies of the stage stored in the two PP limbs can be written in terms of the selected generalized coordinates.

Substituting the kinetic and potential energies into Lagrange’s equation:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q_i}}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = F_i$$

(41)

with $i = 1$ and 2, allows the generation of the dynamic equation describing a free motion of the stage:

$$M\ddot{q} + Kq = 0$$

(42)

where the $2\times2$ equivalent mass and stiffness matrices take on the diagonal forms $M = \text{diag}\{M\}$ and $K = \text{diag}\{K\}$, respectively, along with

$$M = m_1(1 + A_2^2)/2 + m_2(1 + 16A_2^2)/2$$

$$+ (m_3 + m_4 + 16m_5/3 + 5m_6/4 + 2m_7)A^2$$

(43)

$$K = K_{in}$$

(44)

where the masses $m_1$ to $m_7$ are denoted in Fig. 1(c).

In this section, the derived models for the evaluation of amplification ratio, input stiffness, output stiffness/compliance, load capacity, and resonance frequency of XY stage are verified by static and modal analyses carried out with FEA software package ANSYS™.

The architectural parameters of the stage [see Figs. 1(a), 5 and 6] are tabulated in Table I, and the physical and mechanical parameters of the alloy Al-7075 are: Young’s modulus = 71.7 GPa, yield strength = 503 MPa, Poisson’s ratio = 0.33, Density = 2810 kg/m³. A 3-D finite element model is established with the 20-node element SOLID186. In addition, the mesh model is created with a medium mesh size and refined at the flexure hinges to obtain smaller mesh size therein so as to obtain more accurate simulation results. Besides, zero displacements are assigned on the surfaces of the mounting holes to immobilize the mechanism.

V. Model Validation with FEA

In this section, the derived models for the evaluation of amplification ratio, input stiffness, output stiffness/compliance, load capacity, and resonance frequency of XY stage are verified by static and modal analyses carried out with FEA software package ANSYS™.

Therefore, the in-plane resonance frequency of the stage can be obtained as

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$

(45)

which is in unit of Hertz.

A. Amplification Ratio and Input Stiffness

First, a static structural FEA is conducted by applying forces on the input ends of the amplifier 2, which produces a translational motion along the $y$-axis direction as illustrated in Fig. 8. By extracting the input and output displacements,
the amplification ratio and input stiffness of the stage can be derived as 6.45 and 6.99 N/μm, respectively. By comparing the maximum parasitic translation along the $x$-axis with the primary output motion in $y$-axis, it can be observed that the parasitic translation is less than 0.5% and the maximum in-plane rotation is less than 0.023 mrad, which indicates a well-decoupled property of the XY stage.

Theoretically, the pseudo-rigid body model (PRBM 1) results (12) and (13) predict that the above ratio and stiffness are 8 and 7.24 N/μm. In contrast, the two performances assessed by the PRBM 2 results (25) and (24) are 7.29 and 6.59 N/μm, respectively. For comparison, the performances are tabulated in Table II, where the model errors are calculated with respect to FEA results, i.e., (PRBM_1-FEA)/FEA×100%. It is observed that although both of the two models can predict the input stiffness with a deviation less than 6%, the PRBM 1 overestimates the amplification ratio with a large deviation of 24%, which is almost twice the PRBM 2 result (13%). This explains the reason why the beam bending deformation of the amplifier input rod is considered in PRBM 2 to derive a more accurate kinematics model in Section III-B. It is noticeable that due to the meshing and round-off errors of finite element simulation, FEA has certain limitations concerning the evaluation of the actual error. The errors described in Table III (as well as Tables IV and V) are calculated only with respect to the FEA results.

In addition, all the approaches show that the input stiffness is less than the minimum stiffness of PZT actuator (14–208 N/μm±20%), which means that the PZT can be operated normally to actuate the XY stage.

### Table III

<table>
<thead>
<tr>
<th>Method</th>
<th>$K_x$ (N/μm)</th>
<th>$t_{\text{max}}$ (mm)</th>
<th>$F_{\text{max}}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRBM</td>
<td>0.111</td>
<td>1.11</td>
<td>123.95</td>
</tr>
<tr>
<td>FEA</td>
<td>0.119</td>
<td>1.35</td>
<td>156.88</td>
</tr>
<tr>
<td>% error</td>
<td>−6.7</td>
<td>−17.8</td>
<td>−20.09</td>
</tr>
</tbody>
</table>

### Table IV

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRBM (Hz)</td>
<td>99.63</td>
<td>99.63</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>FEA (Hz)</td>
<td>94.20</td>
<td>95.49</td>
<td>118.71</td>
<td>124.67</td>
<td>173.80</td>
<td>196.98</td>
</tr>
<tr>
<td>% error</td>
<td>5.8</td>
<td>4.3</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

The table shows the resonance frequencies (Hz) of the XY stage.

---

**B. Output Stiffness and Compliance**

Second, the in-plane stiffness and compliance properties are simulated through static FEA by applying a force $F_x$ on the output platform. The corresponding output platform motion is extracted from FEA results for the calculation of the stage properties. The stage performance evaluated by the analytical model (30) and FEA is shown in Table III. It is observed that the model error with respect to the FEA result is within 7%, which confirms the accuracy of the established stiffness model.

Besides, the out-of-plane stiffness ($K_z$) is also simulated by applying a force ($F_z$) at the center of the top platform and recording the caused displacement ($t_z$) therein. It is calculated that $K_z = 0.794$ N/μm. Comparing the in-plane and out-of-plane values, it can be seen that the out-of-plane stiffness is nearly seven times stiffer than the in-plane one, which means that the XY stage can be operated with less out-of-plane deflections.

### C. Travel Limit and Load Capacity

Third, the travel limit and load capacity under external payloads are tested via static FEA. By trial and error, a force $F_{\text{max}} = 156.88$ N is applied on the stage output platform, which drives the maximum von-Mises stress in the stage to the yield stress of the material. Under such condition, the maximum travel of the stage can be derived as $t_{\text{max}} = 1.35$ mm. Moreover, these properties evaluated by the analytical model results (39) and (40) are also described in Table III, which show that the PRBM model underestimates the travel and load limits of the stage with deviations of 17.8% and 21.0%, respectively. In view of model (40), it is observed that the prediction error for the load capacity is dependent on both stiffness model and travel limit errors. This is one of the reasons why the model for the load capacity predication has a larger error (21.0%) with respect to FEA result. The above travel limit and load capacity values are obtained without considering the actuator’s force, which is actually variable in practice. If the force produced by the PZT is concerned, the performances under different actuation forces should be evaluated in order to perform a more detailed characterization of the stage. Such a comprehensive study calls for a further detailed research.

In addition, the out-of-plane travel and load capacities of the XY stage are also assessed with FEA by applying a vertical force on the top platform. The simulation reveals that the out-of-plane deflections of the platform are mainly attributed to the torsional and bending deformations ($\theta_x$ and $\theta_y$, see Fig. 5(a)) of the amplifier hinges. It is derived that the stage can undergo the maximum out-of-plane load of 566.19 N with the maximum travel of 0.695 mm. This implies that the top platform of the stage can support the maximum mass of 57.77 kg with a small vertical deflection. This is the payload limit of the XY stage in Z direction within elastic deformation range of the material. More payload will cause inelastic deformation of the stage and failure of the material, and instability of XY motion in consequence.

### D. Resonance Frequency

Furthermore, the resonance frequency of the stage is simulated by the modal analysis with FEA. The first six natural frequencies are described in Table IV. The resonance frequency calculated by the dynamic model (45) is also shown in the table. It is observed that the analytical model only predicts the first two natural frequencies since two generalized coordinates ($q_1$ and $q_2$) are selected. Comparing to the FEA, the dynamic model overestimates the resonance frequency with a deviation less than 6%, which validates the effectiveness of the derived model. Besides, the FEA results show that the first two natural frequencies (94.20 Hz and 95.49 Hz) are almost identical.
and the mode shapes are translations along the \(x\)- and \(y\)-axes, respectively. This indicates that the XY stage has similar dynamic behaviors in the two axes. Additionally, the FEA simulation shows that the out-of-plane vibration occurs on the fifth mode shape at higher frequency of 173.80 Hz.

It is observed from (45) that the resonance frequency of the stage structure can be magnified by increasing the input stiffness or reducing the equivalent mass of the stage. For instance, the material with a thinner thickness can be used for fabrication and unnecessary mass of the moving parts can be removed to achieve a resonance frequency higher than 100 Hz. In the following discussions, a prototype of the XY stage is developed and experimentally tested for precision positioning.

VI. Prototype Development and Preliminary Experiments

In this section, a prototype of the XY stage is fabricated and preliminary open-loop testing is conducted to demonstrate the performance of the stage.

A. Experimental Setup

An overview of the experimental testing apparatus is graphically depicted in Fig. 9(a), where the close views of the XY stage with sensor and actuator and its top mounting platform are further detailed in Figs. 9(b) and 9(c), respectively.

The two PP limbs are fabricated by the wire-EDM process using two pieces of Al-7075 alloy. Two 20\(\mu\)m-stroke PZTs (PAS020 from Thorlabs, Inc.) are inserted into the two limbs for actuation. The two limbs are then assembled together with the output platform via bolts, and fixed at the base in accordance with the assembly procedures shown in Fig. 4. The PZTs are driven by a two-axis piezo amplifier and driver (BPC002 from the Thorlabs) with a voltage ranging from 0 to 75 V. In addition, each actuator is inserted into the mechanical amplifier and preloaded through the screw mounted at the tip of the actuator. This produces interference fits between the PZT and amplifier. Thus, no clearances exist during the operation thanks to elastic deformations of the material. The displacements of the stage output platform are measured by two laser displacement sensors (Microtrak II, head model: LTC-025-02 from MTI Instruments, Inc.) with a measuring range of 2.5 mm. The analogy output (bounded within \(\pm 5\) V) of the position sensors are read simultaneously by a host computer through a PCI-based data acquisition (DAQ) board (PCI-6143 with 16-bit A/D converters, from National Instruments Corp.). It is calculated that the resolution of the position detection system is 0.038 \(\mu\)m.

B. Open-Loop Performance Testing Results

First, the open-loop static properties of the XY stage are experimentally tested. With a low-frequency (0.05 Hz) triangular-wave voltage ranging from 0 to 75 V applied to the two PZTs, the two axial translations of the XY stage are recorded, respectively. As expected, with the open-loop voltage-driven strategy, the PZT exhibits nonlinear effects mainly attributed to the hysteresis. Thus, the relationships between the stage output displacements and input voltages of PZTs are also nonlinear as illustrated by the curves in Fig. 10. Actually, the hysteresis of the whole system involves both piezoelectric hysteresis introduced by PZTs and elastic hysteresis induced by the flexure mechanism, where the latter deflection is much smaller than the former one.

The least-squares linear fits of the \(x\)- and \(y\)-axes hysteresis-loop data are adopted to obtain the amplification ratio of the stage in the two axes. Assume that the maximum stroke 20 \(\mu\)m
corresponds to the voltage input 75 V of the PZT. In view of the slope of the linear fits, the amplification ratios for x- and y-axes can be derived as 6.6 and 6.3, respectively. It implies that the stage workspace range is around 132 µm x 126 µm.

Besides, from Fig. 10(a) [10(b)], it is also observed that when the stage moves along the x-axis (y-axis), a parasitic translation exists in the y-axis (x-axis). The maximum parasitic motions in the y- and x-axes are 5.96 µm and 3.98 µm, which indicate that the parasitic translations account for 4.5% and 3.2% of the primary motions, respectively. The crosstalk between the two axes mainly comes from manufacturing errors of the stage and assembly errors of the displacement sensors. The experimental results demonstrate the low-level parasitic motions of the XY stage.

Moreover, compared to the amplification ratio evaluated by FEA in Section V-A, it is obvious that the x-axis experimental result is larger than the simulated one. The reason mainly comes from the preloading effect of the PZT mounting. Because the two PZTs are inserted into the two mechanical amplifiers and preloaded using the screws, the preloading influences both the input displacement of PZT and architecture parameters of the flexure mechanism. As a result, the input displacement is decreased, the initial values of the parameters $l_x$ and $l_y$ [see Fig. 5(b)] are increased and deceased, respectively. Hence, the ratio of $l_x/l_y$ is greater than the nominal value. With reference to variation tendency of the amplification ratio as shown in Fig. 7, an amplification ratio larger than the expected value is achieved.

Furthermore, in order to realize a micro-scale positioning, the piezoelectric hysteresis effects are compensated by a controller constructed in the subsequent discussions.

VII. CONTROLLER DESIGN AND EXPERIMENTAL TESTING

In this section, the hysteresis effects induced by two PZTs are compensated so as to obtain a submicron positioning accuracy for the XY stage. Specifically, the hysteresis is modeled using the Bouc-Wen model, and an inverse hysteresis model-based feedforward plus feedback control scheme is employed for the stage. Moreover, because the stage is well decoupled as verified above, the two axial motions can be treated independently, just like a serial stage. Thus, two single-input-single-output (SISO) controllers are implemented for the x- and y-axes of the micropositioning stage, respectively. For brevity, only the controller design procedure for the x-axis motion is presented below.

A. Hysteresis Modeling and Identification

The dynamic model of the entire micropositioning system with hysteresis can be established as follows [9], [33]:

$$ m\ddot{x} + b\dot{x} + kx = k(du - h) $$  \hspace{1cm} (46)

$$ \dot{h} = ad\dot{u} - \beta|\dot{u}|h|^{n-1} - \gamma|\dot{u}|h^n $$  \hspace{1cm} (47)

where the parameters $m$, $b$, $k$, and $x$ represent the mass, damping coefficient, stiffness, and x-axis displacement of the XY stage, respectively; $d$ is the piezoelectric coefficient, $u$ denotes the input voltage, and $h$ indicates the hysteretic loop in terms of displacement whose magnitude and shape are determined by parameters $\alpha$, $\beta$, $\gamma$, and the order $n$, where $n$ governs the smoothness of the transition from elastic to plastic response. For the elastic structure and material, $n = 1$ is assigned in (47).

In order to identify the Bouc-Wen hysteresis model, the seven parameters $(m, b, k, d, \alpha, \beta, \gamma)$ of the system are identified simultaneously. In particular, particle swarm optimization (PSO) is adopted in the current problem due to its superiority of optimization performance over other methods such as the direct search approach and genetic algorithm (GA) [34], [35]. In addition, the fitness function is chosen as follows:

$$ F(m, b, k, d, \alpha, \beta, \gamma) = \frac{1}{c} \sum_{i=1}^{c} (x_i - x_i^{BW})^2 $$  \hspace{1cm} (48)

where $c$ is the total number of samples and $x_i - x_i^{BW}$ represents the error of the $i$th sample which is calculated as the difference between the experimental result ($x_i$) and Bouc-Wen model output ($x_i^{BW}$).

---

TABLE V

<table>
<thead>
<tr>
<th>Parameter</th>
<th>x-axis value</th>
<th>y-axis value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>$m$</td>
<td>0.1072</td>
<td>0.1072</td>
<td>kg</td>
</tr>
<tr>
<td>$b$</td>
<td>$1.5548 \times 10^3$</td>
<td>$1.6539 \times 10^3$</td>
<td>N/s/m</td>
</tr>
<tr>
<td>$k$</td>
<td>$1.9595 \times 10^5$</td>
<td>$1.9579 \times 10^5$</td>
<td>N/m</td>
</tr>
<tr>
<td>$d$</td>
<td>$2.0296 \times 10^{-6}$</td>
<td>$1.9283 \times 10^{-6}$</td>
<td>m/V</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4448</td>
<td>0.3722</td>
<td>–</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.0294</td>
<td>0.0381</td>
<td>–</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0381</td>
<td>0.0082</td>
<td>–</td>
</tr>
</tbody>
</table>
and experimental result are illustrated in Figs. 11(b) and 11(c), forward item can be derived in view of (46), i.e., strategy is employed in this research.

a feedforward (FF) combined with feedback (FB) control the control scheme is shown in Fig. 12, which indicates that the trajectory of the XY stage is given. The block diagram of the input voltages applied to PZTs once the desired position is experimentally tested. With a 10-μm consecutive step input. To compensate for the model errors and other uncertainties, a feedforward plus feedback controller is designed below.

B. Motion Controller Design

The purpose of motion controller design is to determine the input voltages applied to PZTs once the desired position trajectory of the XY stage is given. The block diagram of the control scheme is shown in Fig. 12, which indicates that a feedforward (FF) combined with feedback (FB) control strategy is employed in this research.

Based on the inverse Bouc-Wen hysteresis model, the feedforward item can be derived in view of (46), i.e.,

\[ u_{FF}(t) = \frac{1}{kd} \left[ m\ddot{x}_d(t) + b\dot{x}_d(t) + kx_d(t) \right] + \frac{h(t)}{k} \]  

(49)

where \( t \) represents the time variable, \( x_d \) is the desired position trajectory, and the hysteresis term \( h \) can be calculated from (47) by numerical approaches (e.g., the fourth-order Runge-Kutta method) using Matlab™/Simulink™ software. More details about its implementation can be referred to [33].

Considering the model errors as revealed in the previous discussions, the hysteresis effects can not be totally compensated by the feedforward control (49). Therefore, an additional feedback control is adopted to compensate for the model imperfection and other disturbances of the system. Specifically, a PID feedback controller is used due to its robustness and easy-to-implement properties. The feedback control input can be written as:

\[ u_{FB}(t) = K_p \left[ e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{de(t)}{dt} \right] \]  

(50)

where the tracking error is defined as \( e(t) = x_d(t) - x(t) \) with \( x \) denoting the measured position. Besides, the three control parameters \( K_p, T_i, \) and \( T_d \) are proportional gain, integral time, and derivative time, respectively.

By adopting an incremental PID algorithm, the overall control signal can be derived in a discretized form:

\[ u(k) = u_{FF}(k) + u_{FB}(k) \]

\[ = u_{FF}(k) + u(k-1) + K_p[e(k) - e(k-1)] \]

\[ + \frac{K_p T_i}{T_i} e(k) + \frac{K_p T_d}{T_d} [e(k) - 2e(k-1) + e(k-2)] \]  

(51)

where \( k \) is the index of time series, \( T \) represents the sampling time interval (\( T = 0.002 \) s), \( u(k-1) \) is the control command in the previous time step, and the feedforward term \( u_{FF}(k) \) is generated from a lookup table which is established off-line based on the inverse Bouc-Wen hysteresis model.

Besides, in view of the limits of input voltage to PZT, a saturation function is added to restrict the controller output to the range of 0 to 75 V, as shown in Fig. 12. In what follows, the performance of the XY stage with the constructed controller is experimentally tested.

C. Closed-Loop Experimental Testing

1) Single-Axis Tracking: First, the single-axis tracking performance is tested. With a 10-μm consecutive step signal applied to PZT 1, the \( x \)-axis motion responses are generated and plotted in Fig. 13. It is seen that a quick response with an
overshoot less than 5% is produced. In addition, the steady-state error is maintained within the range of ±0.11 µm as shown in the magnified view in Fig. 13. From the steady-state error of the step responses [36], the positioning resolution of the XY stage can be obtained as 0.22 µm.

Besides, the y-axis motion tracking performance is also examined. With an input rate of 0.2 Hz, the sinusoidal motion tracking results of a 40-µm peak-to-peak amplitude sine-wave signal are described in Fig. 14. It is observed that the stage can track the commanded motion accurately with the maximum tracking error less than 0.41 µm, which also implies that the plant hysteresis has been compensated to a negligible level.

2) Biaxial Contouring: Second, to discover the two-axis cooperative tracking performance of the XY stage for 2-D motion, biaxial contour tracking tests are executed. Different from the trajectory tracking error, i.e., the difference between the desired and the actual positions, contouring error is defined as the minimum distance between the actual position and desired trajectory along an orthogonal direction to the trajectory.

First of all, by actuating the two axes simultaneously, a linear contour along a 45-degree line is conducted to verify the contouring capability of the XY micropositioning stage. The stage is commanded to move from the home position (0, 0) to (100 µm, 100 µm) and then return to the home. For a feed rate of 20 µm/s, the contouring results and two axes tracking and contouring errors are shown in Figs. 15(a) and 15(b), respectively. It is observed that the tracking errors of the two axes (with the maximum values of 3.29 µm and 2.47 µm occurring at the turning point) cancel each other and produce a much lower magnitude of contouring errors (with the maximum value of 0.82 µm). This indicates that the XY stage has similar tracking capabilities in the two working axes. Precluding the turning point of the reciprocating linear motion, the contouring error has a small magnitude of 0.36 µm.

Moreover, a circular contouring test is performed with different input rates for a circle of 20-µm radius centered at the workspace. For instance, with the input frequencies of 0.1 Hz, 0.5 Hz, and 2.0 Hz, the contouring results and errors are plotted in Figs. 16(a) and 16(b), respectively. It is seen that as the rising of the input rate, less time is required to complete the contouring task. However, it is at the cost of a larger contour error since the maximum error increases from 0.51 µm, 0.78 µm, to 3.58 µm. The large error occurring at the higher input rate 2.0 Hz results in a shrunk shape of the circle as shown in Fig. 16(a), which is mainly caused by the bandwidth limit of the implemented digital controller. In addition, because the x- and y-axes motions have worse tracking at the peak and valley points of the sine waves during higher frequency contouring, the contour errors are not uniformly distributed over one period.

D. Discussions on Stage Performance

For a clear presentation, the performance of the current system is summarized in Table VI. The resolution of the stage is mainly dominated by the quantization of data acquisition devices and resolution of displacement sensors. From Table VI, one can observe that the resolution is much smaller than the tracking and contouring error of the stage. The reason lies
in that the resolution is obtained from the step response of the stage, while the control errors described in Table VI are generated for more complicated sine-wave (0.2 Hz) tracking and linear and circular contouring tasks instead. The sinusoidal tracking errors is heavily dependent on the frequency of input signal, and it increases as rising of the input rate. Besides, the contour errors rely on feed rates greatly. As the increasing of the feed rates, both linear and circular contour errors increase as well. Therefore, the resolution is much smaller than the tracking and contouring errors. The same conclusion has been demonstrated in previous works, e.g., [37].

In addition, the assembled XY stage has a dimension of $116 \times 116 \times 46 \text{ mm}^3$. It may not be compact enough to be inserted into a scanning electron microscope. Even so, it is much more compact than a monolithic one fabricated from only a piece of material in [16], which achieves almost the same size of workspace with a much larger size of profile dimension of $250 \times 250 \times 15 \text{ mm}^3$. Moreover, if PZTs with smaller size and leaf-spring flexures instead of right-circular flexure hinges are adopted, the proposed stage can be fabricated with much more compact dimension.

Comparing with a commercially available XY piezo stage produced by Physik Instrumente, e.g., model P-541.2CD$^1$, which has a dimension of $150 \times 150 \times 16.5 \text{ mm}^3$, a closed-loop workspace $100 \times 100 \text{ mm}^2$ with a closed-loop resolution of 0.2 nm, the developed stage has a more compact planar dimension with a larger workspace size, while owns a much worse resolution. The limitation of the fabricated stage mainly arises from the displacement sensors which have a not-high resolution of 38 nm. The sensors are the bottle-neck in ultra-precision positioning stages. Using high-performance sensors with subnanometer-level resolution, much better resolution can be achieved with the proposed stage.

It is remarkable here that the above 1-D tracking and 2-D contouring experiments for different amplitudes of input signals are conducted with the same set of PID control parameters for both x- and y-axes ($K_p = 5.4 \times 10^{-2}, T_i = 2.7 \times 10^{-5} \text{ s}$, and $T_d = 1.5 \times 10^{-5} \text{ s}$, respectively). Additional tracking experiments reveal that, for a desired motion with a specific input frequency and larger amplitude, better control performance can be obtained by tuning the PID parameters to bigger ones. From this point of view, ideal PID parameters should be designed in accordance with the amplitude of reference input. This is the limitation of PID controller with fixed parameters. Thus, more sophisticated control strategies will be explored in the further research.

It is also noticeable that two analogy low-pass filters with cut-off frequency of 40 Hz are used in the above experiments to eliminate high-frequency noise of the sensor readings. In view of the achieved performance of the positioning system with the hardware available, the stage may be more suitable for micromanipulation of microscopic objects such as biological cells with the size of tens of micrometers. For such a specific application, the proposed stage is more advantageous than a monolithic one owing to the more compact dimension, and it can be applied more suitable in a limited space especially multiple stages are required to perform a cooperative manipulation. Experiments will be conducted to demonstrate the performance of the stage for such a micromanipulation task in the future as well.

### VIII. Conclusion

This research work reveal that a piezo-driven XY micropositioning stage can be designed with integrated parallel, decoupled, and stacked kinematics structure to achieve the performances in terms of high load-carrying capacity, identical dynamic behaviors of X and Y axes, decoupled output motion, high accuracy, and compact size for micro-/nanopositioning. The kinematics and statics problems of the stage are solved based on the PRB model with the consideration of key elements’ bending deformations. The output stiffness and compliance, maximum stress and load capacity, and resonance frequency of the stage are also calculated analytically. The conducted FEA simulations not only confirm the accuracy of the derived models but also demonstrate the nice mechanical performances of the XY stage. Moreover, the experimental studies show that a submicron accuracy single-axis motion tracking and biaxial linear and circular contouring have been achieved by the micropositioning stage, which validate the effectiveness of the proposed control scheme employing the inverse Bouc-Wen hysteresis model-based feedforward combined with the incremental-type PID feedback control. Considering the achieved positioning performance, the stage may be more suitable for micromanipulation of microscopic objects such as biological cells. In the future works, the designed stage will be employed for nanopositioning applications using high-resolution displacement sensors. Moreover, the stage can also be miniaturized into micro- and meso-scale for pertinent micro-/nanomanipulation applications.

### References


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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.