A Practical PID-based Scheme for the Collaborative Driving of Automated Vehicles

Packiaraj Xavier and Ya-Jun Pan

Abstract—For automated vehicles operated in platoons, it is important to see what information is necessary to be communicated between vehicles to ensure safety and performance in maneuvering, and how complicated the controllers are to be implemented. In this paper, we address the platoon problem by using the decentralized proportional, integral and derivative (PID) control approach for the applications of autonomous vehicles, which are modelled as an interconnected system in the form of the well known bicycle model. The control inputs for each vehicle are the traction force and steering angle. The practical collaborative driving approach consists of two main scenarios: the leader-follower platoon and the overtaking maneuver. Only the relative position and following angle between two adjacent vehicles are required for the controller design. Simulation results and dynamic visualization using virtual reality toolbox are demonstrated to show the effectiveness of the simple and practical approach.

I. INTRODUCTION

Over the past decade, increased attention has been paid to the research theme of automated vehicles in transportation systems [1]. This was motivated by various advantages for moving vehicles in platoons such as increasing road capacity and efficiency, reducing congestion energy consumption and pollution, and enhancing safety and comfort [2]. Several good examples of recent efforts are as follows in the literature. The work in [3] (PATH - The California Partners for Advanced Transit and Highways) proposed an integrated longitudinal and lateral control system for the operation of automated vehicles in platoons with the experiment of using eight vehicles and magnetometers on a two-lane freeway. In [1], five automated vehicles equipped with inter-vehicle communication devices were demonstrated by using differential GPS and dedicated short-range communications, in which the platoon was able to conduct maneuvers such as stop-and-go, platooning, merging and obstacle avoidance on an oval shaped test track.

For the task of vehicles in platoons, from the control aspect, it can be treated as an interconnected system with finite number of subsystems. The control objective is to keep a safe distance between any vehicles and its neighbors in the platoon while tracking the trajectory either generated by the leading vehicle or by the desired formation trajectories. Several decentralized control strategies have been proposed for collaborative driving systems [4]. The platoon setup consists of longitudinal and lateral controllers. The longitudinal controller determines the speed of the vehicle and the following distance of trailing vehicles. The lateral control falls in dealing with the lane change and the path following tasks. As well, overtaking tasks have been discussed in [5]. Overtaking is one of the most dangerous highways maneuvers. When overtaking the vehicles, the vehicle completes the maneuver vehicle performs three actions. The first action involves the overtaking vehicle moving out of the current lane, the second is to actually overtake the previous vehicle and finally the vehicle must return to the original lane.

While the efforts in the literature for coordinated maneuvering have different focuses based on inter-vehicle communications, such as the string stability, sensor issues and the feasibility of cooperative driving cases in limited conditions. In this paper, we aim to propose a practical collaborative driving approach based on simple PID controllers and well known bicycle models for simulations. The decentralized control approach for the application of autonomous vehicles requires minimum information (the relative position and following angle) from each other to design the local controller for each vehicle. The practical collaborative driving approach consists of the following two categories: the leader-follower platoon and overtaking maneuver.

In the overtaking maneuvers, we introduced a virtual vehicle strategy to overcome the influence of the non-holonomic constraint of vehicles in the lane change maneuver based on the relative model. Thus the controller designs are same as that of the leader-follower formation which again simplified the lateral control design and minimize the number of the inter-vehicle communication variables. Simulation results further confirm the effectiveness of the proposed approach.

II. PROBLEM FORMULATION

A. Absolute and Relative Bicycle Model Formulations

According to the model formulation in [4] and [6], the leading vehicle in the absolute coordinate (as shown in Fig.1) can be described as the following bicycle model:

\[
\begin{align*}
\dot{x}_1 &= \underline{v}_1 \cos(\theta_1^e + \beta_1) \\
\dot{y}_1 &= \underline{v}_1 \sin(\theta_1^e + \beta_1) \\
\dot{\theta}_1^e &= \dot{\omega}_1 \\
M \dot{\omega}_1 &= \bar{f}_v^1 \cos(\beta_1) + \bar{f}_l^1 \sin(\beta_1) \\
\beta_1 &= -\frac{1}{\underline{M} \omega_1} \frac{\bar{f}_v^1 \sin(\beta_1)}{\bar{M}_1} \cos(\beta_1) - \omega_1 \\
\dot{\omega}_1 &= \frac{\bar{f}_v^1}{\bar{M}_1}
\end{align*}
\]

where the parameters are defined as in Appendix A. For any following vehicles with the relative position related to the
The following terms as,

\[
\tan(\beta_i^f) = \tan(\beta_i) + \frac{L_f \omega_i}{v_i \cos(\beta_i)} \Rightarrow \beta_i^f \approx \beta_i + \frac{L_f \omega_i}{v_i} \\
\tan(\beta_i^r) = \tan(\beta_i) - \frac{L_r \omega_i}{v_i \cos(\beta_i)} \Rightarrow \beta_i^r \approx \beta_i - \frac{L_r \omega_i}{v_i}.
\]

The control inputs are as follows: \( F_i^f \) - the front wheel traction force, \( F_i^r \) - the rear wheel traction force, \( \delta_i^f \) - the front wheel steering angle, \( \delta_i^r \) - the rear wheel steering angle.

In this paper, it is assumed that the rear wheel is not used for driving and steering, which means \( F_i^r = 0 \) and \( \delta_i^r = 0 \). In the relative model, there are external inputs needed from the vehicle ahead in the immediate neighborhood.

### B. Absolute Positions of the Following Vehicles in Platoons

Since in the relative model (2), the absolute position of the following vehicles are not shown. However, it can be derived according to the relative model formulation. For any following vehicles, the general formula for the absolute position of the \( i^{th} \) vehicle (starting from the \( 2^{nd} \) vehicle) in the platoon can be represented as,

\[
\begin{align*}
  x_i &= x_{i-1} + R_i \cos \alpha_i, \\
  y_i &= y_{i-1} + R_i \sin \alpha_i, \\
  i &= 2, 3, \cdots, n,
\end{align*}
\]

where \( x_i \) and \( y_i \) are the absolute coordinates of the \( i^{th} \) vehicle (Fig.1); \( \alpha_i \) is the angle described as follows. For the \( 2^{nd} \) vehicle, it is calculated as \( \pi - [\theta_2 - \theta_1 - \phi_2] \), where \( \theta_1 \) is the absolute orientation of Vehicle 1, \( \theta_2 \) is the absolute orientation of Vehicle 2, and \( \phi_2 \) is the following angle of Vehicle 2 with respect to Vehicle 1. For any vehicles with \( i = 3, 4, \cdots, n \), \( \alpha_i = \alpha_{i-1} - \phi_{i-1} - \theta_i + \phi_i \), where \( \theta_i \) is the absolute orientation of Vehicle \( i \) and \( \phi_i \) is the following angle of Vehicle \( i \) with respect to Vehicle \( i - 1 \).

### III. PID-BASED COLLABORATIVE DRIVING APPROACH

Two situations are considered in this section. The first case is on the vehicle following control with one leading vehicle and finite numbers of following vehicles. The second case is on the lane overtaking collaborative driving approach for the dynamic lane switching maneuvering.

The control objective is to maintain a safe inter-vehicle spacing within a lane, and the control of a vehicle entering and exiting from the platoons (overtaking task). The controller to be used for the platoon is assumed to be fully decentralized with limited communication existing between the vehicles. Each vehicle can only measure the relative distance and the following angle between itself and the leading vehicle. Assuming that only the front wheel steering and front wheel drives are measured. The schematic diagram is as shown in Fig.3.

#### A. Following Controller Design

The controllers designed in the following control tasks are decentralized PID controllers. It consists of two separate controllers, one is the longitudinal controller and the another one is the lateral controller. Define the desired space between adjacent vehicles as \( R_d \) and the desired following angle as
For the following control case, \( \phi_d = 0 \) holds. Then the error signals are given as
\[
e_i^\phi = \phi_i - \phi_d, \quad e_i^R = R_i - R_d, \quad i = 2, \ldots, n, \tag{4}
\]
where \( \phi_i \) and \( R_i \) are the measured following angle and relative distance between the adjacent vehicles.

The longitudinal controller of the following vehicles are designed as a simple PID controller of the form,
\[
F_i^l = \left( K_{P,i}^R + \frac{K_{T,i}^R}{s} + K_{D,i}^R s \right) e_i^R, \quad i = 2, \ldots, n, \tag{5}
\]
where \( K_{P,i}^R, K_{T,i}^R \) and \( K_{D,i}^R \) are designed gains, to control the velocity as well as to maintain the constant distance. The longitudinal controller has two main tasks. One is the steady state maneuvers of speed control or vehicle following and the other is the transient maneuvers of splitting from a platoon and joining the platoon.

The lateral controller assists the vehicles in lane change and curvilinear path following. In the curvilinear path following task, the lateral control (e.g. the steering angle input) ensures the following vehicle to follow the curvilinear path that the leading vehicle is undergoing.

The lateral control input is also designed as a PID controller
\[
\delta_i^l = \left( K_{P,i}^R + \frac{K_{T,i}^R}{s} + K_{D,i}^R s \right) e_i^\phi, \quad i = 2, \ldots, n, \tag{6}
\]
where \( K_{P,i}^R, K_{T,i}^R \) and \( K_{D,i}^R \) are designed gains.

### B. Collaborative Overtaking Maneuvers

The collaborative overtaking maneuvers needs a systematic strategies in a high level controller. The information exchanged between the adjacent vehicles are not enough to accomplish this task. In the proposed practical approach, the following coordination scenarios are considered.

As shown in Fig.4 and Fig.5, the splitting maneuver strategy can be summarized as: Vehicle \( i \) sends message to Vehicle \( i - 1 \) and Vehicle \( i + 1 \) to initiate the splitting task. Then Vehicle \( i + 1 \) communicates with Vehicle \( i - 1 \). Vehicle \( i \) tries to track a virtual vehicle for its splitting task. The virtual vehicle’s control inputs \( F \) and \( \delta \) would be designed according to the actual lane width etc. After it arrives on the straight lane, it accelerates.

As shown in Fig.6 and Fig.7, the merging maneuver strategy can be summarized as: Vehicle \( i \) sends message to Vehicle \( i - 1 \) and Vehicle \( i + 1 \) to initiate the merging task. Then Vehicle \( i + 1 \) communicates with Vehicle \( i - 1 \). Vehicle \( i \) tries to track a virtual vehicle for its merging task. The virtual vehicle’s control inputs \( F \) and \( \delta \) would be designed according to the actual lane width etc. After it arrives on the straight lane, it accelerates.

Then Vehicle \( i + 1 \) communicates with Vehicle \( i - 1 \) and to leave enough space for Vehicle \( i \) to join the platoon. Vehicle \( i \) tries to track a virtual vehicle for its merging task. The virtual vehicle’s control inputs \( F \) and \( \delta \) would be designed according to the actual lane width as well.
vehicle must overtake the second vehicle, it can be done by combining splitting and merging maneuvers. While splitting, instead of following the second vehicle, the third vehicle should follow the virtual vehicle which guides it into the adjacent lane. Now the second vehicle and the third vehicle are at the same position (parallel) in different lanes. The second vehicle is commanded to decrease the speed so that the following distance between the second and the leading vehicle is increased. At the same time the third vehicle increases its speed so that it overtakes the second vehicle. After overtaking, the third vehicle must return to the original lane. In our coordination approach, the third vehicle is again following the virtual vehicle and merge to the original lane.

\[ K_{p,i}^R = K_{i,2}^R = 1, \quad K_{i,2}^\phi = 0.5, \quad K_{i,2}^D = 0.2, \]
\[ K_{p,i}^R = K_{p,i-1}^R + 1, \quad K_{i,i}^R = K_{i,i}^R + 1, \]
\[ K_{p,i}^\phi = K_{p,i-1}^\phi + 0.2, \quad K_{i,i}^\phi = K_{i,i}^\phi + 0.1, \] (7)

with \( i = 3, \cdots, 5 \); and all derivative gains are zeros.

The second set is for the situation when the following angle \( \phi_d \neq 0 \):

\[ K_{p,i}^R = 4, \quad K_{i,2}^R = 0.01, \quad K_{D,i}^R = 5, \]
\[ K_{p,i}^\phi = 2, \quad K_{i,2}^\phi = 0.01, \quad K_{D,i}^\phi = 3, \]
\[ K_{p,i}^R = K_{p,i-1}^R + 1, \quad K_{i,i}^R = K_{i,i}^R + 0.01, \]
\[ K_{D,i}^R = K_{D,i-1}^R + 1, \quad K_{i,i}^\phi = K_{i,i}^\phi + 1, \]
\[ K_{D,i}^\phi = K_{D,i}^\phi + 0.01, \quad K_{D,i}^\phi = K_{D,i}^\phi + 1, \] (8)

with \( i = 3, 4, 5 \). Hence the controller gains in the following vehicles are not the same and the gains are larger while they are close to the end of the platoon.

1) Vehicles in the platoon on a straight lane: In the following example, the platoon of five identical vehicles are considered. The following two parameters are set as \( R_d = 10 m \) and \( \phi_d \) with \( 0^\circ \) and \( 22.5^\circ \) separately. As shown in Fig.9, the following vehicles are following the lead vehicle with the relative distance \( (R_d) \) of 10 m and the following angle \( (\phi_d) \) of \( 0^\circ \).

When the following angle is not zero but \( 22.5^\circ \) (with gains in (8)), the platoon can also be formed with constant speed and space between each adjacent vehicle in short time as shown in Fig.10. The following angle is \( \phi = 22.5^\circ \) which is specified in the figure. However, at the beginning of the platoon, there is a transient time that the five vehicles are settling down to the steady state. Note that the fifth vehicle takes longer time because its motion is affected by the motions of the four vehicles in front.

2) Vehicles in the platoon on a curvilinear lane: A platoon of five identical vehicles on a curvilinear lane is considered as well. Similarly as in the straight lane case, the following vehicles are required to track the desired trajectory with \( R_d = 10 m \) and \( \phi_d = 22.5^\circ \).

In the simulation results, the following vehicles are following the leading vehicle with the relative distance \( R_d = 10 m \) as shown in Fig.11 and Fig.12 with \( \phi = 22.5^\circ \) and gains in (8). Specifically in Fig.12, even though it is a curvilinear lane, the five vehicles can form the platoon with constant distance and desired following angle after a short time.

During the maneuvers, the controllers are same as in (5) and (6). The desired following angle between the virtual vehicle and the vehicle undertaking maneuvers is always set as zero. This is actually an in-line tracking instead of side tracking in which the performance is limited by the non-holonomic constraint of the bicycle model for the vehicle. Note that the lane width should be known as a prior.

Remark 1: Since the controller depends on the PID controller design, how to tune the gains is a very important issue. The control gains for each vehicle should not be the same because the vehicles at the end of the platoon is influenced by the accumulated space errors of the vehicles in the string ahead [8]. In this paper, the gains in the \( i \)th vehicle is larger than the gains in the \((i-1)\)th vehicle. How to get a set of optimal control gains needs to be further investigated.

IV. SIMULATION RESULTS

In this section, two examples are basically illustrated to show the effectiveness of the proposed approach. In the first example, five identical vehicles are required to accomplish the platoon task. While in the second example, three identical vehicles are shown to illustrate the overtaking maneuver. In both cases, the parameters of the individual vehicle are set as follows: \( L^f = 0.8 m, \quad L^r = 1 m, \quad M = 50 Kg, \quad B = 40 \frac{N}{m/s^\cdot \cdot}, \quad J = 500 \frac{m^2}{kg}, \quad J = 100 \frac{m^2}{rad/s^2}, \quad \mu_f = 50 \frac{N}{rad/s^2}, \quad \mu_r = 450 \frac{N}{rad/s^2} \). This set of parameters are chosen for simulation purpose for vehicles with four wheels steering and driving, autonomous vehicles with intelligent sensors.

A. Vehicle Following Example

In this section, the simulation is performed by using five identical vehicles to show the effectiveness of the controller proposed in this paper. Two cases are considered here. The first one is to drive the vehicles to form the platoon on a straight line while the second case considered is required to work on a curvilinear path.

The longitudinal controller and the lateral controller designed in (5) and (6) are used for the following vehicles with \( i \geq 2 \). There are two sets of control gains.

The first set is for the situation when the following angle is as \( \phi_d = 0 \):

\[ K_{p,i}^R = 4, \quad K_{i,2}^R = 1, \quad K_{i,2}^\phi = 0.5, \quad K_{i,2}^D = 0.2, \]
\[ K_{p,i}^R = K_{p,i-1}^R + 1, \quad K_{i,i}^R = K_{i,i}^R + 1, \]
\[ K_{p,i}^\phi = K_{p,i-1}^\phi + 0.2, \quad K_{i,i}^\phi = K_{i,i}^\phi + 0.1, \]

with \( i = 3, \cdots, 5 \); and all derivative gains are zeros.

The second set is for the situation when the following angle \( \phi_d \neq 0 \):

\[ K_{p,i}^R = 4, \quad K_{i,2}^R = 0.01, \quad K_{D,i}^R = 5, \]
\[ K_{p,i}^\phi = 2, \quad K_{i,2}^\phi = 0.01, \quad K_{D,i}^\phi = 3, \]
\[ K_{p,i}^R = K_{p,i-1}^R + 1, \quad K_{i,i}^R = K_{i,i}^R + 0.01, \]
\[ K_{D,i}^R = K_{D,i-1}^R + 1, \quad K_{i,i}^\phi = K_{i,i}^\phi + 1, \]
\[ K_{D,i}^\phi = K_{D,i}^\phi + 0.01, \quad K_{D,i}^\phi = K_{D,i}^\phi + 1, \]

with \( i = 3, 4, 5 \). Hence the controller gains in the following vehicles are not the same and the gains are larger while they are close to the end of the platoon.
desired platoon task cannot perform well, which motivated the work of using a virtual leading vehicle for the overtaking maneuvers in this example. Hence the control gains are selected as in (7).

In this example, we chose three vehicles to demonstrate the overtaking maneuver. As shown in Fig.13, Vehicle 3 leaves the platoon and follows the virtual vehicle (not drawn in the diagram) and then switches to another lane. The platoon keeps the same motion forward.

Fig.14 shows that the merging action of Vehicle 3. The vehicle can merge to the platoon successfully by following a virtual leading vehicle with zero following angle. As shown in Fig.15.(a), Vehicle 3 sends a message (intention to merge) to Vehicle 2 after they are on the same position (x-dir) on the two lanes and ready to merge. Then Vehicle 2 slows down to leave more space \( R_d = 20 \) m for Vehicle 3 to merge. Vehicle 3 only starts the motion of merging after Vehicle 2’s action. Fig.15.(b) shows the moment of Vehicle 3 entering the team. After a while, the desired space between Vehicle 1 and Vehicle 3 is \( R_d = 10 \) m, the same as the distance between Vehicle 3 and Vehicle 2.

V. DYNAMIC VISUALIZATION RESULTS

In this section, 3D animation by using virtual reality toolbox in Matlab is studied by using three identical vehicles to show the effectiveness of the controller proposed in this paper. This would help to see the performance of the simulation results dynamically, i.e., making sure there are no possible collisions between vehicles. The vehicles and
the virtual world are developed by using solid works and by virtual reality toolbox in Matlab. Only the collaborative driving case is shown here due to limited space.

Fig. 16 shows the overtaking maneuvers of the straight lane platoon. Vehicle 3 leaves the platoon and follows the virtual vehicle (not drawn in the diagram) and then switches to another lane. The platoon keeps the same motion forward. The initial positions of all the three vehicles are shown as in (a). Fig.16(b), (c) and (d) show the splitting and merging maneuvers of the vehicles. In Fig.16(e) and (f), the front and side views of the final position clearly show all the vehicles in the platoon maintains the $R_d = 10\, m$ and $\phi_d$ with $0^\circ$. During the overtaking maneuvers, Vehicle 3 sends a message to Vehicle 2 for its intention of overtaking. The results shown in the animation explain the effectiveness of the proposed practical approach for overtaking tasks as well.

In this paper, a practical and simple decentralized control scheme is designed for automated vehicles in platoons. The virtual vehicle tracking method is proposed for overtaking maneuver tasks. Fundamental PID controllers are used for the longitudinal and lateral controllers. The coordination between the vehicles depends on the measurement of relative distance and the following angle. Simulation and animation results are carried out and bicycle models are adopted for vehicles as well as in the simulations.

**Appendix A: Vehicle Parameters and Descriptions**

- $x^a$ Absolute $x$ position
- $y^a$ Absolute $y$ position
- $\theta^a$ Absolute orientation
- $v$ Longitudinal velocity of the center of mass
- $\omega$ Angular velocity of the center of mass
- $\beta$ Sideslip angle
- $F_f^r$ Front wheel traction force input
- $F_f^r$ Rear wheel traction force input
- $f_f^r$ Front wheel lateral force (function)
- $f_f^r$ Rear wheel lateral force (function)
- $\mu_f^r$ Front wheel lateral friction coefficient (scalar)
- $\mu_f^r$ Rear wheel lateral friction coefficient (scalar)
- $L_f^r$ Distance between the front wheel and the center of mass
- $L_r^r$ Distance between the rear wheel and the center of mass
- $\delta_f^r$ Front wheel steering angle input
- $\delta_r^r$ Rear wheel steering angle input
- $M$ Mass of the vehicle
- $R$ Linear viscous Damping ratio (Air resistance)
- $J$ Moment of inertia about the center of mass
- $b$ Angular damping ratio
- $R_i$ Relative distance from the previous agent
- $\Phi_i$ Angle between the heading direction of a vehicle and vehicle-to-vehicle connection line
- $\theta_i$ Relative orientation

**References**


