Network Lifetime of Application-Specific Randomly Deployed Wireless Sensor Networks in Arbitrary Sensor Density

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Abstract

In many cases, sensors are randomly deployed in Wireless Sensor Networks (WSN), called Sensor-Randomly-Deployed WSN (SRD WSN). Several cluster-based routing protocols are provided to maximize network lifetime of SRD WSN in different sensor densities. LEACH performs better than direct routing in the density of 0.01. BCDCP excels LEACH in the density of 0.05. DMSTRP outperforms LEACH and BCDCP in the density of. However, simulation results under one or two kinds of sensor densities are not strong enough to prove the optimum of the routing protocols. In this paper, we give the general formulas to compute the network lifetimes of the above three routing protocols, discuss their optimal number of clusters, and compare their optimal network lifetime in arbitrary sensor densities. These formulas can provide more general design guidelines applicable to SRD WSN than simulation results under only one or two kinds of sensor densities.

1. Introduction

Recent advances in wireless communications and electronics have enabled the development of low-cost, low-power, small-sized wireless sensor nodes. Large numbers of these wireless sensor devices are feasible to connect to the physical world and construct a Wireless Sensor Network (WSN) for collecting useful user-cared data to the Base Station (BS). [1] Each sensor node has limited sensing, computation, and wireless communication capabilities.

Driven by batteries that has limited energy resource and cannot be recharged, sensor nodes must adopt energy efficient methods. Several cluster-based routing protocols are provided for energy efficiency. In direct routing, sensors all send their sensed data directly to BS. LEACH [1] provides a better routing than direct routing by grouping sensors into several clusters. In [1], simulation results show LEACH performs better than direct routing in the density of 0.01 with 100 nodes deployed in the network area of 100m × 100m. Another work proposes BCDCP routing protocol to improve LEACH further. In [2], simulation results show BCDCP excels LEACH in the density of 0.05 with 500 nodes deployed in the network area of 100m × 100m. Also in [3], it gives another routing protocol based on two-level Minimal Spanning Tree (MST) named DMSTRP, which use MST to replace club structure in BCDCP. Simulation results show DMSTRP outperforms LEACH and BCDCP in the density of less than 0.0011 with 100 nodes deployed in the network area of 300m × 300m.

However, simulation results under one or two kinds of sensor densities are not strong enough to prove the optimum of the routing protocols. Furthermore, except LEACH, both BCDCP and DMSTRP have not discussed the optimal number of clusters, which in fact directly affects the optimal network lifetime. In this paper, we give the general formulas to compute the network lifetimes of the above three routing protocols, discuss the optimal number of clusters, and compare the optimal network lifetime in arbitrary sensor densities. These formulas can provide more general design guidelines.

The paper is organized as follows: The basic assumptions and models are provided in section 2. In section 3, analysis of routing topologies for maximizing network lifetime in SRD WSN is given. In section 4, the optimal numbers of clusters in the three protocols of LEACH, BCDCP and DMSTRP are discussed and the maximum network lifetime is given. Finally, section 5 concludes this paper.

2. A model of SRD WSN
2.1. Assumptions

To simplify the problem, some assumptions are given as follows:

(1) All sensors are within the wireless communication range when they communicate with each other or with the BS. (2) All sensors have homogeneous sensing, computing and communication capabilities. (3) All sensors are randomly deployed in WSN, called Sensor-Randomly-Deployed WSN (SRD WSN). (4) BS is far away from the sensor networks and BS has infinity energy resource. (5) All sensors in the network have the same initial energy resource just like in LEACH, BCDCP and PEGASIS, and dissipate their energy resource at the same rate. (6) Network lifetime is defined as the time span from the deployment to the instant when the first sensor dies (or when the entire sensors die). According to (5), all the sensors would exhaust their energy resource at the same time. (7) Both the energy dissipation of sensing data and the energy dissipation for clustering are neglected. Compared with the power consumption of CPU and Radio, the power consumption of sensor part is so small that can be neglected [4]. Also, we suppose that all the clustering algorithms are run on the BS and no energy dissipation of clustering on sensor nodes. (8) The time span that BS collects the information from all the sensors once is defined as a round. In a round, each sensor has only one sensed data with the same packet size. (9) The sensors that receive the data combine one or more packets to produce a same-size resultant packet, and by this way, the number of data that need to send by radio is reduced. This is reasonable, because it is generally used to the scenario that there is much correlation among the data sensed by the different sensors. In [4], it shows that the wireless communication consumes the most of the energy. Thus, it is an elegant solution to use the processing capabilities of CPU to reduce the number of data that need to rout back to the BS through wireless communication. In data fusion, the number of data packets determines the amount of energy dissipation on CPU. (10) The energy dissipation of fusing one bit data is a constant value. In LEACH, BCDCP, and PEGASIS, they all adopt the constant value according to application experiments. In [5], it gives the method to fuse n packets of sound data into one packet data. Latter, in [6], it proves that the energy dissipation of fusing one bit sound data is about a constant value. Also, in [7], it gives the experiment results that the energy dissipation of fusing one bit image data is approximately a constant value.

2.2. A Model of Maximum Network Lifetime

In this paper, there are 3 energy consumption modes: transmitting data, receiving data, and fusing data.

We use a radio energy model the same as that in [1,2], in which the energy dissipation \( E_T(k,d) \) of transmitting \( k \)-bit data between two nodes separated by a distance of \( d \) meters is given as follows:

\[
E_T(k,d) = \begin{cases} 
  k(E_{elec} + \epsilon_{FS} \times d^2) & (d < d_0) \\
  k(E_{elec} + \epsilon_{MP} \times d^4) & (d > d_0)
\end{cases}
\]  

(1)

where \( d_0 = \sqrt{\frac{\epsilon_{FS}}{\epsilon_{MP}}} \), \( E_{elec} \) denotes electronic energy, \( \epsilon_{FS} \) and \( \epsilon_{MP} \) denote transmit amplifier parameters corresponding to the free-space and the two-ray models. The energy cost incurred in the receiver of the destination sensor node is given by \( E_R(k) = k \times E_{elec} \).

Also, the energy dissipation of fusing \( k \)-bits data is given by

\[
E_F(k) = k \times E_{df}.
\]

(3)

where \( E_{df} \) is the energy dissipation of fusing 1 bit data.

Given \( N \) sensors randomly deployed in the network area of \( S = M \times M \), and the average distance between any sensor and the BS is \( d_{nBS} \). Each sensor has initial energy resource of \( E_0 \). The network lifetime of WSN can be denoted as the number of rounds given by

\[
L = \frac{NE_0}{E_{av}}
\]

(4)

where \( NE_0 \) is the total energy resource, and \( E_{av} \) is the average network energy dissipation in a round.

3. Analysis of routing topologies for maximizing network lifetime

3.1 LEACH routing protocol

In Direct routing, all the sensors directly transmit sensed data to the BS. Because the BS is far away from the WSN, the energy dissipation of transmitting data in the WSN is too big to be accepted, when compared to LEACH in [1].

LEACH’s general topology is shown in Fig. 1, in fact the topology will be changed dynamically in each round. LEACH groups sensors into local clusters, and chooses one (Cluster Head) CH in each cluster. A club connects all the sensors intra a cluster, and the CHs and the BS are connected by a club, too. LEACH reduces the transmitting distances of each sensor, also the CH collect all the data intra cluster and fuse the data into only one packet to reduce the number of data
that need to transmit to the BS. Therefore, LEACH gets better energy efficiency than Direct Routing.

According to [1], the average transmitting square distance of non-CH sensors in a club area of $Q$ can be given as

$$E_{club}(d_{nach}) = Q / 2\pi.$$  

Thus, given $M$ sensors in a club area of $Q$, the energy dissipation of this club is given by

$$E_{club}(k,M,Q) = k(2(M-1)E_{elec} + ME_{df} + \frac{e_{fs}Q}{2\pi}(M-1)).$$  

In [1], given $N$ sensors deployed in WSN area of $S$, it gives the optimal number of CHs by

$$N_{ch} = \sqrt{\frac{e_{fs}SN}{2\pi\epsilon_{MP}}} / d_{obs}^2.$$  

Thus, we can get the maximum network lifetime of LEACH as follows:

$$L_{LEACH-opt} = E_{c} / k((2E_{elec} + E_{df} + d_{obs}^2 \sqrt{\frac{2e_{MP}S}{N\pi}})).$$  

Fig. 1 Club-club routing in LEACH.

### 3.2 BCDCP routing protocol

BCDCP gives a better solution for energy efficiency. The club-MST structure of BCDCP is shown in Fig. 2. In BCDCP, it adopts club structure intra cluster like LEACH, connects CHs with MST, and choose a leader among CHs to send the data to the BS.

Energy dissipation formula of club in BCDCP is the same as that in LEACH given in eq. (8). If we can get the energy dissipation formula of MST, the network lifetime of BCDCP can be given.

#### 3.2.1 Energy Dissipation Formula of MST

1) **Average value of edges**

We suppose that sensor nodes are independently and uniformly distributed. According to [8], the sum of edge lengths in a MST, $L_{MST}$, is given by

$$\lim_{n \to \infty} \frac{L_{MST}}{n^{(k-1)/k}} = \beta_{MST}(k)$$  

where $k$ is the number of dimensions. In this paper, let $k = 2$, then

$$\bar{d} = \frac{L_{MST}}{n-1} = \lim_{n \to \infty} \frac{\beta_{MST}(2)n^{1/2}}{n-1} = \lim_{n \to \infty} \frac{\beta_{MST}(2)}{n^{1/2}}.$$  

Also, let $S = M \times M$, the sensor density is

$$\sigma = \frac{n}{M^2} \Rightarrow n = \sigma M^2.$$  

From eq. (10) and eq. (11),

$$\bar{d} = \lim_{n \to \infty} \frac{\beta_{MST}(2)}{M^{1/2}}.$$  

2) **Number of leaf nodes**

Using Monte Carlo to estimate of frequencies, it gives the number of leaf nodes in [10] as

$$L \to 2N / 9.$$
3) Formula of energy dissipation in MST

All the non-leader sensors in MST transmit their sensed data along the MST-connected path to reach the leader sensor. We provide a parallel scheduling transmitting algorithm in [3]. Each sensor receives the data transmitted from others and then fuses the received data with its own. Thus, suppose the total number of sensors is \( N \) and the number of leaf sensors is \( L \), the energy dissipation of fusing \( k \)-bits data is

\[
E[ED] = (2N - L - 1)kE_{df}. \tag{16}
\]

The receiving energy dissipation in MST is

\[
E[ER] = (N - 1)kE_{elec}. \tag{17}
\]

The transmitting energy dissipation in MST is

\[
E[ET] = (N - 1)kE_{elec} + \varepsilon_{FS} k \sum_{i=1}^{N-1} d_i^2. \tag{18}
\]

Thus, the whole network energy dissipation is

\[
E = E[ED] + E[ER] + E[ET]. \tag{19}
\]

Write \( \phi(N) = \left( \sum_{i=1}^{N-1} d_i^2 \right)/(N - 1) \). According to eq. (14), eq. (15) and eq. (19), given \( N \) sensors deployed in the MST-connected cluster area of \( S \), the formula of energy dissipation in each round is given by

\[
E_{\text{MST}}(k, N, S) = k(2(N - 1)E_{elec} + \frac{16N}{9} - 1)E_{df} + (N - 1)\varepsilon_{FS}A^2 S / N. \tag{20}
\]

3.2.2 Network Lifetime of BCDCP

In BCDCP, the non-CH sensors send data to CHs in \( N_{CH} \) clubs, CHs transmit, fuse, and receive data in one MST, and the leader of CHs sends final data to BS, thus the energy dissipation of BCDCP is given by

\[
E_{\text{BCDCP}} = N_{CH} E_{\text{club}}(k, \frac{N}{N_{CH}}, S / N_{CH}) + E_{\text{MST}}(k, N_{CH}, S) + E_{\text{FS}}(k, d_{\text{obs}}) \tag{21}
\]

where

\[
\alpha = (2N - 1)E_{elec} + (N - 1)E_{df} + \varepsilon_{MP} d_{\text{obs}}^4 + \varepsilon_{FS} S(A^2 - 1/(2\pi)), \quad \beta = 16E_{df} / 9, \quad \gamma = \varepsilon_{FS} S(N/(2\pi) - A^2). \quad \text{The optimal number of clusters can be found by setting the derivative of } E_{\text{BCDCP}} \text{ with respect to } N_{CH} \text{ to zero,}
\]

\[
N_{CH} = \sqrt{\frac{\gamma}{\beta}} = \sqrt{\frac{9\varepsilon_{FS}S(N/(2\pi) - A^2)}{16E_{df}}}. \tag{22}
\]

4. Maximum network lifetime

We now give some cases to explain the formulas of both maximum network lifetime and optimal number of clusters in LEACH, BCDCP, and DMSTRP. And suppose the conditions as follows: the total energy resource on each sensor is \( E_0 = 2J \), a packet of data is \( k = 20 \text{kbits} \) and \( d_{\text{obs}} = 125 \text{m} \). Similar to [1,2], let \( E_{df} = 5nJ \), \( E_{FS} = 10pJ \text{ / bit} / m^2 \), \( E_{elec} = 50nJ \), and \( \varepsilon_{MP} = 0.0013 \text{ pJ / bit} / m^2 \).
4.1 Optimal number of clusters

Through analysis of the maximum network lifetime under different dynamic network structure, it shows that the optimal number of clusters determines the maximum network lifetime. We compare the theoretical values computed by eq. (22) to the simulation results of the optimal number of clusters in BCDCP as table 1 shows. It proves that the theoretical value of optimal number of clusters in BCDCP is similar to the simulation results.

<table>
<thead>
<tr>
<th>N</th>
<th>M(m)</th>
<th>100</th>
<th>80</th>
<th>60</th>
<th>40</th>
<th>20</th>
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<td>10.5</td>
<td>7.9</td>
<td>5.3</td>
<td>2.6</td>
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<tr>
<td></td>
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<td>11</td>
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<td>3</td>
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<tr>
<td>200</td>
<td>Theory</td>
<td>18.8</td>
<td>15.0</td>
<td>11.3</td>
<td>7.5</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
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<td>19</td>
<td>16</td>
<td>12</td>
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</tr>
<tr>
<td>300</td>
<td>Theory</td>
<td>23.0</td>
<td>18.4</td>
<td>13.8</td>
<td>9.2</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
<td>24</td>
<td>19</td>
<td>14</td>
<td>11</td>
<td>5</td>
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<tr>
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<td>Theory</td>
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<td>21.3</td>
<td>16.0</td>
<td>10.7</td>
<td>5.3</td>
</tr>
<tr>
<td></td>
<td>Simulation</td>
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<td>22</td>
<td>18</td>
<td>12</td>
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</tr>
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<td>17.9</td>
<td>11.9</td>
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<tr>
<td></td>
<td>Simulation</td>
<td>30</td>
<td>24</td>
<td>19</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

The optimal number of clusters in BCDCP is very useful. In [2], it proposes BCDCP protocol, but no optimal number of clusters is discussed. In a SRD WSN of $N = 500$ sensors deployed in network area of $S = 10^4 m^2$, using eq. (22), the optimal number of clusters in BCDCP is 29. Also, without using the optimal number of clusters, the simulation results of network lifetime is limited. For example, with $N = 100$ and $S = 9 \times 10^4 m^2$ in [3], it adopts 9 clusters in BCDCP to compare the network lifetime of BCDCP with DMSTRP, but the optimal number of clusters in BCDCP under this condition is 39.

By using eq. (8), eq. (21), and eq. (24), Fig. 4 shows the comparison curves of LEACH, BCDCP and DMSTRP under the condition of $N = 100$ and $S = 9 \times 10^4 m^2$, because in this condition DMSTRP outperforms BCDCP without considering the optimal number of clusters. Also, Fig 4 shows the average network energy dissipation varies with the number of clusters.

In Fig. 4, the curve of BCDCP and the curve of DMSTRP are intersected at the point of $N_{CH} = 17$ computed by eq. (23) and eq. (24) when setting $E_{BCDCP} = E_{DMSTRP}$. Thus, if $N_{CH} < 17$, then $E_{BCDCP} > E_{DMSTRP}$. This can explain the results in [3].

4.2 Discussion of maximum network lifetime in arbitrary sensor densities

Without losing generality, we give three cases under three different representative sensor densities: modest sensor density of 0.01 ($N = 100, S = 100 \times 100 m^2$), high sensor density of 0.05 ($N = 500, S = 100 \times 100 m^2$), and low sensor density of 0.001 ($N = 100, S = 300 \times 300 m^2$). Maximum network lifetimes of LEACH, BCDCP and DMSTRP are compared under above sensor densities in Fig.5-Fig.7.

The comparison curves of LEACH, BCDCP and DMSTRP in Fig.5-Fig.7 give the following evidences:

1. In arbitrary sensor density, the optimal network lifetime of BCDCP outperforms both that of LEACH and DMSTRP, and achieves the maximum network lifetime.
2. According to eq. (9), the optimal network lifetime of LEACH is only related to sensor density, and the curves of LEACH are always horizon lines in the three figures.
3. Generally, DMSTRP performs better than LEACH in terms of network lifetime, except a few cases in Fig. 6.
4. The sensor density is lower, the DMSTRP curve is nearer to BCDCP curve; the sensor density is higher, the DMSTRP curve is nearer to LEACH curve.

In Fig. 4, the lowest point in the curve of BCDCP is lower than the lowest point in the curve of LEACH. Also, DMSTRP has only highest point, and the curve of DMSTRP is near to a horizon line. In a word, the optimal network energy dissipation in BCDCP is the best compared with LEACH and DMSTRP in the sensor density of 0.001 ($N = 100, S = 300 \times 300 m^2$).
5. Conclusions

This paper presents the network lifetime formulas of three main cluster-based routing protocols in sensor randomly deployed wireless sensor networks. The optimal number of clusters is computed and is taken as a very important factor to get the maximum network lifetimes under different network topologies. General formulas to compute maximum network lifetimes of LEACH with Club-Club topology, BCDCP with Club-MST topology, and DMSTRP with MST-MST topology are provided. Three cases under different representative sensor densities show that Club-MST network topology in BCDCP is the best one to maximize the network lifetime, compared with LEACH and DMSTRP. The formulas of both optimal number of clusters and maximum network lifetimes in this paper can provide more general design guidelines applicable to SRD WSN than simulation results under only one or two kinds of sensor densities.

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