Multi-UAV-based Optimal Crop-dusting of Anomalously Diffusing Infestation of Crops

Jianxiong Cao\textsuperscript{a,b}, YangQuan Chen\textsuperscript{b,*}, Changpin Li\textsuperscript{a}

\textsuperscript{a}Department of Mathematics, Shanghai University, Shanghai 200444, China
\textsuperscript{b}School of Engineering, University of California, Merced, 5200 North Lake Road, Merced, CA 95343, USA

Abstract
This paper presents a UAV-based optimal crop-dusting method to control anomalously diffusing infestation of crops. Two anomalous diffusion models are considered, which are, respectively, time-fractional order diffusion equation and space-fractional order diffusion equation. Our problem formulation is motivated by real-time pest management by using networked unmanned cropdusters where the pest spreading is modeled as a fractional diffusion equation. We attempt to solve the optimal dynamic location of actuators by using Centroidal Voronoi Tessellations. A new simulation platform (FO-DiffMAS-2D) for measurement scheduling and controls in fractional order distributed parameter systems is also introduced in this paper. Simulation results are presented to show the effectiveness of the proposed method as well as the role of fractional order in the overall control performance.

Keywords: Anomalous diffusion process, Centroidal Voronoi Tessellation, Distributed parameter system simulation, Fractional diffusion equation

1. Introduction

It is well known that diffusion model is one of the most commonly used mathematical approaches for the description of transport dynamics, and the normal diffusion equation is used to govern the corresponding process. However, it fails to describe some transport dynamics in various complex systems.

*Corresponding author
Email addresses: caojianxiong2007@126.com (Jianxiong Cao),
ychen53@ucmerced.edu (YangQuan Chen ), lcp@shu.edu.cn (Changpin Li )

Preprint submitted to Elsevier November 12, 2014
In the past few decades, fractional calculus has attracted an increasing interests of researchers, and by using fractional calculus, anomalous diffusion models are developed to describe transport process in complex dynamic system [1]. Due to the nonlocal and hereditary properties of fractional operators, fractional diffusion equations are used to simulate those anomalous diffusion transport dynamics in complex systems. In this paper, we study the complex transport dynamic behavior of pest infestation of crops. One of the main problems is to locate the diffusion source, give optimal dynamic actuator location according to the measured data from sensors, and then take action to the anomalous infestation. Many researchers focused on seeking source problem. In [2], the authors used mobile robots to measure diffusion source with gradient climbing method. Parameter estimation algorithm [3] was also used to identify a moving diffusion source. Recently, with the rapid development of unmanned aerial vehicle (UAV), multiple UAV are used for source seeking [4]. However, there is not enough information for controlling a diffusion process only by knowing its source. Qiang Du et al. [5] proposed Centroidal Voronoi Tessellations in coverage control, and then CVT method was extended to a diffusion and spray control [6]. Here, inspired by the properties of less time with higher efficiency while using UAV, we try to use low-cost UAVs as cropdusters/actuators to control anomalous diffusion of pest spreading.

It can be easily seen that detecting and controlling a diffusion process will be viewed as an optimal sensor/actuator location problem in a distributed parameter system. Basically, CVT algorithm provides a non-model-based method for coverage and diffusion control by using a group of robots. Therefore, people use CVT to generate some desired actuator positions.

This paper tries to solve the problem how to locate a group of UAVs to sense and control anomalous diffusion process of pest infestation. What we most concern here is the minimal impact to the soil and agriculture caused by pesticide and pest. The pest has severe negative impact on agricultural industry, but the pesticide may also have bad effect on the soil. So, the main objective for us is to deploy the UAVs and control the pest diffusion in an optimal way to minimize any negative impact on the crops and also the soil. That is to say, the pesticide should be released in such a way that the diffusion of the pest is bounded so that the heavily affected area is kept as small as possible. These all depend on how the UAVs positions are chosen, how they move and what control law is used to release the pesticide.

Motivated by real-time pest management by using unmanned cropdusters
and the application of CVT in optimal placement of resource \cite{3}, we pro-
posed a practical algorithm based on CVT to solve the problem of actuator
motion planning to spray the pest. In our experiment, the pest/pollution
density is given by the sensors that cover the area and form a mesh. Based
on Diff-MAS2D \cite{8}, we develop a new simulation platform FO-Diff-MAS2D
for fractional diffusion system measurement and control with mobile sensors
and mobile actuators. Our proposed algorithm has been implemented on
FO-Diff-MAS2D. Simulation results access the effectiveness of our algorithm
as well as the role of fractional order in the overall control performance
changes.

The remainder of this paper is organized as follows. In Section 2, the
models of anomalous diffusion process and the corresponding control prob-
lems are formulated. In Section 3, Centroidal Voronoi Tessellations (CVT)
based optimal actuator location algorithm is briefly introduced. The main
features of our simulation platform FO-Diff-MAS2D for fractional diffusion
system measurement and control are introduced in Section 4. In Section 5,
we imply simulation experiments to show the effectiveness of our proposed
methods. Finally, we conclude the paper and give future research efforts in
last section.

2. Mathematical models and problem formulation

In this section, we give two anomalous diffusion models to describe infe-
sitation problem. The two models are described by time fractional diffusion
equation and space fractional diffusion, respectively.

Suppose that two anomalous diffusion processes evolve in a convex poly-
tope \( \Omega : \Omega \in \mathbb{R}^2 \). The time domain is \( t \geq 0 \). Let \( \rho(x, y) : \Omega \rightarrow \mathbb{R}_+ \) represent
the pest density over \( \Omega \). The corresponding dynamic process can be modeled
by using the following time fractional diffusion equation

\[
C D_{0,t}^\alpha \rho(x, y, t) = k_\alpha \left( \frac{\partial^\alpha \rho}{\partial x^\alpha} + \frac{\partial^\alpha \rho}{\partial y^\alpha} \right) + f_d(\rho, x, y, t) + f_c(\tilde{\rho}, x, y, t),
\]

and space fractional diffusion equation below

\[
\frac{\partial \rho(x, y, t)}{\partial t} = k_\beta \left( \frac{\partial^\beta \rho}{\partial |x|^\beta} + \frac{\partial^\beta \rho}{\partial |y|^\beta} \right) + f_d(\rho, x, y, t) + f_c(\tilde{\rho}, x, y, t),
\]
where $\rho(x, y, t)$ is the pest density to be controlled, $k_\alpha$ and $k_\beta$ are positive constants representing the diffusion rate, $f_d(\rho, x, y, t)$ is source, $\hat{\rho}$ is the measured data of $\rho$ from the sensors, $f_c(\hat{\rho}, x, y, t)$ is the control input by mobile actuators to spray the pest, and its exact form depends on the closed-loop control law designed by the user based on certain control performance requirement. $C^{D}_0 \rho$ is Caputo fractional derivative of order $\alpha$ ($0 < \alpha \leq 1$) defined by [9]

$$
C^{D}_0 \rho = \begin{cases} 
\frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} \rho(x, y, \tau) d\tau, & 0 < \alpha < 1, \\
\frac{\partial \rho}{\partial t}, & \alpha = 1.
\end{cases} \tag{3}
$$

The operators $\frac{\partial^\beta \rho}{\partial |x|^{\beta}}$ and $\frac{\partial^\beta \rho}{\partial |y|^{\beta}}$ are Riesz fractional derivatives, which are, respectively defined as [9]

$$
\frac{\partial^\beta \rho}{\partial |x|^{\beta}} = \begin{cases} 
-c_\beta \left( RL^\beta D_{a,x} \rho + RL^\beta D_{x,b} \rho \right), & 1 < \beta < 2, \\
\frac{\partial^2 \rho}{\partial x^2}, & \beta = 2.
\end{cases} \tag{4}
$$

where $RL^\beta D_{a,x}$, $RL^\beta D_{x,b}$ are left/right Riemann-Liouville derivatives for variable $x$ defined in [9]

$$
\frac{\partial^\beta \rho}{\partial |y|^{\beta}} = \begin{cases} 
-c_\beta \left( RL^\beta D_{c,y} \rho + RL^\beta D_{y,d} \rho \right), & 1 < \beta < 2, \\
\frac{\partial^2 \rho}{\partial y^2}, & \beta = 2.
\end{cases} \tag{5}
$$

where $RL^\beta D_{c,y}$, $RL^\beta D_{y,d}$ are left/right Riemann-Liouville derivatives for variable $y$. The parameter $c_\beta$ in (3) and (4) is $c_\beta = \frac{1}{2 \cos(\beta \pi/2)}$.

Denote by $P = (p_1, \cdots, p_n)$ be the location of $n$ actuators and $| \cdot |$ be the Euclidean distance. Each actuator at position $p_i$ will receive information from sensors and then move and release the pesticide chemical by some control law. The objectives are:

- Control the infestation diffusion to a confined area.
- Cropdusting in a time optimal way while not making the area overdosed.
• Minimize the crops area that is heavily affected.

$n$ sensors will divide $\Omega$ into a set of $n$ polytopes $\mathcal{V} = \{V_1, \cdots, V_n\}$, $p_i \in V_i$, $V_i \cap V_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^n V_i = \bar{\Omega}$ ($\bar{V}_i = V_i \cup \partial V_i$ and $\bar{\Omega} = \Omega \cup \partial \Omega$). In order to control the diffusion process and minimize the heavily affected area, the actuators should be close to those areas with high pest concentrations so that the pest can be killed timely and does not diffuse further. But confining all actuators very close to the pollution source is not a good strategy. To decide the positions of the actuators, we consider the minimizing of the following cost function

$$K(P, \mathcal{V}) = \sum_{i=1}^n \int_{V_i} \rho(q) |q - p_i|^2 dq \text{ for } q \in \Omega.$$  \hspace{1cm} (6)

One can see that to minimize $K$, the distance $|q - p_i|$ should be small when the pest density $\rho(q)$ is big. It is the density function $\rho(q)$ that determines the optimal positions of the actuators. A necessary condition for $K$ to be minimized is that $\{p_i, V_i\}^k_{i=1}$ is a CVT of $\Omega$ [10]. Our algorithm is based on a discrete version of [10] and the density information comes from the measurements of the static, low-cost sensors.

3. optimal actuator algorithm

In this section, we give the algorithm to compute the locations of actuators by using CVT. Analogue to [6, 11, 12, 13], we also use Lloy’s method to determine CVT. The algorithm is described as follows [10]:

Give a region $\Omega$, a density function $\rho(x)$ defined for all $x \in \bar{\Omega}$, and a positive integer $k$

1. select an initial set of $k$ points $\{z_i\}^k_{i=1}$ as the generators,
2. construct the Voronoi sets $\{V_i\}^k_{i=1}$ associated with generators $\{z_i\}^k_{i=1}$,
3. determine the mass centroids of the Voronoi sets $\{V_i\}^k_{i=1}$; these centroids form the new set of points $\{z_i\}^k_{i=1}$,
4. If the new points meet some convergence criterion, terminate; otherwise, return to step 2).

We assume that an actuator can communicate with the sensors and other actuators within radius $R_i$. $R_i$ can be changed. Here we introduce a distributed algorithm from [6] with mild modification. At the first execution of step 2) in the above Lloy’s algorithm, each UAV will do the following:
1. Assign its detection range \( R_i \) with a small initial value, detect all its neighboring sensors with radius \( R_i \).
2. Construct its own Voronoi cell within the radius \( R_i \).
3. For every sensor \( q_i \), compute \( d_i = \max |q_i - p_i| \).
4. If \( R_i > 2 \times d_i \), stop. Otherwise set \( R_i = 2 \times R_i \), go to step 2).

\( R_i \) obtained at the first step will be used as the initial values for the following steps. For example, if successive 3 updates, the \( R_i \) remains unchanged, then \( R_i \) can be decreased, \( R_i = R_i - \Delta r \) for some \( \Delta r > 0 \). This improvement on the algorithm from \([6, 14]\) helps to reduce the computation requirements.

The mobile actuators are treated as virtual particles and obey the second-order dynamical equation:

\[
\ddot{p}_i = -k_p(p_i - \bar{p}_i) - k_v \dot{p}_i.
\]  

(7)

The first term of (7) on the right hand side is a proportional control law used as the force input to control the motion, where \( \bar{p}_i \) is the computed mass centroid of the current Voronoi cell. The second term is the viscous friction artificially introduced \([15]\). \( k_v \) is the friction coefficient and \( \dot{p}_i \) denotes the velocity of the actuator \( i \).

4. FO-DiffMAS-2D platform

In this section, we introduce the simulation platform FO-Diff-MAS2D for fractional diffusion model system measurement and control with mobile sensors and mobile actuators.

The following two types of boundary conditions at boundary \( \partial \Omega \) can be used in the problem formulation.

- **Dirichlet boundary condition**

\[
\rho = C,
\]  

(8)

where \( C \) is a real constant.

- **Neumann boundary condition**

\[
\frac{\partial \rho}{\partial n} = C_1 + C_2 \rho,
\]  

(9)

where \( C_1 \) and \( C_2 \) are two real constants; \( n \) stands for the outward direction.
FO-Diff-MAS2D uses finite difference method to discretize the spatial derivative in (1), and fractional central difference [16] to approximate space fractional derivative in (2), then leaves the time domain integration to Mat-
lab/Simulink. Specifically, FO-Diff-MAS2D is used to solve a two dimensional fractional diffusion equation. See Appendix for the numerical verifications. As an extension of Diff-MAS2D [8], the main features of FO-Diff-MAS2D are listed as follows:

- Sensors and actuators can be collocated or non-collocated.
- Disturbances can be movable and time-varying.
- The mobility platform dynamics of sensors and actuators can be modeled as either first order or second order.
- Movement of sensors and actuators can be open-loop or closed-loop.
- Arbitrary control algorithms can be applied in $f_c(\check{\rho}(x, y, t), x, y, t)$.

5. Simulation experiments

In this section, we consider two different fractional diffusion models to describe the anomalous diffusion infestation of crops. FO-Diff-MAS2D is used as the simulation platform for our realization. The area concerned is given by $\Omega = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

**Example I** The complex transport dynamic system with control input is modelled as the following time fractional diffusion equation

$$cD^{\alpha}_{0,t}\rho(x, y, t) = 0.01 \left( \frac{\partial^2 \rho(x, y, t)}{\partial x^2} + \frac{\partial^2 \rho(x, y, t)}{\partial y^2} \right) + f_c(x, y, t) + f_d(x, y, t),$$

with homogeneous Neumann boundary, given by

$$\frac{\partial u}{\partial n} = 0.$$

The pest source is modeled as a point disturbance $f_d$ to the fractional diffusion system (11) with its position at $(0.8, 0.2)$ and

$$f_d(t) = 20e^{-t}|_{(x=0.8, y=0.2)}.$$
The pest source begins to move at $t = 0$ to the area $\Omega$, 4 actuators which can release the pesticide are deployed with initial positions at $(0.33, 0.33), (0.33, 0.66), (0.66, 0.33), (0.66, 0.66)$, respectively. There are $29 \times 29$ sensors evenly distributed in a square area $(0,1)^2$ and they form a mesh over the area. In our simulation, we suppose that once deployed, the sensors remain static. Fig. 1 shows the initial positions of the actuators, the positions of the sensors and the position of the pollution source.

To check the validity of time fractional diffusion model, we give the simulation result in [6] for comparison, the results are displayed in Fig. 2. The y axis is the sum of the sensors measurements. It is clear that the result by using time fractional diffusion model is in accordance with integer order diffusion model when order $\alpha \approx 1$.

In order to illustrate the different diffusion behavior and control effectiveness with different time fractional derivative order, we choose $\alpha = 0.7, 0.8, 0.9, 1$, respectively. The simulation results are shown in Fig. 3 but the system is divergent when $\alpha < 0.7$, $\alpha = 0.7$ is the best derivative order in equation (10). One can see that the peak of the pollution becomes lower as $\alpha$ decreases. It can be deduced that the pest diffusion may be a process of subdiffusion.

We now fix $\alpha = 0.7$ as the optimal time derivative order in equation (10). First, for designing the actuator motion control law, the viscous coefficient is fixed by $k_v = 1$, in order to obtain optimal control strategy, letting $k_p$ change.
Figure 2: Evolution of the amount of pollutants

Figure 3: Comparision different evolution with different values of $\alpha$

from 1 to 9, as shown in Fig. $k_p = 6$ is the best gain. Then, we give the following control input

$$\ddot{p}_i = -6(p_i - \bar{p}_i) - \dot{p}_i. \quad (11)$$

We choose the simulation time to $t = 6s$. And the step size is chosen as $\Delta t = 0.002s$. The actuator recomputes its desired position every 0.1s. To show how the actuators can control the diffusion of the pests, the UAVs begin to react at $t = 0.4s$. The system evolves under the effects of diffusion of pests
and diffusion of pesticide released by UAVs.

In Fig. 5, the y axis is the sum of the sensors measurements. It shows that the amount of pests decreases to 19% of its peak value at the end of the simulation. And the decreasing process is monotonic. The evolution of the amount of pests without control is also shown in Fig. 5. we can tell that the pest infestation of crops has been well controlled.

Figure 4: Evolution of the amount of pollutants with different control law ($\alpha = 0.7$)

Figure 6 shows the evolution of the diffusion process with 4 actuators controlling one pest source diffusing through the region. The blue circles represent the actuator positions. The red circles represent the desired actuator positions using CVT.

It can be seen that the pollution density is becoming lower when the actuators are approaching their desired positions. It can be concluded that the pollution has been well controlled by equation (10) with fractional diffusion equation combining actuator control law (11).

Example II The system is modelled by the following space fractional diffusion equation

$$
\frac{\partial \rho(x, y, t)}{\partial t} = 0.01\left(\frac{\partial^3 \rho}{\partial |x|^{3}} + \frac{\partial^3 \rho}{\partial |y|^{3}}\right) + f_d(\rho, x, y, t) + f_c(\tilde{\rho}, x, y, t),
$$

(12)

combining with the homogeneous Dirichlet boundary condition

$$
\rho = 0.
$$

10
The pest source is modeled as a point disturbance $f_d$ to the fractional diffusion system \( \text{(12)} \) with its position at \((0.75, 0.35)\) and
\[
 f_d(t) = 20e^{-t}|_{(x=0.75,y=0.35)}.
\]

The pest source begins to move at \( t = 0 \) to the area \( \Omega \), 4 robots which can release the pesticide are deployed with initial positions at \((0.33, 0.33)\), \((0.33, 0.66)\), \((0.66, 0.33)\), \((0.66, 0.66)\), respectively. There are \( 29 \times 29 \) sensors evenly distributed in a square area \((0, 1)^2\) and they form a mesh over the area. In our simulation, we assume that once deployed, the sensors remain static. Fig. 7 shows the initial positions of the robots, the positions of the sensors and the position of the source.

In order to illustrate the different diffusion behavior and control effectiveness with different space fractional derivative order, we choose \( \beta = 2, 1.9, 1.7, 1.6, 1.5 \), respectively, and simulation time is \( t = 4 \) s. The simulation results are shown in Fig. 11. From the figure, we can see that \( \beta = 1.7 \) is the best spatial derivative order in equation \( \text{(12)} \) to model the space fractional diffusion process. Thus, we fix \( \beta = 1.7 \) in \( \text{(12)} \), and use the same control law in case I, the numerical simulation result is shown in Fig. 9. One can see that the space fractional diffusion of pest infestation is well controlled.
Figure 6: Evolution of $\rho(x, y, t)$, mobile robots positions and desired positions
Figure 7: Initial layout of actuators, sensors and obstacle.

Figure 8: Comparison of different evolution with different values of $\beta$. 

\begin{align*}
\beta &= 2 \\
\beta &= 1.9 \\
\beta &= 1.7 \\
\beta &= 1.6 \\
\beta &= 1.5
\end{align*}
6. Conclusions

In this paper, we consider the optimal dynamic location problem for a group of UAVs which can release pesticide chemicals, known as “mobile actuator” or “cropdusters” for neutralizing a pest diffusion process which are governed by time fractional order diffusion equation and space fractional diffusion equation. A new simulation platform has been built to prove the effectiveness of our algorithm and the problem of optimal dynamic motion of mobile actuators has been solved by CVT with better effectiveness. We also obtain the optimal value of derivative order for both time fractional order and space fractional order, when they are used to describe the corresponding anomalous diffusion process. In the future, we will extend our UAV-based optimal crop-dusting problem by using fractional order control law.

APPENDIX

To verify that FO-DiffMAS-2D works better for solving the anomalous diffusion problem, we use it to numerically solve the following two dimensional fractional diffusion equation.

**Example 1** Consider the following two dimensional time fractional dif-
fusion equation

\[ cD_{0,t}^{\alpha}u(x, y, t) = 0.01\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + f(x, y, t), \quad 0 \leq x \leq 1, 0 \leq y \leq 1, \ t \geq 0, \]  

(13)

with initial and boundary conditions

\[ u(x, y, 0) = 0, \quad 0 < x < 1, \ 0 < y < 1, \]  

(14)

\[ u(0, y, t) = u(1, y, t) = 0, \]  

(15)

where

\[ f(x, y, t) = \frac{2t^2 - \alpha}{\Gamma(3 - \alpha)}(x - x^2)(y - y^2) + 0.02t^2(x - x^2 + y - y^2). \]

The exact solution of the above equations is

\[ u(x, y, t) = t^2x(1 - x)y(1 - y). \]

Then, let \( h_x = h_y = \frac{1}{20} \) as spatial step size, \( \tau = \frac{1}{1250} \) as time step length, the comparisons between numerical and exact solutions at time \( t = 0.5 \) with different \( \alpha \) are displayed in the followings.

From Figs. 10-13, we can see that the numerical results are in good accordance with exact solutions.

**Example 2** Consider the space fractional diffusion equation

\[ \frac{\partial u(x, y, t)}{\partial t} = 0.01\left(\frac{\partial^\beta u}{\partial |x|^\beta} + \frac{\partial^\beta u}{\partial |y|^\beta}\right) + f(x, y, t), \quad 1 < \beta < 2, \ 0 \leq x \leq 1, 0 \leq y \leq 1, \ t \geq 0, \]  

(16)

together with initial and boundary conditions

\[ u(x, y, 0) = 0, \quad 0 < x < 1, 0 < y < 1, \]  

(17)

\[ u(0, y, t) = u(1, y, t) = 0, \]  

(18)

\[ u(x, 0, t) = u(x, 1, t) = 0, \]
where

\[
f(x, y, t) = 2t(x - x^2)(y - y^2) + \frac{0.02t^2}{2 \cos(\beta \pi/2)} \left\{ (y - y^2) \right. \\
\left. \frac{x^{1-\beta} + (1 - x)^{1-\beta}}{\Gamma(2 - \beta)} - \frac{2x^{2-\beta} + 2(1 - x)^{2-\beta}}{\Gamma(3 - \beta)} \right\} \\
+ (x - x^2) \left[ \frac{y^{1-\beta} + (1 - y)^{1-\beta}}{16 \, \Gamma(2 - \beta)} - \frac{2y^{2-\beta} + 2(1 - y)^{2-\beta}}{\Gamma(3 - \beta)} \right].
\]

Figure 10: Comparison of numerical and exact solution when \( \alpha = 0.6 \)

Figure 11: Comparison of numerical and exact solution when \( \alpha = 0.7 \)
The exact solution of above equation is

\[ u(x, y, t) = t^2 x(1 - x)y(1 - y). \]

Letting \( h_x = h_y = \frac{1}{20} \) as spatial step size, \( \tau = \frac{1}{250} \) as time step length, the comparisons between numerical and exact solutions at time \( t = 1 \) with different orders \( \beta \) are shown as follows.

We can see that the numerical solutions are agree with the exact solutions.
From the above two examples, it is easy to see that FO-DiffMAS-2D is efficient in solving two dimensional diffusion equations.
Figure 16: Comparison of numerical and exact solution when $\beta = 1.7$

Figure 17: Comparison of numerical and exact solution when $\beta = 1.9$

References


