Abstract—SC-OFDM (single carrier orthogonal frequency division multiplexing) has demonstrated excellent performance in multipath fading channels with very low peak to average power ratio (PAPR). Similar to other multi-carrier transmission technologies such as MC-CDMA and OFDM, SC-OFDM also suffers significant performance degradation to inter-carrier interference (ICI) when there is a frequency offset between the transmitter and receiver or in a high mobility environment. In this paper, we analyze the effect of ICI on SC-OFDM system and propose a novel modulation scheme called Magnitude Shift Keying (MSK) for SC-OFDM system. The MSK modulation provides SC-OFDM system immunity to ICI and significantly outperforms SC-OFDM system and OFDM system with Phase Shift Keying (PSK) modulations in severe ICI environment. Simulation results over AWGN channel and multi-path fading channels with different ICI confirm the effectiveness of the proposed system.

I. INTRODUCTION

The single carrier orthogonal frequency division multiplexing (SC-OFDM) [1] technique has received a lot of attention as the alternative transmission technique to the conventional OFDM due to its better performance and lower peak to average power ratio (PAPR) property as compared with the conventional OFDM. Single Carrier OFDM, or similar technology, was independently developed by multiple research groups almost simultaneously. For example, single carrier transmission with frequency domain equalization (SCFDE) [2][3][4][5] or carrier interferometry orthogonal frequency division multiplexing (CI/OFDM) [6][7][8] are essentially the same technology by combining the benefit of multi-carrier transmission and single carrier transmission together, by employing cyclic prefix to single carrier transmission and frequency domain processing.

In any multi-carrier transmission technology such as OFDM, it is crucial to maintain orthogonality among all the subcarriers. Otherwise, intercarrier interference (ICI) will occur and lead to significant performance degradation. SC-OFDM also experiences ICI while employing frequency domain equalization when there is a frequency offset observed between the transmitter and receiver, due to either the frequency offset between the local oscillators at the transmitter and receiver or Doppler shift introduced by high mobility. Nevertheless, it is of great interest to study the performance of SC-OFDM with the presence of ICI in mobile environment. Many studies have been conducted in evaluating the BER performance of OFDM system and MC-CMADA system with ICI [9] [10] and how to improve the performance by reducing ICI for OFDM [11][12][13][14] or by estimation the carrier frequency offset (CFO) [15][16][17]. Not much research has been done for SC-OFDM (or SCFDE, or CI/OFDM) in the presence of severe ICI.

In this paper, we analyze SC-OFDM [1] with ICI and show a unique diagonal property of SC-OFDM with ICI. We then propose a novel modulation scheme called Magnitude Shift Keying (MSK) for SC-OFDM. This new modulation scheme provides SC-OFDM with immunity to ICI, i.e., the BER performance of a SC-OFDM system with MSK does not depend on the ICI. While the proposed SC-OFDM with MSK modulation performs worse than SC-OFDM with PSK modulation when there is no ICI, the new system significantly outperforms SC-OFDM with PSK in severe ICI environment. Compared with some ICI cancellation schemes or CFO estimation schemes, the proposed modulation technique does not need to sacrifice the data rate via employing training sequence, instead it is totally immune to the ICI. Simulation results confirm the benefits and the immunity of the proposed system.

The rest of the paper is organized as follows: In section II, we review the system model of SC-OFDM. Section III provides the analysis of ICI in SC-OFDM and presents an important diagonal property of ICI matrix in SC-OFDM. We then propose MSK modulation for SC-OFDM which is immune to ICI and also analyze the theoretical BER performance in section IV. Section V shows the simulation results which confirm our analysis, and conclusion is given in Section VI.

II. SYSTEM MODEL

A. Transmitter

A conceptual representation of the SC-OFDM [3][4] transmitter is shown in Fig. 1. The transmitted signal corresponding to the \( k^{th} \) data symbol in a single OFDM block is

\[
s_k(t) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} a_k \beta_i e^{j 2\pi i f \Delta f t} p(t)
\]

where \( a_k \) is the \( k^{th} \) data symbol; \( \beta_i \) is the \( i^{th} \) element of the \( k^{th} \) data symbol’s spreading code; \( \Delta f \) is the spacing between subcarriers; and \( p(t) \) is a rectangular pulse shape that time limits the signal to one OFDM symbol duration, \( T \). To assure the orthogonality among the subcarriers, we have \( \Delta f = \frac{1}{T} = \frac{1}{N T_c} \) where \( T_c \) is the data symbol period. In SC-OFDM, we employ an orthogonal set of complex spreading codes which correspond to the normalized DFT matrix, and the code set corresponds to \( \beta_i = \frac{1}{\sqrt{N}} \exp(-j \frac{2\pi i k}{N}) \).
The total SC-OFDM signal with all N symbols corresponds to

$$s(t) = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{k=0}^{N-1} a_k e^{-j \frac{2\pi}{N} ik} e^{j 2\pi (i+\epsilon) \Delta f t} e^{j 2\pi f_c t} p(t)$$

(2)

Similar to an OFDM system, SC-OFDM can also be implemented using IFFT and FFT.

B. Receiver

Assuming transmission through an multi-path fading channel, the received signal corresponds to

$$r(t) = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{k=0}^{N-1} a_k e^{-j \frac{2\pi}{N} ik} e^{j 2\pi (i+\epsilon) \Delta f t} e^{j 2\pi f_c t} p(t) + n(t)$$

(3)

where \(n(t)\) is the additive white gaussian noise (AWGN), \(\alpha_i\) is the fading gain on the \(i^{th}\) subcarrier, and \(f_0\) is the frequency offset. Let’s denote the normalized frequency offset as \(\epsilon = f_0/\Delta f\). The received SC-OFDM signal can then be represented as

$$r(t) = \frac{1}{N} \sum_{i=0}^{N-1} \alpha_i \sum_{k=0}^{N-1} a_k e^{-j \frac{2\pi}{N} ik} e^{j 2\pi (i+\epsilon) \Delta f t} e^{j 2\pi f_c t} p(t) + \alpha(t)$$

(4)

At the receiver (Fig. 2), the SC-OFDM demodulator detects the \(k^{th}\) symbol by: (1) decomposing the received signal \(r(t)\) onto \(N\) orthogonal subcarriers (via application of FFT), and (2) applying the \(k^{th}\) symbols spreading code.

Using Equal-Gain Combining (EGC) scheme, the received signal vector \(\tilde{r}\) can be shown as:

$$\tilde{r} = \tilde{A} F \tilde{C} F^* \bar{n}$$

where \(\tilde{A}\) is the data vector:

$$\tilde{A} = [a_0, a_1, ..., a_{N-1}]$$

(6)

and \(\tilde{n}\) is the noise vector:

$$\tilde{n} = [n_0, n_1, ..., n_{N-1}]$$

(7)

Matrix \(F\) is the normalized DFT matrix and \(F^*\) is the conjugate transpose of matrix \(F\).

$$F(n, k) = \frac{1}{\sqrt{N}} e^{-j \frac{2\pi}{N} kn}, \quad k, n \in [0, N-1]$$

(8)

Matrix \(A\) is the fading gain matrix:

$$A = \text{diag}(\alpha_0, \alpha_1, ..., \alpha_{N-1})$$

(9)

Matrix \(C\) is the ICI coefficient matrix:

$$C(k, n) = \frac{\sin(\pi (k + \epsilon - n))}{N \sin(\pi (k + \epsilon - n))} \cdot \exp(j \pi (1 - \frac{1}{N})(k + \epsilon - n))$$

(10)
III. ANALYSIS OF SC-OFDM WITH ICI

A. ICI Coefficient

Using eigen-decomposition, it is easy to prove that the ICI coefficient matrix $C$ can be expressed as

$$C = F^*ΨF$$  (11)

where $Ψ$ is a diagonal matrix $Ψ = diag[ψ_0, ψ_1, ..., ψ_{N-1}]$ and $ψ_n$ is the $n^{th}$ eigenvalue of ICI coefficient matrix $C$. Here $ψ_n = exp(j2πnN)$.

It is important to note that all eigenvalues $ψ_n$ have a magnitude of 1.

B. ICI in SC-OFDM

1) AWGN Channel: Now the fading gain on each subcarrier equals to one. Hence we can rewrite the received signal vector of the SC-OFDM system with ICI in AWGN channel as

$$\hat{r} = \hat{A}PCF^{*} + \hat{n}F^{*}$$
$$= \hat{A}F^{*}ΨF^{*} + \hat{n}F^{*}$$
$$= \hat{A}Ψ + \hat{n}F^{*}$$  (12)

Notice that $Ψ$ has unit magnitude, the ICI effect on SC-OFDM data symbols $\hat{A}$ is simply a (different) phase offset on each and every data symbol $a_k$. Therefore, if there is no noise, $|r_k| = |a_k|$ no matter what $\epsilon$ is.

2) Multi-Path Fading Channel: In multi-path fading channel, considering the fading gain matrix $A$, we can rewrite the received signal vector of the SC-OFDM system with ICI as

$$\hat{r} = \hat{A}FACF^{*} + \hat{n}F^{*}$$
$$= \hat{A}FA^{*}ΨF^{*} + \hat{n}F^{*}$$
$$= \hat{A}F^{*}Ψ + \hat{n}F^{*}$$  (13)

Similar as the procedure in AWGN channel, we can also using the magnitude of $\hat{r}$ to make decision without the impact of the $\epsilon$.

IV. MSK FOR SC-OFDM

A. MSK for SC-OFDM in AWGN Channel

After the observation of the ICI coefficients property, we find $FCF^{*}$ is a diagonal matrix. Hence, the ICI effect on the SC-OFDM symbol is not changing the magnitude of each and every data symbol. As a direct result, we introduce a novel digital modulation scheme called Magnitude Shift Keying (MSK) to be applied to SC-OFDM. Specifically, we will only use the magnitude to carry digital symbols. For example, a binary MSK is the same as binary OOK (on off keying). Note that MSK is different than ASK (amplitude shift keying) where MSK is a non-coherent modulation scheme and doesn’t require phase reference.

According to Eq. (12), the decision of the $k^{th}$ data symbol in AWGN channel corresponds to

$$a_k = |r_k|$$  (14)

For 2MSK, the BER performance is exactly the same as the BER performance of OOK with non-coherent detection, which is

$$BER = BER(a_k = 0) * P(a_k = 0) + BER(a_k = 1) * P(a_k = 1)$$
$$= BER(a_k = 0) + BER(a_k = 1)$$
$$= Q_1(0, \sqrt{SNR}) + 1 - Q_1(2/\sqrt{SNR})/2$$  (15)

where $Q_1$ is the Marcum Q-Function.

For L-MSK, using the same process we can easily derive the symbol error ratio (SER) performance as:

$$SER = \sum_{m=0}^{m=L-1} BER(a_k = m) * P(a_k = m)$$
$$= \sum_{m=0}^{m=L-1} \frac{BER(a_k = m)}{L}$$
$$= \frac{Q_1(0, \sqrt{SNR})}{L} + \sum_{m=1}^{m=L-1} \frac{(1 - Q_1(m, 0.5)\lambda)}{L} + \frac{Q_1((L - 1)\lambda, (L - 1 - 0.5)\lambda)}{L}$$  (16)

where $\lambda = \frac{\text{Amplitude}}{\sqrt{SNR}} = \sqrt{\frac{12 \text{snr} \log_2(L)}{(L-1)(2L-1)}}$.

And the BER performance using Gray Code can be approximated by $BER = SNR/\log_2(L)$.

B. MSK for SC-OFDM in Multi-path Fading Channel

In multi-path fading channel, using the same property of ICI coefficient matrix, the decision of $K$ data symbols $\hat{A}$ can be determined by

$$\hat{A} = \arg\max_B [|BF^{*}F^{*}] - |r|^{2}]$$  (17)

It is obvious that since the digital information is only carried on the magnitude, and the ICI effect is not changing the magnitude of the received signal, a SC-OFDM with MSK modulation is immune to ICI. In other words, no matter how large the ICI is, the SC-OFDM with MSK will provide identical performance at the same SNR. Although the BER performance of the SC-OFDM system with MSK is worse than that of the SC-OFDM system with PSK (phase shift keying) modulation when there is no ICI, the proposed MSK modulation offers SC-OFDM significantly better performance in a severe ICI environment.

V. SIMULATION RESULTS

In this section, we examine the BER performance of the proposed SC-OFDM with MSK modulation. Specifically, we compare (1) SC-OFDM with binary MSK versus SC-OFDM with BPSK modulation, and (2) SC-OFDM with 4MSK versus SC-OFDM with QPSK with ICI.
The simplest way to examine the effectiveness of the proposed ICI immune SC-OFDM via MSK modulation is to transmit signals through a AWGN channel with a constant frequency offset between the transmitter and receiver. Fig. 3 to Fig. 4 and Fig. 5 to Fig. 6 illustrate the BER versus SNR simulation results for 2MSK and 4MSK respectively. In these figures, the systems are applied with normalized CFO $\epsilon = [0.1, 0.2, 0.3, 0.4]$. When $\epsilon$ increases, performances of SC-OFDM with BPSK/QPSK modulation significantly degrades, while SC-OFDM with 2MSK/4MSK outperforms the SC-OFDM with BPSK/QPSK when SNR increases. On the other hand, the simulation results also confirm the analysis of theoretical BER performance in Eq. (15) and Eq. (16).

Fig. 7 and Fig. 8 show the BER v.s. normalized CFO $\epsilon$ when $SNR = 10dB$. From these figures, it is evident that the BER performance of SC-OFDM with 2MSK/4MSK remains constant when $\epsilon$ increases, while the BER performance of traditional SC-OFDM with BPSK/QPSK becomes worse when $\epsilon$ increases.

In a practical mobile multipath radio channel, time-variant multipath propagation leads to Doppler frequency shift which is a random variable. Here we measure the performance of the proposed ICI Cancellation method in multipath fading channels. As a measure of Doppler frequencies, we use the normalized maximum Doppler spread $\epsilon_{\text{max}}$, which is defined as the ratio between the channel maximum Doppler spread $f$ and the subcarrier bandwidth. Here, we assume a 4-fold multipath fading channel, whose coherence bandwidth is:

$$BW = N \cdot \Delta f = 4 \cdot (\Delta f)_c$$  \hspace{1cm} (18)

where $BW$ is the total bandwidth of the system and $(\Delta f)_c$ is the coherence bandwidth of the channel.

The BER performances of BPSK with OFDM/SC-OFDM and 2MSK with SC-OFDM in multi-path fading channel are compared in Fig. 9 and Fig. 10. Fig. 11 shows the BER v.s. $\epsilon_{\text{max}}$ when $SNR = 15dB$. It is easily to get the same comparison results as in AWGN channel. The performance of 2MSK with SC-OFDM does not change due to the $\epsilon_{\text{max}}$, while the BER performances of BPSK with OFDM and SC-OFDM become much worse with the increment of $\epsilon_{\text{max}}$. In much more severe environment with ICI, the BER performance
of 2MSK with SC-OFDM performs much better than the traditional BPSK with OFDM or SC-OFDM.

VI. CONCLUSION

In this paper, we analyze the effect of ICI on SC-OFDM receiver and propose a novel modulation scheme called Magnitude Shift Keying (MSK) for SC-OFDM system. Taking advantage of the unique property of the ICI coefficient matrix, we showed that the ICI effect on SC-OFDM received signal is simply a phase offset on each and every data symbol, while keeping the magnitude of the SC-OFDM data the same. Hence, by carrying digital information only on the magnitude of the SC-OFDM signal, a novel modulation scheme called MSK is proposed to be applied to SC-OFDM. The MSK modulation provides SC-OFDM system immunity to ICI and significantly outperforms traditional SC-OFDM system and OFDM system with Phase Shift Keying (PSK) modulations in severe ICI environment. Simulation results of both binary and quadrature MSK over many scenarios confirm our analysis.

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Fig. 11. The BER v.s. $\epsilon_{max}$ when $N = 16, SNR = 10dB$ in multi-path fading channel


