Self-Tuning PI Rate Controller for AQM Router
Supporting Best-Effort Traffic

Yang Hong and Oliver W.W. Yang
SITE, University of Ottawa
Ottawa, Ontario K1N 6N5, Canada
E-mail: yhong@site.uottawa.ca

Abstract- In this paper, we propose a self-tuning PI (Proportional-Integral) rate controller for an AQM (Active Queue Management) Router that would support best-effort traffic in the Internet. Our design employs classical control theory that allows the users to achieve good stability robustness of AQM control system by assigning a proper interval of gain and phase margins. Our self-tuning PI controller will self-tune only when the gain margin or phase margin drifts outside the specified interval. Every controlled source node always sends the data to the network at the maximum allowed transmission rate, thus maximizing the utilization of network resources. The self-tuning PI controller located in the router can calculate the advertised transmission rate for source nodes based on the instantaneous queue length of the buffer. Our simulations demonstrate good stability robustness of AQM control system with our self-tuning PI controller.

Keywords: PI Control, Active Queue Management, Queue Length, Gain Margin, Phase Margin, Best-Effort Traffic, Self-Tune

1. INTRODUCTION

Congestion control for IP networks has been a critical factor in the robustness of the Internet. TCP (Transmission Control Protocol) as a window-based control mechanism is widely implemented for end-to-end congestion control in the Internet [1]. However, it can cause severe sending rate fluctuation and is not suitable for streaming media transmission e.g. [2, 3].

Rate-based control allows the sources to adjust their sending rate to support best-effort service traffic and make the maximum use of network resources, thus offering the most effective solution in achieving high QoS (Quality of Service) for streaming media transmission. Some rate-based control methods have been proposed recently for the AQM (Active Queue Management) router supporting best effort service traffic. For example, TFRC (TCP friendly rate control) scheme was proposed for the equation-based congestion control of unicast traffic e.g. [3,4]. The RAP (Rate Adaptation Protocol) employed a simple AIMD (Additive Increase and Multiplicative Decrease) scheme for real-time stream transmission [5]. A rate-based control scheme was presented to improve the end-to-end TCP performance by controlling the source transmission rate [6]. A two-state adaptive rate control mechanism was proposed for streaming media in [2]. These papers yielded a unique solution for rate-based controller, but did not allow any flexibility to adjust to the great variations in the load level of the Internet, which may result in slow response and cause possible excessive buffer overflow. It is known from classical control theory that gain and phase margins can used to measure the stability robustness for the control system [7]. Based on these frequency domain specifications, some controllers for industrial application have been proposed e.g. [8].

In this paper, we propose a network model for the typical AQM router in the high-speed (e.g. link bandwidth > T4) Internet where the self-tuning PI rate controller is located in the AQM router and can calculate the advertised source transmission rate based on the instantaneous queue length of the buffer so that the source node can regulate transmission rate to prevent congestion in the network. Instead of specifying a fixed value (thus requiring a unique solution), we assign a proper interval of gain and phase margins to design the self-tuning PI rate controller for the AQM of the router. Self-tuning PI controller will self-tune only when the change in the number of the active source nodes or in the uniform round trip time drifts the gain margin or phase margin outside the interval. The PI controller can make the steady queue length equal to the target buffer occupancy, thus preventing excessive buffer overflow in the router. We can demonstrate by simulations that our control scheme serves both guaranteed traffic and best-effort traffic and also has maximum bandwidth utilization.

2. NETWORK MODEL AND ASSUMPTIONS

We consider a data communication network consisting of a number of geographically distributed source/destination nodes. Packets generated by a source node are delivered to their destination through a sequence of routers. The PI rate controller is located in the router and can calculate a desirable source transmission rate \( \hat{q}_s(t) \) based on the instantaneous queue length of the buffer in the router. Then the router advertise it to the source through its IP packet and its ACK packet so that the source can regulate its current transmission rate \( q_s(t) \) to provide the best-effort service traffic. There is also an uncontrolled guaranteed service traffic flows (both locally and the upstream router) with input rate \( v(t) \) into the router.

![Figure 1. System Model of the AQM Router](image-url)
The AQM router under consideration accumulates multiple best-effort service traffic streams and one guaranteed service traffic stream, as shown in Figure 1. There are $N$ controlled source nodes that transmit the packets routed through the AQM router. The AQM router has a finite buffer space $K$ to store the incoming packets and an output link to serve them at a constant data rate of $\mu$. The buffer in the router is organized into two logical queues for the best-effort service traffic and the guaranteed service traffic respectively. It is assumed that the service for each queue is FIFO (First-In-First-Output) and both queues share the same bandwidth of the output link. The queue for the guaranteed service traffic has a higher priority for the service than the queue for the best-effort service traffic. There are two types of time delays: (1) the variable forward time delay $\tau_{fi}$ from the controlled source node $i$ to the router; which includes the propagation delay, the queuing delay and the processing delay, (2) the variable feedback time delay $\tau_{bi}$ of both from the router to the destination and from the destination to the controlled source node $i$, which also includes the same types of delays as $\tau_{fi}$. We also let $\tau_{st}=\tau_{fi}+\tau_{bi}$ be the variable round trip time of the controlled source node $i$.

From the model of the AQM router in Figure 1, the change of the queue length in the buffer of the router is the sum of the input rates of both $N$ best-effort service traffic streams and guaranteed service traffic stream minus the service rate. It can be written as

$$\dot{x}(t) = \sum_{i=1}^{N} q_i(t - \tau_{fi}) + v(t) - \mu .$$  

(1)

### 3. THE SELF-TUNING PI RATE CONTROLLER

We propose a self-tuning PI rate controller that can achieve zero queue deviation in the router and avoid excessive buffer overflow. Every controlled source node is allowed to send the data into the network at its maximum allowed transmission rate in order to utilize the slack bandwidth not used by the guarantee service traffic. Let $\mu_i$ be the maximum transmission rate for the controlled source node $i$, and $x_0$ be the target buffer occupancy of the buffer in the AQM router. Based on the instantaneous queue length $x(t)$ of the buffer, the advertised transmission rate $\hat{q}_i(t)$ for the controlled source node $i$ will be calculated by the following PI control algorithm.

$$\hat{q}_i(t) = \mu_i + K_P e(t) + K_I \int_0^t e(\tau)d\tau$$

$$= \mu_i + K_P (x_0 - x(t)) + K_I \int_0^t (x_0 - x(t))d\tau ,$$  

(2)

where $K_P$ and $K_I$ are the proportional and the integral gain of PI controller respectively. The advertised transmission rate $\hat{q}_i(t)$ must be restricted under the range of 0 and $\mu_i$, i.e. $0 \leq \hat{q}_i(t) \leq \mu_i$, as shown in the performance evaluation later on. It can be easily shown that the transfer function between the advertised source transmission rate $\hat{q}_i(t)$ and the queue deviation $e(t)$ is given as

$$C(s) = K_P + K_I / s .$$  

(3)

It can be seen from the control structure in Equation (2) that the advertised source rate $\hat{q}_i(t)$ is adjusted based on the instantaneous queue length of the buffer. The advertised transmission rate $\hat{q}_i(t)$ then becomes the source rate $q_i(t)$ after the feedback time delay $\tau_{fi}$, which can be expressed as

$$q_i(t) = \hat{q}_i(t - \tau_{bi}) .$$  

(4)

Therefore, by substituting Equation (4) into Equation (1), we can obtain the dynamic of the router as

$$\dot{x}(t) = \sum_{i=1}^{N} \hat{q}_i(t - \tau_{fi} - \tau_{bi}) + v(t) - \mu = \sum_{i=1}^{N} q_i(t - \tau_{fi}) + v(t) - \mu.$$  

(5)

Since the round trip times $\tau_i$ of all controlled source nodes are different, we can make them uniform by letting $\tau_{max} = \tau_{bi}/2 \tau_{max} = \tau_{fi}$, so that after the source node $i$ receives the advertised transmission rate $\hat{q}_i(t)$ from the ACK packet, it will adjust its current transmission rate after a time delay $\tau_{fi}$.

Under this adjustment, we can rewrite Equation (4) as

$$q_i(t) = \hat{q}_i(t - \tau_{bi} - \tau_{fi} + \tau_{fi}).$$  

(6)

By substituting Equation (6) into Equation (1), we can obtain

$$\dot{x}(t) = \sum_{i=1}^{N} q_i(t - \tau_{bi} + \tau_{fi}) + v(t) - \mu .$$  

(7)

Therefore, the transfer function between the instantaneous queue length $x(t)$ and the current advertised source transmission rate $\hat{q}_i(t)$ is given as

$$P(s) = \frac{\sum_{i=1}^{N} e^{-\tau_{fi} s}}{s} = \frac{N e^{-\tau_{fi} s}}{s} .$$  

(8)

![Figure 2. AQM control system](image-url)

**Figure 2. AQM control system**

### 3.1 Design Based on Gain Margin and Phase Margin

We will design self-tuning PI controller based on its gain and phase margins. Figure 2 is the AQM control system we consider for our PI controller design. In this AQM control system, $P(s)$ is the AQM control plant (a router with $N$ controlled source nodes) and $C(s)$ is the PI controller for the AQM. We can define the open-loop transfer function of AQM control system as

$$G(s) = P(s)C(s) = \frac{Ne^{-\tau_{fi} s}}{s} \left( K_P + K_I \right).$$  

(9)

It is a common practice in modern control to set the interval of gain margin to $2 \leq A_m \leq 4$ and the interval of phase margin to $30^\circ \leq \phi_m \leq 60^\circ$ for good response, because too small a gain margin or a phase margin will make the AQM control system sensitive to large load changes and cause instability, while too large a gain margin or a phase margin will result in sluggish response to load changes [9]. We will further specify the nominal gain margin $A_m$ as 3 (the mean of 2 and 4), and the nominal phase margin $\phi_m$ as $45^\circ$ (the mean of $30^\circ$ and $60^\circ$), so that the self-tuning PI controller can clamp the gain and phase margins of the AQM control system to within the above
desirable ranges even upon any big traffic change or any great fluctuation in round-trip time.

From the definition on the gain margin and phase margin of $G(s)$ [9], we can obtain

$$p(j\omega_p)\left[ K_p - j \frac{K_I}{\omega_p} \right] = \frac{Ne^{-j\phi_m}}{Ja_p} = -\frac{1}{A_m} \quad (10)$$

$$p(j\omega_g)\left[ K_p - j \frac{K_I}{\omega_g} \right] = \frac{Ne^{-j\phi_m}}{Ja_p} = -e^{j\phi_g} \quad . \quad (11)$$

where $\omega_p$ and $\omega_g$ are the phase and gain crossover frequencies of the AQM control system. Our objective is to determine the parameters $K_p$ and $K_I$ such that the gain and phase margins $A_m$ and $\phi_m$ are achieved, i.e. Equations (10) and (11) are satisfied. It is noted that there are altogether four unknowns, namely, $K_p$, $K_I$, $\omega_p$, $\omega_g$ in Equations (10) and (11).

Since both equations are complex, it can be analyzed into four independent real equations, from which we can obtain a unique solution for $K_p$ and $K_I$. We adopt the method proposed by [9] to obtain the parameters of PI controller as follows. Splitting Equations (10) and (11) into their real and imaginary parts yields respectively

$$K_p = \text{Re} \left[ \frac{-j\omega_p}{A_mNe^{-jo\omega_p}} \right] = \text{Re} \left[ \frac{-j\omega_g\phi_m}{Ne^{-jo\omega_p}} \right] \quad (12)$$

$$K_I = \omega_p \text{Im} \left[ \frac{-j\omega_p}{A_mNe^{-jo\omega_p}} \right] = \omega_g \text{Im} \left[ \frac{j\omega_g\phi_m}{Ne^{-jo\omega_p}} \right] \quad . \quad (13)$$

Since the frequency points $\omega_p$ and $\omega_g$ are unknown, we shall define the following two complex functions:

$$f_p(\omega) = \text{Re} \left[ \frac{-j\omega}{A_mNe^{-jo\omega}} \right] + j\omega \text{Im} \left[ \frac{-j\omega}{A_mNe^{-jo\omega}} \right] - \frac{\pi}{2} < \angle f_p(\omega) < -\pi \quad (14)$$

$$f_I(\omega) = \text{Re} \left[ \frac{-j\omega\phi_m}{Ne^{-jo\omega}} \right] + j\omega \text{Im} \left[ \frac{j\omega\phi_m}{Ne^{-jo\omega}} \right] + \frac{\pi}{2} < \angle f_I(\omega) < -\pi + \phi_m \quad . \quad (15)$$

The graphs $f_p(\omega)$ and $f_I(\omega)$ are then plotted in the same complex plane. An intersection of these two graphs means that they have the same real and imaginary parts, and thus both Equations (12) and (13) are satisfied so that the intersection is a solution of $K_p$ and $K_I$, and the corresponding $\omega_p$ and $\omega_g$. Obviously no solution exists if they do not intersect. In summary, there will exist a solution in the PI controller design for AQM control system if we specify the proper gain and phase margins such as $2\leq A_m \leq 4$ and $30^0 \leq \phi_m \leq 60^0$.

When the parameters N (the number of active controlled source nodes) and $\tau$ (the uniform round-trip time) of the network changed so greatly that the system gain margin or phase margin fell outside its specified interval, the PI controller need to self-tune to give new $A_m$ and $\phi_m$. This is done by real-time monitoring the gain and phase margins of the AQM control system using the following Equations (16), (17), (18) and (19).

According to the definition of gain margin and phase margin [10], we can obtain

$$A_m = \frac{\omega_p^2}{N \sqrt{K_p^2 + K_I^2}} \quad (16)$$

with

$$\arctan \left[ \frac{K_p\omega_p}{K_I} \right] = \omega_p \tau \quad , \quad (17)$$

and

$$\phi_m = \arctan \left[ \frac{K_p\omega_g}{K_I} \right] - \omega_g \tau \quad . \quad (18)$$

where the smallest positive real root of the Equation (19) will be the default value of gain crossover frequency $\omega_p$ for the calculation of the phase margin of the AQM control system by Equation (18). We now can further consider Equation (17) in the following 2 different cases:

Case (1): when $K_p\omega_p/K_I > 1$ (that is, $\omega_p > K_I/K_p$).

Based on the approximation that $\arctan(x) = \pi/2 - \pi/(4x)$ when $x > 1$, we can obtain

$$\frac{\pi}{2} - \pi \frac{K_p}{2\tau} \omega_p + \frac{\pi K_I}{4 K_p} = 0 \quad . \quad (19)$$

The smallest positive real root of the Equation (20) will be the default value of phase crossover frequency $\omega_p$ for the calculation of the gain margin of the AQM control system by Equation (16).

Case (2): when $0 < K_p\omega_p/K_I < 1$ (that is, $\omega_p < K_I/K_p$).

We can draw the graphs of $\arctan(K_p\omega_p/K_I)$ and $\omega_p$ in the same plane within the range of $0 < \omega_p \leq K_I / K_p$. The intersection of the two graphs is the solution of Equation (17), which is the phase crossover frequency $\omega_p$ for the calculation of the gain margin of the AQM control system by Equation (16).

In summary, we first use Equation (20) to obtain $\omega_p$. If we find $K_p\omega_p/K_I < 1$, then we can obtain $\omega_p$ using the method proposed by Case (2).

### 3.2 Self-tuning PI Controller Design Procedure

To implement the PI rate-based control algorithm in the IP networks, we will define three fields in the header of IP packets. The first field is to carry the maximum source transmission rate $\mu_i$. The second field is to carry the round trip time $\tau_i$ estimated by the source and will be replace with the uniform round trip time $\tau$ in the router. Finally the third field is to carry the advertised transmission rate $\hat{\mu}_i(t)$, whose original value is set as $\mu_i$.

We will design the self-tuning PI controller for the AQM router and regulate the source transmission rate according to the following procedure:

**Step 1:** Specify the gain margin interval to be $2 \leq A_m \leq 4$, the nominal gain margin $A_m$ to be $3$ (mean of 2 and 4), the phase margin interval to be $30^0 \leq \phi_m \leq 60^0$ and the nominal phase margin to be $45^0$ (mean of $30^0$ and $60^0$).
Step 2: Extract the maximum source transmission rate $\mu_i$ and the round trip time $\tau_i$ from the header of the IP packet.

Step 3: Obtain the uniform round trip time $\tau=\max\{\tau_1,\ldots,\tau_N\}$ and then replace $\tau_i$ with $\tau$ in the IP header.

Step 4: Plot the graphs of $f_p(\omega)$ and $f_i(\omega)$ in the complex plane based on Equations (14) and (15). From the intersection of the two graphs, we can obtain the proportional gain $K_p$ and integral gain $K_i$ of PI controller.

Real-time Monitoring:

Step 5: Compute the gain margin and phase margin using Equations (16), (17), (18) and (19) based on the current network information in the router: i.e. the number of the active controlled sources $N$ and the uniform round trip time $\tau$.

Step 6: (a) If the phase margin or gain margin falls outside the specified interval, go to Step 4. (b) If both the gain margin and the phase margin are still inside the interval, the PI controller will remain unchanged, go to Step 5.

PI controller online implementation:

Step 7: Apply the PI controller to calculate the advertised transmission rate $\hat{q}_{i, \text{new}}$ based on the instantaneous queue length $q(t)$ of the buffer and the maximum source transmission rate $\mu_i$. (7a) Compare the current advertised transmission rate $\hat{q}_{i, \text{new}}$ with the advertised transmission rate field $\hat{q}_{i, \text{old}}$ of the IP header and set $\hat{q}_{i, \text{old}} = \hat{q}_{i, \text{new}}$ if we find $\hat{q}_{i, \text{new}} < \hat{q}_{i, \text{old}}$; (7b) Otherwise, the advertised transmission rate field $\hat{q}_{i, \text{old}}$ remains unchanged. Therefore, we can prevent all the AQM routers from congestion in the Internet.

Step 8: Extract the advertised transmission rate $\hat{q}_i(t)$ and the uniform round trip time $\tau$ from the header of the IP packet and then integrate them into the ACK packet when IP packet arrives in the destination.

Step 9: Extract the advertised transmission rate $\hat{q}_i(t)$ and the uniform round trip time $\tau$ from the ACK packet and then regulate the current source transmission rate after the source receives ACK packet.

4. PERFORMANCE EVALUATION

Since our PI controller can regulate the source rate over time for AQM in the router and provide the best-effort service traffic in the network, we are particularly interested in analyzing the transient behaviors of the network. Transient behaviors such as the duration of the response time and the maximum transient queue length are the main concerns in the performance evaluation. Simulations have been performed using a router with an output link capacity (service rate) of $\mu=400Mbps$. The buffer size $K$ of the AQM router is $300Mb$. The target buffer occupancy is specified as $100Mb$.

In order to prevent all the AQM routers from congestion, we consider 8 controlled source nodes that transmit the packets through a most congested AQM router in the experiment. That is, the advertised transmission rate calculated by the most congested AQM router for the IP packets is smaller than the advertised transmission rate field in the IP packet. The maximum transmission rates $\mu_i$, the forward time delays $\tau_{fi}$, the feedback time delays $\tau_{bi}$ and the round trip times $\tau_i$ for the controlled source node $i$ ($i=1,2,\ldots,7,8$) are listed in the Table 1. The input rate of the uncontrolled guaranteed service traffic fluctuates between 10% and 20% of the service rate of the router, as shown in Figure 5(a). From time $t=0s$ to $t=60s$, 4 controlled sources ($i=1,2,3,4$) transmitted the packets into the router; after time $t=60s$, other 4 controlled sources ($i=5,6,7,8$) were added. We follow the design procedure in Section 3.2 to implement our simulation.

<table>
<thead>
<tr>
<th>Node</th>
<th>Max. Rate $\mu_i$ (Mbps)</th>
<th>Forward Delay $\tau_{fi}$ (ms)</th>
<th>Feedback Delay $\tau_{bi}$ (ms)</th>
<th>Round-Trip Time $\tau_i$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>110</td>
<td>300</td>
<td>300</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>100</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>200</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>300</td>
<td>400</td>
<td>700</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>200</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>200</td>
<td>400</td>
<td>600</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>100</td>
<td>300</td>
<td>400</td>
</tr>
</tbody>
</table>

Figure 4. The graph of $f_p$ and $f_i$.

In the beginning of the experiment, we can obtain the uniform round-trip time for 4 controlled sources $\tau=\max\{\tau_1,\tau_2,\tau_3,\tau_4\}=600ms$. Thus the open loop transfer function of AQM control system can be obtained as $G(s)=\frac{4e^{-0.6s}}{s}\left(\frac{K_p+K_i}{s}\right)$. By the method proposed in Section 3.1, the graphs of $f_p$ and $f_i$ were drawn and shown in Figure 4. Then PI controller was given as $C(s)=0.2023+0.049/s$. The self-tuning PI controller started to calculate the advertised transmission rates and integrated it into the IP packets from the controlled sources so that the sources can regulate their transmission rates to provide the best-effort service traffic.
Figures 5(b) to 5(d) show the transient behaviors of the AQM router with 8 controlled sources regulated by our self-tuning PI controller. At the time $t=0s$, the uncontrolled guaranteed traffic flowed into the AQM router with the rate of $800Mbps$, and the 4 controlled source nodes ($i=1,2,3,4$) started to transmit the data at their maximum transmission rate $\mu_i$. The instantaneous queue length (Figure 5(d)) increased till its peak is reached at the time about $4s$. Then it decreased and approached to its target buffer occupancy ($1000Mbps$), until it reached its steady state ($100Mbps$) around $15s$. At time $t=30s$, the uncontrolled guaranteed traffic reduced its transmission rate. The instantaneous queue length then decreased linearly till it reached about $52Mb$. It then increased again and approached a steady state value equal to the target buffer occupancy of $x_0=100Mb$. At the time $t=60s$, another 4 controlled sources ($i=5,6,7,8$) started to send the packets to the destination through the router. At this moment, both the phase margin and the gain margin of AQM control system had fallen outside the specified interval, so the self-tuning PI controller self-tuned and gave new $A_m$ and $\phi_m$. The new PI controller began to calculate the advertised source transmission rates. The instantaneous queue length increased till reached the buffer limit ($300Mbps$) and caused a slight buffer overflow for $4s$. Then it linearly decreased till it reached the target buffer occupancy ($100Mbps$) at the time $t=80s$. When the uncontrolled guaranteed traffic increased at $t=90s$, instantaneous queue length increased till reached about $155Mb$, before dropping back very quickly and returning to the steady value of $x_{\infty} x_0=100Mb$ at the time $t=103s$.

5. CONCLUSIONS

We have developed a self-tuning PI rate controller for a router supporting best-effort service traffic in the high-speed networks with high bandwidth delay product. We have employed the gain and phase margins in classical control theory to make our controller self-tuning for active queue management. Instead of specifying a fixed value, we assign proper interval of gain and phase margins to achieve the good stability robustness of AQM control system. Once the change of network environment drifts the phase margin or gain margin outside the specified interval, the self-tuning PI controller will self-tune. The equations for obtaining the PI controller as well as those for real-time monitoring the gain margin and phase margin of AQM control system have been derived. The PI controller is located in the AQM router and calculates the advertised source transmission rate based on the instantaneous queue length of the buffer, which makes the steady-state value of the queue length equal to the specified target buffer occupancy. We have demonstrated by simulations that with our self-tuning PI controller applied, the network exhibits good transient behavior. Our self-tuning PI rate-based controller is suitable for the rate control of streaming media transmission in the Internet.

Currently, we are updating our rate control algorithm to improve the fairness among different controlled sources. We will do the simulations under multiple router scenarios in our future work.

ACKNOWLEDGMENT

We want to appreciate the financial support from NSERC Postgraduate Scholarship (PGS-B) and NSERC Research Grant (#RGPIN42878).

REFERENCES


![Figure 5 Transient behavior of the AQM router](image_url)