Total Inter-Carrier Interference Cancellation for MC-CDMA System in Mobile Environment

Xue Li¹, Ruolin Zhou¹, Steven Hong², and Zhiqiang Wu¹
Dept of EE, Wright State University¹, Dept of EE, Stanford University²

Abstract—Multi-carrier code division multiple access (MC-CDMA) has been considered as a strong candidate for next generation wireless communication system due to its excellent performance in multi-path fading channel and simple receiver structure. However, like all the multi-carrier transmission technologies such as OFDM, the inter-carrier interference (ICI) produced by the frequency offset between the transmitter and receiver local oscillators or by Doppler shift due to high mobility causes significant BER (bit error rate) performance degradation in MC-CDMA system. Many ICI cancellation methods such as windowing and frequency domain coding have been proposed in the literature to cancel ICI and improve the BER performance for multi-carrier transmission technologies. However, existing ICI cancellation methods do not cancel ICI entirely and the BER performance after ICI cancellation is still much worse than the BER performance of original system without ICI. Moreover, popular ICI cancellation methods like ICI self-cancellation reduce ICI at the price of lowering the transmission rate and reducing the bandwidth efficiency. Other frequency-domain coding methods do not reduce the data rate, but produce less reduction in ICI as well. In this paper, we propose a novel ICI cancellation scheme that can eliminate the ICI entirely and offer a MC-CDMA mobile system with the same BER performance of a MC-CDMA system without ICI. More importantly, the proposed ICI cancellation scheme (namely Total ICI Cancellation) does not lower the transmission rate or reduce the bandwidth efficiency. Specifically, by exploiting frequency offset quantization, the proposed scheme takes advantage of the orthogonality of the ICI matrix and offers perfect ICI cancellation and significant BER improvement at linearly growing cost. Simulation results in AWGN channel and multi-path fading channel confirm the excellent performance of the proposed Total ICI Cancellation scheme in the presence of frequency offset or time variations in the channel, outperforming existing ICI cancellation methods.

I. INTRODUCTION

Multi-carrier CDMA (MC-CDMA) is a strong candidate for next generation wireless communication due to its high spectral efficiency, large system capacity, high flexibility in data rate and easy implementation using the fast Fourier transform (FFT) device without increasing the transmitter and receiver complexities [1]. However, MC-CDMA system is not suitable for mobile communication systems due to the frequency offset between the transmitter and receiver or introduced by Doppler shift in high mobility environment. With this frequency offset, the orthogonality among all the subcarriers is lost and inter-carrier interference (ICI) is generated.

Many methods have been proposed in the literature to mitigate the frequency-offset problem to cancel the ICI for OFDM system. Most of such methods use signal processing and/or coding to reduce the sensitivity of the OFDM system to the frequency offset. For example, in [2], authors developed low-complexity minimum mean-square error and decision-feedback equalizer receivers to suppress ICI based on the fact that the ICI power mainly comes from a few neighboring subcarriers. Some researchers also proposed a lot of schemes to estimate the frequency offset, including data aided estimation [3][4] and blind estimation [5][6][7]. In the light of the same statement, an effective methods known as the ICI self-cancellation scheme has been proposed in [8] where copies of the same data symbol are modulated on L adjacent subcarriers using optimized weights. In [9], a generalized ICI self-cancellation scheme has been proposed. In [10], a ICI self-cancellation using data-conjugate method is proposed. There are only a few ICI cancellation technologies for MC-CDMA system. [11] provides the analysis of the MAI and ICI for MC-CDMA system with frequency offset. In our previous work [12], we have applied the ICI self-cancellation method [8] to MC-CDMA systems and shown the performance gain in high mobility environment.

However, all existing ICI cancellation methods are not without their drawbacks. Even though all the existing ICI cancellation methods reduce ICI and improve BER performance for the system, the performance improvement is very limited. The BER performance after ICI cancellation is still significantly worse than the original system without ICI. More important, most of the existing ICI cancellation methods achieve the ICI reduction and BER performance improvement at the cost of lowering the transmission rate and reducing the bandwidth efficiency. There do exist some methods that do not reduce the date rate, however, such methods produce even less reduction in ICI. Nevertheless, only a few of these ICI cancellation methods are applied for MC-CDMA system with carrier frequency offset to improve the performance of mobile MC-CDMA system.

It has been observed that the ICI coefficient matrix is an orthonormal matrix. Hence, from the receiver side, a MC-CDMA with ICI can be considered as a new MC-CDMA system with a new spreading code matrix. However, since the frequency offset is time varying and unknown at the receiver side, the spreading code matrix of the equivalent MC-CDMA system is unknown. Hence, it has been proposed to transmit training sequences to estimate the frequency offset and cancel ICI via the estimated frequency offset. Of course, by doing so, some data rate needs to be allocated for the training sequence and complicated frequency offset estimation algorithms need to be implemented at receiver side.

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In this paper, we propose a new approach to solve the ICI problem in mobile MC-CDMA system without estimating frequency offset through training symbols and without data rate reduction. Specifically, we propose to quantize the normalized frequency offset into $M$ discrete values, leading to $M$ spreading code matrices as candidates. Next, by decoding the received signal using these $M$ spreading code matrices, $M$ decisions are made on the data symbols. Using these $M$ data symbols to recreate the received signal with ICI and measuring the Euclidean distance of the $M$ recreated signals with the actual received signal, the best normalized frequency offset is chosen and the best corresponding data symbols are determined. Simulation results over AWGN channel and mobile multi-path fading channel demonstrate that not only the proposed system effectively eliminates ICI entirely but offer the best BER performance available which matches the BER performance of MC-CDMA system without ICI, it achieves the superior performance with reasonable computational complexity. It is shown that the complexity of the proposed system is linearly growing with the number of quantization levels $M$, and $M$ does not have to be a huge number to achieve the best performance. Simulation results show that we only need to pick $M = 7$.

The rest of the paper is organized as follows: Section II re-examines the ICI of mobile MC-CDMA systems and proves that the received MC-CDMA signal with the presence of ICI can be considered as an orthogonal MC-CDMA system with new spreading codes. Part III describes the proposed Total ICI Cancellation method. Part IV provides simulation results over AWGN channel and multi-path fading channels. Conclusion follows.

II. ICI OF MC-CDMA SYSTEMS AND ORTHOGONALITY

It is well known that the received MC-CDMA signal on subcarrier $i$ in AWGN channel with ICI is

$$R(i) = \sum_{k=0}^{K-1} X(k) \beta_k^i S(0) + \sum_{l=0, l \neq i}^{N-1} \sum_{k=0}^{K-1} X(k) \beta_k^i S(l-i) + n_i,$$

where $N$ is the total number of the subcarriers, $X(k)$ denotes the transmitted symbol ($X(k) \in \{+1, -1\}$ if BPSK is employed, for example) for the $k^{th}$ user, $\beta_k^i$ is the $k^{th}$ user’s spreading code on $i^{th}$ subcarrier, and $n_i$ is the additive Gaussian noise sample on $i^{th}$ subcarrier. The sequence $S(l-i)$ is the ICI coefficient from $l^{th}$ subcarrier to $i^{th}$ subcarrier:

$$S(l-i) = \frac{\sin(\pi(\varepsilon + l - i))}{N \sin(\frac{\pi}{N}(\varepsilon + l - i))} \cdot \exp\left(\frac{j \pi}{\Delta F} \left(1 - \frac{1}{N}\right)(\varepsilon + l - i)\right)$$

where $\varepsilon$ is the normalized frequency offset given by $\varepsilon = \frac{\Delta F}{\Delta f}$, $\Delta F$ is the carrier frequency offset, $\Delta f$ is the subcarrier bandwidth of the system. It is reasonable to assume that $0 \leq \varepsilon < 1$.

Now, denote vector $\vec{X}$ as the transmitted symbol for $K$ users $\vec{X} = \{X(0), X(1), \ldots, X(K-1)\}$, vector $\vec{r}$ as the received signal vector on $N$ subcarriers $\vec{R} = \{R(0), R(1), \ldots, R(N-1)\}$, $\vec{n} = \{n_0, n_1, \ldots, n_{N-1}\}$, and matrix $\mathbf{C}$ as the spreading matrix with $\mathbf{C}(k,i) = \beta_k^i$, we have:

$$\vec{R} = \vec{X} \mathbf{C} + \vec{n} \quad (3)$$

where $\mathbf{S}$ is the ICI coefficient matrix, and the $p^{th}$ row and $q^{th}$ column element of $N \times N$ matrix $\mathbf{S}$ is

$$S_{p,q} = S(p-q) \quad (4)$$

and the matrix $\mathbf{S}$ corresponds to

$$\mathbf{S} = \begin{bmatrix}
S(0) & S(-1) & \ldots & S(1-N) \\
S(1) & S(0) & \ldots & S(2-N) \\
\vdots & \vdots & \ddots & \vdots \\
S(N-1) & S(N-2) & \ldots & S(0)
\end{bmatrix} \quad (5)$$

From Eq. (3), it is obvious that the received signal on all the subcarriers can be viewed as a new MC-CDMA signal with a new spreading code matrix $\mathbf{CS}$.

Now, it is important to note that the ICI coefficient matrix $\mathbf{S}$ is an orthogonal matrix, i.e.,

$$\mathbf{SS}^* = \mathbf{I} \quad (6)$$

where $\mathbf{S}^*$ is the conjugate transpose of matrix $\mathbf{S}$ and $\mathbf{I}$ is the identity matrix.

Hence, the MC-CDMA signal with ICI at receiver side can be considered as a new orthogonal MC-CDMA system with spreading code matrix $\mathbf{CS}$. As a direct result, the ICI can be totally removed from the MC-CDMA signal if we apply a matrix multiplication to the received signal vector $\vec{R}$:

$$\vec{Y} = \vec{R} \mathbf{S}^* \mathbf{C}^* = \vec{X} + \vec{n} \mathbf{S}^* \mathbf{C}^* \quad (7)$$

Next, we can simply make decision of $\vec{X}$ based on the sign of $\vec{Y}$. Since $\mathbf{S}^*$ is also an orthonormal matrix, the noise vector $\vec{n} \mathbf{S}^*$ has the same covariance matrix as that of $\vec{n}$. Hence, the entire ICI is eliminated and the BER performance would be the same of a MC-CDMA system without ICI.

Of course, the problem is: the receiver does not know the spreading code matrix $\mathbf{S}$ because the normalized frequency offset $\varepsilon$ is unknown. Hence, it has been proposed to estimate the normalized frequency offset $\varepsilon$ through some training symbols. Of course, by doing so, some data rate needs to be allocated for the training symbols, and sophisticated frequency offset estimation algorithms need to be implemented at receiver side.

III. TOTAL ICI CANCELLATION FOR MC-CDMA SYSTEM

A. Analysis in AWGN Channels

Here, we propose the Total ICI Cancellation scheme to eliminate ICI on mobile MC-CDMA systems without transmitting any training symbols (and reducing data rate). While
the normalized frequency offset $\varepsilon$ is unknown to the receiver, we can quantize $\varepsilon$ into $M$ equally spaced values:

$$
\varepsilon'_m = m \cdot \Delta \varepsilon, m = 0, 1, \ldots, M - 1
$$

(8)

where $\Delta \varepsilon$ is the quantization level of normalized frequency offset, and $M$ is the number of quantization levels:

$$
\Delta \varepsilon = \frac{1}{M}
$$

(9)

One of these $M$ quantized $\varepsilon$'s is closest to the true $\varepsilon$.

Now, we can build $M$ parallel branches at the receiver. Each branch uses one of the $M$ quantized $\varepsilon$'s to create the corresponding ICI coefficient matrix $\tilde{S}$. Hence, we have $M$ ICI coefficient matrices $\tilde{S}_0, \tilde{S}_1, \ldots, \tilde{S}_{M-1}$ where the $m$th matrix corresponds to:

$$
\tilde{S}_m = \begin{bmatrix}
S_m(0) & S_m(-1) & \ldots & S_m(1-N) \\
S_m(1) & S_m(0) & \ldots & S_m(2-N) \\
\vdots & \vdots & \ddots & \vdots \\
S_m(N-1) & S_m(N-2) & \ldots & S_m(0)
\end{bmatrix}
$$

(10)

and

$$
S_m(l-k) = \frac{\sin(\pi (\varepsilon'_m + l-k))}{N \sin(\frac{\pi}{N} (\varepsilon'_m + l-k))} \exp \left( j\pi \left(1 - \frac{1}{N}\right)(\varepsilon'_m + l-k) \right)
$$

(11)

Using these $M$ matrices, we can have $M$ decisions on the transmitted data vector $\tilde{X}$ where the $m$th branch will make decision on the estimation of $\tilde{X}$ as:

$$
\tilde{X}_m = \text{sgn}(\tilde{R}\tilde{S}_m^*C^*)
$$

(12)

where $\text{sgn}(X)$ presents the sign of $X$.

Next, with the data vector estimation $\tilde{X}_m$, each branch can reproduce the received signal $\tilde{Y}_m$ by using the data vector estimation $\tilde{X}_m$, the ICI coefficient matrix of that branch $\tilde{S}_m$:

$$
\tilde{R}_m = \tilde{X}_m C \tilde{S}_m
$$

(13)

It is easy to understand that the one branch whose $\varepsilon'_m$ is closest to the true value of $\varepsilon$ should reproduce the received signal $\tilde{R}_m$ also closest to the received signal vector $\tilde{R}$. Hence, we only need to calculate and compare the Euclidean distances between the $M$ reproduced received signal vectors $\tilde{R}_m$ and the truly received signal vector $\tilde{R}$, and pick the one with the minimum distance to be the best branch and use that branch’s estimated data vector as the final decision:

$$
\hat{X} = \text{argmin}\{||\tilde{R}_m - \tilde{R}||^2\}
$$

(14)

where $||\tilde{R}_m - \tilde{R}||^2$ represents the Euclidean distance between vector $\tilde{R}_m$ and vector $\tilde{R}$. Fig. 1 shows the $||\tilde{R}_m - \tilde{R}||^2$ versus $(\varepsilon'_m - \varepsilon)$ for different SNR, and it is clear that when there is no noise, $||\tilde{R}_m - \tilde{R}||^2$ reaches to the optimum when $(\varepsilon'_m - \varepsilon) = 0$. Meanwhile, at high SNR, the optimum also occurs when $|\varepsilon'_m - \varepsilon|$ is very small.

![Fig. 1. $||\tilde{R}_m - \tilde{R}||^2$ versus $(\varepsilon'_m - \varepsilon)$](image)

It is important to note that the complexity of the proposed Total ICI Cancellation method is linearly growing with the quantization level $M$, keeping the computational complexity at reasonable range. The increased complexity is not significant, especially when $M$ is small. As we will show in the next Section, we don’t need to use a huge $M$ to achieve the best performance. In all the cases, $M = 8$ is good enough to provide perfect ICI cancellation and superb BER performance matching the lower bound.

### B. Analysis in Multipath Fading Channels

In a multipath fading channel, let’s denote the complex fading gain on the $k$th subcarrier is $\alpha_k$. Then the received signal vector after transmission through such a fading channel with frequency offset is:

$$
\tilde{R} = \tilde{X}C\alpha S + \tilde{n}
$$

(15)

where $\alpha$ is a diagonal matrix $\alpha = \text{diag}\{\alpha_0, \alpha_1, \ldots, \alpha_{N-1}\}$.

Similar to the analysis in AWGN channel, the received signal represented in Eq. (15) can also be viewed as a new MC-CDMA system. Hence, if the spreading code matrix $S$ is known, we can eliminate the ICI by multiplying $S^*$ to the received vector $\tilde{R}$.

So the Total ICI Cancellation schemes works the same way as in AWGN channel with only one exception: the fading channel characteristics $\alpha$ needs to be estimated at the receiver side (which is required for MC-CDMA transmission). Consider the fading effects, we can use combining technology to improve estimation performance of $\hat{X}$:

$$
\hat{X}_m = \text{sgn}(\tilde{R}\tilde{S}_m^*W^*)
$$

(16)

where $W$ is a diagonal matrix containing the combining weights, e.g. $W$ becomes an identity matrix if equal gain combining (EGC) are applied. The reproduced received signal vector now also has to consider the fading effects:

$$
\hat{R}_m = \hat{X}_m C \alpha \tilde{S}_m
$$

(17)
The block diagram of the proposed Total ICI Cancellation scheme is shown in Fig. 2.

![Block Diagram of Total ICI Cancellation](image)

**Fig. 2. Block Diagram of the Total ICI Cancellation**

### IV. SIMULATION RESULTS

In this section, we use numerical simulation results to present the effectiveness of the proposed Total ICI Cancellation scheme. We provide BER simulation results for the proposed Total ICI Cancellation scheme in both AWGN channel and multi-path fading channels compared with Self-Cancellation schemes for MC-CDMA system [12]. All the systems are assumed to have $N = 64$ subcarriers, full/half loaded and employ BPSK/QPSK modulation.

#### A. AWGN Channel with a Constant Frequency Offset

The simplest way to examine the effectiveness of the proposed Total ICI Cancellation scheme is to transmit signals through an AWGN channel with a constant frequency offset between the transmitter and receiver.

![Simulation 1 in AWGN with BPSK modulation and NFO=0.1](image)

**Fig. 3. Simulation 1 in AWGN with BPSK modulation and NFO=0.1**

Fig. 3 illustrates the simulation result when the normalized frequency offset (NFO) $\varepsilon = 0.1$ and Fig. 4 shows the case when $\varepsilon = 0.7$. In the Total ICI Cancellation scheme, we use $M = 10$. In both of the two figures, the blue line shows the BER performance of MC-CDMA without ICI, the magenta line marked with circles represents the performance of MC-CDMA with ICI, the red line marked with square represents that of our proposed Total ICI Cancellation technology, and the green line with triangle shows the performance of self-cancellation scheme. Since we choose $M = 10$, so the quantization level of normalized frequency offset $\Delta \varepsilon = 0.1$. Hence, one of the $M$ branches actually has the perfect ICI coefficient matrix to work with. To prove the effectiveness of our Total ICI Cancellation method in all scenarios, Fig. 5 presents the results when $\varepsilon = 0.485372$ so none of the $M$ branches matches the actual $\varepsilon$, and the BER performance of Total ICI Cancellation scheme also matches with the MC-CDMA system without ICI. To illustrate high modulation, Fig. 6 shows the BER performance when $\varepsilon = 0.38757$ with QPSK modulation of half loaded MC-CDMA system, and the simulation results also confirm the benefit of the proposed Total ICI Cancellation technology.

From these figures, it is evident that the proposed Total ICI Cancellation technology provides MC-CDMA system the same BER performance as that of a MC-CDMA without ICI no matter what $\varepsilon$ is, while other two systems degrades when $\varepsilon$ increases. Meanwhile, the Total ICI Cancellation scheme provides much more gain compared to the other two systems when NFO increases.

![Simulation 2 in AWGN with BPSK modulation and NFO=0.7](image)

**Fig. 4. Simulation 2 in AWGN with BPSK modulation and NFO=0.7**
B. Multipath Mobile Channels

In a practical mobile multipath radio channel, time-variant multipath propagation leads to Doppler frequency shift which is a random variable. Here we measure the performance of the proposed Total ICI Cancellation method in multipath fading channels. As a measure of Doppler frequencies, we use the normalized maximum Doppler spread \( \varepsilon_B \), which is defined as the ratio between the channel maximum Doppler spread to the subcarrier bandwidth. We use the Hilly Terrain (HT) channel models defined by the GSM standard as our channel model. Total number of subcarriers is also assumed to be 64.

Fig. 8 shows the case when \( \varepsilon_B = 0.2 \) and Fig. 9 shows the case when \( \varepsilon_B = 0.4 \). In the Total ICI Cancellation scheme, we use \( M = 10 \). In both of the two figures, the legend is the same as in AWGN channel. It is clear from these figures that the proposed Total ICI Cancellation entirely eliminates the effect of ICI and matches the performance of the MC-CDMA without ICI in fading channels as well, no matter what NFO is; while other two benchmarks have much worse performance, especially when NFO becomes large.

Fig. 10 illustrates the effect of the number of normalized frequency offset quantization levels \( M \) on the performance of the proposed Total ICI Cancellation scheme. In Figure 10, four BER versus \( M \) curves of different SNRs are shown. It is easy to understand that when \( M \) increases, more quantization levels are used and better ICI coefficient matrix estimation is achieved, so the performance of the proposed scheme will also improve. As shown in Figure 10, when \( M \) is very small, the proposed Total ICI Cancellation scheme actually offers pretty bad performance due to the large quantization error. However, when \( M \) increases, the Total ICI Cancellation converges fast and provide ICI cancellation and BER improvement quickly.

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Fig. 9. Simulation 2 in Fading Channel and normalized maximum Doppler spread $=0.4$

When $M$ is larger than 7, there is no noticeable performance gain to increase the quantization level. This can be explained as the following: when the quantization step $\Delta \varepsilon$ is small enough, the Total ICI Cancellation’s ICI cancellation capability is enough to remove all the intercarrier interference and there is no need to decrease $\Delta \varepsilon$ anymore. It is evident from Figure 10 that the computational complexity of the proposed scheme is very reasonable.

Fig. 10. Effect of NFO Quantization Levels, AWGN, NFO=0.1

V. Conclusion

In this paper, we proposed a novel intercarrier interference cancellation scheme called Total ICI Cancellation for MC-CDMA system in mobile environment. Taking advantage of the orthogonality of the ICI coefficient matrix, the proposed ICI cancellation scheme can eliminate the ICI experienced in mobile MC-CDMA systems entirely and provide significant BER improvement which matches the BER performance of MC-CDMA system without ICI at all. The proposed Total ICI Cancellation scheme not only provides perfect performance, it doesn’t reduce the bandwidth efficiency of the system like many existing ICI cancellation methods. Simulations over AWGN channel and multipath fading channel confirm the effectiveness of the proposed scheme. Finally, the Total ICI Cancellation scheme achieves such superb performance at a very reasonable computational complexity which linearly grows with the number of normalized frequency offset quantization.

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References