Adaptive Transmit Beamforming with Space-Time Block Coding for Correlated MIMO Fading Channels

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Abstract-In this paper, we present an open-loop transmit scheme for MIMO communications. We first use the uplink channel correlation matrix (UCCM) to calculate the array weight vectors for downlink beamforming (DLBF). Then based on the symbol error rate (SER) upper bound as design criterion, we derive an algorithm for adaptive power allocation among multiple beams, and thus develop the transmit scheme joint adaptive beamforming (ABF) with space-time block coding (STBC). The main benefit of the scheme is that it can almost achieve the optimal performance with low complexity. Next, using the moment generation function (MGF) approach and the Gauss-Chebyshev integration, we derive a simple and accurate numerical analysis method for the proposed scheme under three widely used modulations. Finally, computer simulation results demonstrate the superiority of the open-loop transmit scheme.

Key Words-Adaptive beamforming, Space-time block coding, Transmit scheme, MIMO communication.

I. INTRODUCTION

Recently, beamforming (BF) and space-time coding (STC) are widely recognized as two key approaches to improve the capacity and reliability of wireless communication systems employing multiple transmit antennas. Although considering the array inter-element spacing and the required knowledge about the channel state information (CSI) at the transmitter, they are different and even contradictory, it was shown in [1] that the benefits of these two techniques can be achieved simultaneously using side information. Besides this work, there are a number of other articles that discuss the combination of BF with STC. For example, in [2] and [3], the authors use either channel mean or channel correlation feedback, and generalized optimal transmit eigen-beamformer for widely used linear modulations, and coupled orthogonal STBC to increase the data rate without compromising the performance. However, these strategies require a considerable amount of feedback bandwidth, which may be a heavy burden in practice. In [4], the authors combined one-dimensional (1-D) beamforming with Alamouti’s STBC to achieve more performance gain for the case of more than two transmit antennas. In [5], the authors proposed a downlink transmission scheme combining two-dimensional (2-D) eigen-beamforming with Alamouti’s STBC to achieve both diversity gain as well as array gain. The analogous ideal was also presented in [6] and [7], where the pure diversity and the pure BF schemes were also analyzed and compared. But the major drawback of these methods is that they can not adaptively allocate the proper power among multiple beams according to the channel parameters, thus their performances are not optimal in a lot of environments. Besides sufficient error performance analysis was not given there.

In this paper, we present an open-loop scheme for correlated MIMO fading channels, which is realistic in cellular communications due to close inter-element spacing of antenna array. We first use the UCCM to calculate the array weight vectors for DLBF. Then based on the model of equivalent scaled additive white Gaussian noise (AWGN) channel induced by the STBC, we obtain the signal-to-noise ratio (SNR) at the receiver and derive the SER upper bound. Using it as the design criterion, we derive an algorithm for adaptive power allocation, and thus develop the transmit scheme combining ABF with STBC. Although the result is a little analogous to the work of [3], that paper did not take the effect of STBC into account for beamformer design. Besides, the derivation and expression for the SER upper bound are also different. Next, utilizing the MGF approach and the Gauss-Chebyshev integration, we derive a simple and accurate numerical analysis method for the proposed scheme under three widely used modulations. Finally, computer simulation results demonstrate that the proposed transmit scheme outperforms the conventional STBC and the other existing related typical strategies.

Notation: $\| \|_F$ denotes the Frobenius norm of a matrix, $E[\cdot]$ denotes expectation, $\text{diag}(\cdot)$ denotes a diagonal matrix, $(\cdot)^*, (\cdot)^T,$ and $(\cdot)^H$ denote conjugate, transpose, and Hermitian transpose, respectively. $\delta(\cdot)$ represents the Kronecker delta function, $Q(\cdot)$ represents the classical definition of the Gaussian $Q$-function, $\tan(\cdot)$ represents the tangent function, $\ln(\cdot)$ represents the natural logarithm function, $\exp(\cdot)$ represents the exponential function, $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ represent the real part and the imaginary part of a complex number respectively.

The rest of the paper is organized as follows. In section II, the problem description is provided. In Section III, a novel method to calculate the array weight vectors for DLBF from UCCM is firstly presented, then based on the criterion of SER upper bound, an adaptive power allocation algorithm is
proposed, thus the space-time transmit scheme is developed. In Section IV, using the MGF approach and the GaussChebyshev integration, a simple and accurate numerical analysis method for the proposed scheme under three widely used modulations is derived. Finally, computer simulations are given in Section V, and conclusion is drawn in Section VI.

II. PROBLEM DESCRIPTION

In this paper, for the purpose of simplicity and without loss of generality, we consider a wireless communication with $N$ transmit antennas and a single receive antenna. However, our results can be extended to multiple receive antennas as in [3]. Figure 1 shows the structure of transmit scheme combining BF with STBC. The serial signal to be transmitted, $s$, is first coded with a space-time block encoder, yielding $P$ ($P \leq N$) branch outputs as [8]

$$
S = \begin{bmatrix}
s_{11} & s_{12} & \cdots & s_{1T} \\
s_{21} & s_{22} & \cdots & s_{2T} \\
\vdots & \vdots & \ddots & \vdots \\
s_{P1} & s_{P2} & \cdots & s_{PT}
\end{bmatrix}
$$

(1)

where $T$ denotes the number of time slots used to encode $K$ symbols, and $s_{ij}$ ($i \leq P, j \leq T$) denotes a linear combination of the signal constellation components and their conjugates. Then the $P$ branch coded signals pass through $P$ beamformers, and transmitted by the $N$ elements simultaneously. Therefore, at a particular time $nT$, the transmitted signal matrix $X_{nT}$ ($N \times T$) can be expressed as

$$
X_{nT} = WFS
$$

(2)

where $W = [w_1, w_2, \cdots, w_P]$ and $F = \text{diag}(f_1, f_2, \cdots, f_P)$, $w_i$ ($N \times 1$) and $f_i$ denote the array weight vector and the corresponding coefficient of the $i$th beamformer. Here, we suppose $\|w_i\|^2_F = 1$ and $\sum_{i=1}^{P} f_i^2 = 1$ to guarantee that the total transmit power keeps constant for fair comparison.

Fig.1 Transmit scheme combining BF with STBC

Suppose the channel is a cluster-based flat fading channel and there are $L$ paths in scatter clusters and the angle-of-departure (AOD) of each path varies around the mean cluster AOD $\theta_c$ within the cluster angle spread $\Delta \theta$, namely, the $L$ multipath signals are uniformly distributed in the range of $[\theta_c - \Delta \theta/2, \theta_c + \Delta \theta/2]$, then the instantaneous downlink channel response vector is given by [5]

$$
\mathbf{h}_d(t) = \sum_{l=1}^{L} \rho_l(t) \mathbf{a}_d^T(\theta_l)
$$

(3)

where $\theta_l$ and $\rho_l(t)$ denote the AOD and the fading coefficient of the $l$th path signal respectively. Without loss of generality, here $\rho_l(t)$ is modeled as complex Gaussian random variable with zero mean and unit variance. If uniform linear array (ULA) is employed at the transmitter, the downlink array steering vector $\mathbf{a}_d(\theta_l)$ is

$$
\mathbf{a}_d(\theta_l) = [1, \exp(i \beta_d \cos \theta_l), \cdots, \exp(i (N - 1) \beta_d \cos \theta_l)]^T
$$

(4)

where $d$ is the inter-element spacing, and $\beta_d$ is the downlink wavenumber. Therefore, the downlink channel correlation matrix (DCCM) $\mathbf{R}_d$ can be obtained from [9]

$$
\mathbf{R}_d = E[\mathbf{h}_d^H(t) \cdot \mathbf{h}_d(t)] = \frac{1}{L} \sum_{l=1}^{L} \mathbf{a}_d^H(\theta_l) \cdot \mathbf{a}_d^T(\theta_l)
$$

(5)

The index of $t$ will be frequently dropped from now on for notation brevity.

From (2), the received signal vector can be written as

$$
\mathbf{y}_{nT} = \mathbf{h}_d \mathbf{w}_i + \mathbf{v}_{nT}
$$

(6)

where $\mathbf{y}_{nT}$ and $\mathbf{v}_{nT}$ denote the $1 \times T$ received signal vector and background noise vector respectively. Let

$$
\mathbf{g}_i = \mathbf{h}_d \mathbf{w}_i \quad (i = 1, 2, \cdots, P)
$$

(7)

and use the model of equivalent scaled AWGN channel induced by the STBC [10], the decoded signal vector considering the code rate $R$ can be expressed as

$$
\mathbf{y}_{nT} = \frac{1}{R} \|\mathbf{G}\|^2_F \mathbf{s}_{nT} + \mathbf{v}_{nT}
$$

(8)

where $\mathbf{y}_{nT}$ denotes the $K \times 1$ complex vector after STBC decoding from the received vector $\mathbf{y}_{nT}$, and $\mathbf{s}_{nT}$ denotes the input $K \times 1$ complex vector. $R$ is the code rate of the STBC, namely, $R = K/T$, and $\mathbf{v}_{nT}$ is the $K \times 1$ complex Gaussian noise with zero mean and $\|\mathbf{G}\|^2_F N_0/(2R)$ variance in each real dimension. Therefore, the effective SNR at the receiver is

$$
\gamma = \frac{R E_s \|\mathbf{G}\|^2_F}{R\|\mathbf{G}\|^2_F N_0} = \gamma_s \|\mathbf{G}\|^2_F
$$

(9)

where $\gamma_s = E_s^2/|N_0| = E_s/N_0$ is the transmit SNR per symbol. Therefore, if we want to get the optimal performance at the receiver, the transmit beamformer must be designed to satisfy

$$
J = \max_{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_P} \frac{E\left[\|\mathbf{G}\|^2_F\right]}{\|\mathbf{G}\|^2_F} = \max_{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_P} \sum_{l=1}^{P} f_l^2 \left(\mathbf{w}_l^H \mathbf{R}_d \mathbf{w}_l\right)
$$

(10a)

s.t. $\mathbf{w}_l^H \mathbf{w}_l = \delta(i - j)$ and $\sum_{l=1}^{P} f_l^2 = 1$

(10b)

Using the method of Rayleigh quotient [11], we can get the
solution of the constrained optimization problem as
\[
J = \sum_{i=1}^{P} \delta_i f_i^2 
\]
where \(\delta_i\) and \(f_i\) denote the \(i\)th eigenvalue and the corresponding eigenvector of the DCCM, that is
\[
\mathbf{R}_d = [v_1, v_2, \ldots, v_n] diag(\delta_1, \delta_2, \ldots, \delta_N) [v_1, v_2, \ldots, v_n]^H 
\]
Here the \(N\) eigenvalues are arranged in a nonincreasing order.

III. PROPOSED TRANSMIT SCHEME

A. Downlink Array Weight Vectors Computation

For a frequency division dupplex (FDD) system, most of the previous works suppose that the DCCM \(\mathbf{R}_d\) is yielded through feedback. However, these strategies are unrealistic in practice due to the heavy feedback burden. Although many matrix transformation methods have been proposed (see [12]-[15], and references therein for example), they cannot be used in our scheme due to estimation accuracy or computation load. So here, we make use of UCCM and present a new method to calculate the array weight vectors for DLBF.

Analogous to (4), the UCCM can be expressed as [9]
\[
\mathbf{R}_u = \mathbf{E}[\mathbf{h}_u(t) \cdot \mathbf{h}_u^H(t)] = \frac{1}{L} \sum_{l=1}^{L} a_u(\theta_l) \cdot a_u^H(\theta_l) 
\]
where \(\mathbf{h}_u(t)\) and \(a_u(\theta_l)\) denote the uplink channel response vector and array steering vector respectively.

In order to calculate the DLBF weight vectors, we first utilize spectral theorem and decompose \(\mathbf{R}_u\) as
\[
\mathbf{R}_u = \mathbf{U} \mathbf{S} \mathbf{U}^H 
\]
where \(\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_N]\) and \(\mathbf{S} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)\), which satisfy \(\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N\). The \(N \times 1\) complex vector \(\mathbf{u}_1\) can be written as
\[
\mathbf{u}_1 = [A_1 \exp(j\phi_1), A_2 \exp(j\phi_2), \ldots, A_N \exp(j\phi_N)]^T
\]
Since a constant phase difference between the components of array weight vectors does not affect the BF performance, we can subtract \(\phi_1\) from \(\phi_i\), and use the periodicity of exponential function simultaneously to obtain
\[
\mathbf{u}_1 = [A_1, A_2 \exp(j\Delta\phi_2), \ldots, A_N \exp(j\Delta\phi_N)]^T
\]
where \(\Delta\phi_i = 0, \Delta\phi_i = \phi_i - \phi_1\), and satisfies
\[
0 \leq \Delta\phi_2 \leq \ldots \leq \Delta\phi_N
\]
\[
0 \leq \left\| \Delta\phi_i - \Delta\phi_{i-1} \right\| < 2\pi
\]
Consider that the frequency difference between uplink and downlink around 10% is typical, and the antenna performance is more sensitive to the phases of array weights, we can do frequency calibration to \(\mathbf{u}_1\) and get the 1st array weight vector as [16]
\[
\mathbf{w}_1 = [A_1, A_2 \exp(-j\Delta\phi_2), \ldots, A_N \exp(-j\Delta\phi_N)]
\]
where \(f_d\) and \(f_u\) denote the carrier frequencies of downlink and uplink respectively. Next, we will employ the orthogonality of eigen-beamforming weight vectors as (19) to calculate \(\mathbf{w}_i\) \((i = 2, 3, \ldots, N)\).

\[
\mathbf{w}_i^H \mathbf{w}_j = \delta(i-j) 
\]
Define the array weight vector space as
\[
\mathbf{E}_i = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_{i-1}] \quad (i = 2, 3, \ldots, N)
\]
its orthogonal complement space can be obtained from
\[
\mathbf{E}_i^\perp = \mathbf{I}_{NN} - \mathbf{E}_i (\mathbf{E}_i^H \mathbf{E}_i)^{-1} \mathbf{E}_i^H
\]
Therefore, if we project \(\mathbf{u}_i^*\) \((i = 2, 3, \ldots, N)\) to \(\mathbf{E}_i^\perp\), we can get the corresponding weight vectors for DLBF, that is
\[
\mathbf{w}_i = \mathbf{E}_i^\perp \mathbf{u}_i \quad (i = 2, 3, \ldots, N)
\]

B. Adaptive Power Allocation

In this section, we will discuss the algorithm of adaptive power allocation. We first suppose that based on the previous mentioned calculation method for the downlink array weight vectors, \(N\) beams are formed, then our goal is to select the \(P\) beams from the \(N\) beams, and determine \(f_i\) simultaneously with the SER upper bound as criterion.

According to [17], based on the equivalent scaled AWGN channel induced by the STBC, the SERs of \(M\)-PSK \((M \geq 4)\), \(M\)-PAM, and \(M\)-QAM \((M \geq 4)\) can be expressed as
\[
P_{S,PSK} = 2E [\sqrt{\sin^2 (\frac{\pi}{M})}] 
\]
\[
P_{S,PAM} = \frac{2(M-1)}{M} E [\sqrt{\frac{6}{M^2 - 1}}] 
\]
\[
P_{S,QAM} \leq 4E [\sqrt{\frac{3}{M-1}}] 
\]
Joint considering (7) and (9), and using the well-known upper bound on the \(Q\)-function as
\[
Q(x) \leq \frac{1}{2} \exp \left( -\frac{x^2}{2} \right)
\]
we can rewrite (23) with a unifying form as
\[
P_{S,bound} = aE [\exp(-b||\mathbf{G}||_2^2)] = aE [\exp(-b\sum_{i=1}^{N} f_i^2 ||g_i||^2)]
\]
where
\[
a = \begin{cases} 
1 & \text{for PSK} \\
\frac{(M-1)}{M} & \text{for PAM} \\
\frac{3}{2(M-1)} & \text{for QAM}
\end{cases} \\
b = \begin{cases} 
\frac{1}{2} & \text{for PAM} \\
3 & \text{for QAM}
\end{cases}
\]
Since \(\mathbf{R}_d\) can be considered as a perturbation of \(\mathbf{R}_u\) due to the frequency difference, we have
\[
E[\mathbf{g}_i \cdot \mathbf{g}_i^*] = \mathbf{w}_i^H \mathbf{R}_d \mathbf{w}_i = \delta_i \approx \lambda_i \quad (i = 1, 2, \ldots, N)
\]
Furthermore, note that the mean of $g_i$ is zero, we can prewhit e $g_i$ as
\[ g_i = \sqrt{a_i} \tilde{g}_i \]  
so that $\tilde{g}_i$ is independent identically distributed complex Gaussian random variable with zero mean and unit variance. Let
\[ z_i = |\tilde{g}_i|^2 \]  
then $z_i$ has a chi-square distribution with 2 degrees of freedom, whose probability density function (pdf) is [17]
\[ p_{z_i}(z_i) = \exp(-z_i) \]  
Substitute (27), (28) and (29) into (25), straightforward computations yield
\[ P_{S,\text{bound}} = a E \left[ \exp \left( -b \gamma z + \sum_{i=1}^{N} \lambda f_i^2 z_i \right) \right] \]
\[ = a \prod_{i=1}^{N} \int_{0}^{\infty} \exp \left( -b \gamma \sum_{i=1}^{N} \lambda f_i^2 z_i \right) \exp(-z_i) \cdots \exp(-z_N) dz_1 \cdots dz_N \]
\[ = a \prod_{i=1}^{N} \frac{1}{1 + b \gamma \lambda f_i^2} \]  
Therefore, if we want to minimize the upper bound on SER to yield the power allocation among the multiple beams, we should solve the following constrained optimization problem
\[ e = \max \limits_{f_i} \prod_{i=1}^{N} (1 + b \gamma \lambda f_i^2) \text{ s.t. } \sum_{i=1}^{N} f_i^2 = 1 \]  
For convenience, we change the cost function $e = \ln(e)$ and define the Lagrangian as
\[ F = \sum_{i=1}^{N} \ln \left( 1 + b \gamma \lambda f_i^2 \right) + \mu \left( \sum_{i=1}^{N} f_i^2 - 1 \right) \]  
Differentiating the Lagrangian with respect to $f_i^2$, and setting it to zero, we have
\[ f_i^2 = -\frac{1}{\mu} - \frac{1}{b \gamma \lambda} \]  
Imposing the constraint, we can calculate $\mu$, which in turn determines $f_i^2$ through (33). However, for a given power $E_s$, this solution may not guarantee $f_i^2 > 0 \\forall i \in [1, N]$, we thus have to reduce the number of operating beamformers from $N$ to $P$. Toward this goal, we substitute the power constraint and obtain
\[ -\frac{1}{\mu} = \frac{1}{b} \sum_{j=1}^{P} \frac{1}{\lambda_j} + \frac{1}{b \gamma} \sum_{j=1}^{P} \frac{1}{\lambda_j} \]  
Then the power coefficient of the $i$th beam is given by
\[ f_i^2 = \frac{1}{b \gamma} \left( 1 + \frac{1}{b \gamma} \sum_{j=1}^{P} \frac{1}{\lambda_j} \right) \]  
Furthermore, consider $f_i^2 > 0$, we can get the following lower bound on the required SNR to turn on $P$ beams
\[ \gamma_s \geq \frac{1}{b} \left( \frac{P}{\lambda_j} - \sum_{j=1}^{P} \frac{1}{\lambda_j} \right) = \gamma_{th, P} \]  
Finally, the proposed open-loop transmit scheme combining ABF with STBC can be summarized as follows.

**Step 1:** Sample the channel estimates $\hat{h}_i(t)$ every $N_b$ blocks, and collect $K$ channel response vectors to estimate UCCM $\mathbf{R}_u$ as $\hat{\mathbf{R}}_u = \frac{1}{k} \sum_{k=1}^{K} \mathbf{h}_u(k) \mathbf{h}_u^H(k)$. Here $N_b$ is chosen large enough to guarantee that $\hat{h}_u(k N_b)$ and $\hat{h}_u((k+1) N_b)$ are uncorrelated.

**Step 2:** Decompose $\hat{\mathbf{R}}_u$ to yield the $N$ eigenvalues in nonincreasing order and the $N$ corresponding eigenvectors. Calculate the array weight vectors for DLBF with the method discussed in Section IIIA.

**Step 3:** Determine the modulation and calculate $\gamma_{th, P}$ $(P=1,2,\cdots,N)$ with (36).

**Step 4:** If $\gamma_s = E_s / N_0$ of the given power $E_s$ is in the interval $[\gamma_{th, P}, \gamma_{th, P+1}]$, set $f_{i+1}^2 = f_{i+2}^2 = \cdots = f_N^2 = 0$ and obtain $f_1^2, f_2^2, \cdots, f_P^2$ through (35).

**Step 5:** With the calculated array weight vectors and the corresponding coefficients, the $P$ adaptive beamformers are designed, and the space-time block codes are determined at the same time.

**IV. ERROR PERFORMANCE ANALYSIS**

This section deals with the error performance analysis of the proposed transmit scheme under the three widely used modulations: $M$-PSK $(M \geq 4)$, $M$-PAM and square $M$-QAM $(M \geq 4)$.

According to (23a), (9) and (27), the SER of $M$-PSK $(M \geq 4)$ constellation signal can be approximated as
\[ P_{S,\text{PSK}} \approx 2 E \left[ \sqrt{2 \sin^2 \left( \frac{\pi}{M} \right)} \sum_{i=1}^{P} \lambda_i f_i^2 |\tilde{g}_i|^2 \right] \]  
Let
\[ \zeta = 2 \sin^2 \left( \frac{\pi}{M} \right) \gamma_s \sum_{i=1}^{P} \lambda_i f_i^2 |\tilde{g}_i|^2 \]  
the SER of $M$-PSK can be simplified to [18]
\[ P_{S,\text{PSK}} \approx 2 E \left[ \zeta^\frac{1}{\Delta} \right] = \frac{1}{2} \int_{-\infty}^{\infty} \Phi_{\zeta}(s) 2s \sqrt{1 - 2z} ds \]  
Here we assume that $c$ is in the region of convergence of $\Phi_{\zeta}(s) (1 - 2z)^{-1/2}$, and a good choice is $c = 1/4$. $\Phi_{\zeta}(s)$ is the MGF of $\zeta$, which is given by
\[ \Phi_{\zeta}(s) = E \left[ \exp(-s\zeta^\Delta) \right] = \prod_{i=1}^{P} \left[ 1 + 2 \sin^2 \left( \frac{\pi}{M} \right) s \lambda_i f_i^2 \gamma_s \right]^{-1} \]  
By setting $\Phi_{\Delta}(s) = \Phi_{\zeta}(s) (1 - 2z)^{-1/2}$ and $s = c + j\omega$, (39)
can be rewritten as
\[ P_{S,PSK} = \frac{1}{\pi} \int_{c-j\infty}^{c+j\infty} \frac{\Phi_\Delta(x)}{c+j\omega} \frac{dx}{c^2+\omega^2} \]
\[ = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{c \text{Re}\{\Phi_\Delta(c+j\omega)\} + \omega \text{Im}\{\Phi_\Delta(c+j\omega)\}}{c^2+\omega^2} d\omega \]
\[ = \frac{1}{2\pi} \int_{-1}^{+1} \Phi_\Delta \left( c + j c \frac{1-x^2}{x^2} \right) \left( 1-x^2 \right) \left( \frac{1-x^2}{x^2} \right) dx \]
\[ + \frac{1}{2\pi} \int_{-1}^{+1} \frac{\sqrt{1-x^2}}{x} \Phi_\Delta \left( c + j c \frac{1-x^2}{x} \right) \left( 1-x^2 \right) \left( \frac{1-x^2}{x^2} \right) dx \]  
(41)
where \( \omega = c\sqrt{1-x^2}/x \). Use a Gauss-Chebyshev numerical integration rule with \( N \) nodes, we have
\[ P_{S,PSK} = \frac{1}{2N} \sum_{k=1}^{N} \left[ \text{Re}\{\Phi_\Delta(\nu(1+j\tau_k))\} + \text{Re}\{\Phi_\Delta(\nu(1+j\tau_k))\} \right] + E_v \]
(42)
where \( \tau_k = \tan((k-1/2)\pi/N) \) and \( E_v \to 0 \) as \( N \to \infty \). In the numerical calculation, \( N = 64 \) is good enough.
Similarly, from (23b) the SER of \( M \)-PAM constellation signal can be expressed as
\[ P_{S,PAM} = \frac{2(M-1)}{M} E \left[ Q \left( \frac{6}{M^2-1} \sum_{i=1}^{P} \lambda_i f_i^2 \left| \tilde{g}_i \right|^2 \right) \right] \]
\[ = \frac{(M-1)}{2M^2} \sum_{k=1}^{N} \left[ \text{Re}\{\Phi_\Delta(\nu(1+j\tau_k))\} + \text{Re}\{\Phi_\Delta(\nu(1+j\tau_k))\} \right] + E_v \]
(43)
where \( \Phi_\Delta(x) = (1-2s)^{-1/2} \prod_{i=1}^{P} \left( \frac{1}{M^2-1} + \frac{6}{M^2-1} - s \lambda_i f_i^2 \gamma_{\nu} \right) \]^{-1}.
For an \( M \)-ary, \( M = 2^n \) (n is even), square QAM signal, since it can be considered as two \( \sqrt{M} \)-PAM signals on quadrature carries, its SER is given by [17]
\[ P_{S,QAM} = 1 - \left( 1 - P_{S,PAM} \right)^2 \]
(44)
where \( P_{S,PAM} \) is the SER of \( \sqrt{M} \)-PAM signals, which can be obtained from
\[ P_{S,PAM} = 2 \left( 1 - \frac{1}{\sqrt{M}} \right) E \left[ Q \left( \frac{3}{M-1} \gamma_{\nu} \right) \right] \]
\[ = \frac{(M-1)}{2\sqrt{M}} \sum_{k=1}^{N} \left[ \text{Re}\{\Phi_\Delta(\nu(1+j\tau_k))\} + \text{Re}\{\Phi_\Delta(\nu(1+j\tau_k))\} \right] + E_v \]
(45)
where \( \Phi_\Delta(s) = (1-2s)^{-1/2} \prod_{i=1}^{P} \left( \frac{1}{M^2-1} + \frac{3}{M^2-1} - s \lambda_i f_i^2 \gamma_{\nu} \right) \]^{-1}.

V. COMPUTER SIMULATIONS

In this section, we consider a 6-element ULA with half uplink wavelength inter-element spacing at the transmitter and a single antenna at the receiver, and do simulations to examine the performance of the proposed open-loop scheme. In all the simulations, we suppose \( f_d = 1.1 f_u \), and consider two typical modulations: QPSK and 16-QAM. In addition, our tests focus on two typical cases: Case1 has the angular spread of 10°, and Case2 has the angular spread of 60°.

First of all, with the assumption that the mean AOD is 60°, the SER performances of QPSK as a function of SNR are plotted in Fig.2. As we can see, the SER difference between the proposed open-loop scheme and the eigen-beamforming scheme with perfect knowledge of DCCM, which is denoted with EBF+STBC, can be ignored. The scenario implies that our method can almost provide the optimal performance. In addition, it can also be found that the calculated results match very well with the simulated results in the two angular spread cases, thus the validity of the SER analysis based on MGF approach and the Gauss-Chebyshev integration is verified. However, compared with Monte-Carlo simulation method, it can dramatically reduce the computation time. Furthermore, the SER upper bounds are also included in the figure. The fixed SNR differences between the upper bound and the exact SER demonstrate the effectiveness of minimizing the SER bound.

Next, we suppose the desired user moves from one end of a 120° sector to another to cover all the possible AODs of the sector, and plot the average SER of both Case1 and Case2 versus SNR for QPSK and 16-QAM in Fig.3 and Fig.4 respectively. It can be observed that the proposed scheme outperforms the three other related strategies in the SNR region of interest. The other strategies are the conventional Alamouti’s STBC [19], the 1-D BF combined with STBC as in [4], and 2-D BF combined with STBC as in [5]. With small angular spread, the SER curve of the proposed scheme is almost identical to the 1-D BF strategy at low SNR, whereas for increasing SNR, it approaches the curve of 2-D BF strategy. On the other hand, in large angular spread environment, due to the adaptive power allocation, our scheme can achieve more diversity gain to enhance the SER performance, compared with the other relative typical strategies. Furthermore, we can see that the higher the SNR, the more performance improvement can be obtained by the proposed scheme.

![Fig.2 SER versus SNR for QPSK](image)
Finally, computer simulation results demonstrate that the proposed transmit scheme outperforms the conventional space-time block coding and the other existing related typical strategies.

VI. CONCLUSION

Recently, the study of combing BF with STBC for downlink transmission has been an active research topic in wireless communication systems. In this paper, we present an open-loop scheme for correlated MIMO fading channels. We first use the UCCM to calculate the array weight vectors for DLBF. Then based on the model of equivalent scaled AWGN channel induced by the STBC, we obtain the SNR at the receiver and derive the SER upper bound. Using it as the design criterion, we derive an algorithm for adaptive power allocation, and thus develop the transmit scheme combining ABF with STBC. The main benefit of the scheme is that it can significantly reduce the system complexity while achieve the approximately optimal performance. Next, utilizing the MGF approach and the Gauss–Chebyshev integration, we derive a simple and accurate numerical analysis method for the proposed scheme under three widely used modulations.

REFERENCES