Compression of Hyperspectral Remote Sensing Images by Tensor Approach

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Abstract

Whereas the transform coding algorithms have been proved to be efficient and practical for grey-level and color images compression, they could not directly deal with the hyperspectral images (HSI) by simultaneously consider both the spatial and spectral domains of the data cube. The aim of this paper is to present an HSI compression and reconstruction method based on the multi-dimensional or tensor data processing approach. By representing the observed hyperspectral image cube to a 3-order-tensor, we introduce a tensor decomposition technology to approximately decomposes the original tensor data into a core tensor multiplied by a factor matrix along each mode. Thus, the HSI is compressed to the core tensor and could be reconstructed by the multi-linear projection via the factor matrices. Experimental results on particular applications of hyperspectral remote sensing images such as unmixing and detection suggest that the reconstructed data by the proposed approach significantly preserves the HSI’s data quality in several aspects.

Keywords: hyperspectral image, compression, tensor decomposition, spectral unmixing, target detection

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1. Introduction

Remotely sensed images, which are acquired by the airborne or spaceborne sensors, have been extensively used in earth observation applications. Hyperspectral imaging sensors can collect an image in which each pixel has the contiguous bands of spectra, and these large number of spectral channels provide the opportunity for the detailed analysis of the land-cover materials [1], e.g., endmember extraction [2, 3], spectral unmixing [4, 5], target detection [6, 7, 8], image classification [9, 10, 11], and so on. However, as the hyperspectral image (HSI) is intrinsically a data cube which has two spatial dimensions (width and height) and a spectral dimension, numerous researches have indicated that the redundancy from both inter-pixel and inter-band correlation are very high and thus the data cube could be compressed by some algorithms without a significant loss of the useful information for subsequent HSI analysis [12, 13].

Generally, image compression technologies can significantly reduce the HSI volumes to a more manageable size for storage and communication. In the literature, most of the existing HSI compression algorithms are transform coding based approaches, e.g., Set Partitioning in Hierarchical Trees (SPIHT) and Set Partitioned Embedded bloCK (SPECK) algorithms [14], the progressive 3-D coding algorithm [15], the 3-D reversible integer lapped transform [16], and the discrete wavelet transform coupled with tucker decomposition [17], etc. Also based on the wavelet transform, Du et al. proposed a series of works on using JPEG 2000 ISO standard for HSI compression, the most important of which are JPEG2000 and Principal Component Analysis (PCA) based HSI compression methods [12, 18, 19]. As suggested in the aforementioned papers, the transform coding have been proved efficient and practical for HSI compression. However, most of the transform coding related algorithms were originally designed to process 2-D grey-level images, and then extended to 3-D data cube without the consideration of special characteristics of HSI, which might be problematic when the subsequent image analysis is conducted on the reconstructed HSI cube [12, 20].
In this paper, we propose a method for compression of the HSIs in a novel point of view, which is based on the multi-dimensional or tensor data processing approach [21, 22, 23, 24, 25]. As indicated in some previous works within the hyperspectral imaging area, an HSI data can be intrinsically treated as a 3-order-tensor, by this way, the data structure of both the spatial and spectral domains is well preserved [26, 27]. For the task of HSI compression, by representing the observed HSI data cube to a 3-order-tensor with two spatial modes and an additional spectral mode, we introduce a tensor decomposition technology to decomposes the original tensor into a core tensor with same order while much lower dimensionality multiplied by a matrix along each mode, under the umbrella of multi-linear algebra, i.e., the algebra of tensors. Thus, the HSI is compressed to the core tensor, and the reconstructed HSI is actually a low-rank tensor which could be acquired by the multi-linear backward projection via the factor matrices. HSI compression and reconstruction experiments on two public data sets show that the proposed method not only obtains the highest PSNR value, but also significantly preserves the HSI data quality which is benefit for several subsequent image analysis including the endmember extraction, spectral unmixing, and target detection.

The remainder of this paper is organized as follows. In the following section, we give a brief description of related tensor algebra, and then presents the proposed HSI compression algorithm in detail. After that, the experiments are reported in Section III, followed by the conclusion.

2. The Proposed HSI Compression Algorithm

The notations used in this paper are followed by convention in the multi-linear algebra, e.g., vectors are denoted by lowercase boldface and italic letters, such as \( \mathbf{x} \), matrices by uppercase boldface and italic, such as \( \mathbf{U} \), and tensors by calligraphic letters, such as \( \mathcal{X} \). For a \( K \)-order-tensor \( \mathcal{X} \in \mathbb{R}^{L_1 \times L_2 \times \cdots \times L_K} \), where \( L_i \) shows the size of this tensor in each mode, and the elements of \( \mathcal{X} \) are denoted with indices in lowercase letters, i.e., \( \mathcal{X}_{i_1, i_2, \cdots, i_K} \), in which each \( i_k \)
addresses the $i$-mode of $\mathcal{X}$, and $1 \leq l_i \leq L_i$, $i \in \{1, 2, \ldots, K\}$. Unfolding $\mathcal{X}$ along the $i$-mode is defined by keeping the index $l_i$ fixed and varying the other indices, the result of which is denoted as $\mathcal{X}(i) \in \mathbb{R}^{L_i \times \prod_{j \neq i} L_j}$. The $i$-mode product of a tensor $\mathcal{X}$ by a matrix $U \in \mathbb{R}^{J_i \times L_i}$, is a tensor with entries $(\mathcal{X} \times_i U)_{i_1, \ldots, i_{i-1}, i, i_{i+1}, \ldots, i_K} = \sum_{l_i} \mathcal{X}_{i_1, \ldots, l_i-1, l_i+1, \ldots, i_K} U_{j_i, i}$. The Frobenius norm of a tensor $\mathcal{X}$ is given by $\|\mathcal{X}\| = \sqrt{\sum_{l_1} \sum_{l_2} \cdots \sum_{l_K} \mathcal{X}_{l_1, l_2, \ldots, l_K}^2}$, and the Euclidean distance between two tensors $\mathcal{X}$ and $\mathcal{Y}$ could be measured by $\|\mathcal{X} - \mathcal{Y}\|$. For more detailed information, refer to [21, 28, 29, 30].

As discussed above, in order to preserve the most representative information of the HSI data, we denote the data cube as a 3-order-tensor $\mathcal{X} \in \mathbb{R}^{L_1 \times L_2 \times L_3}$, in which $L_1$, $L_2$, and $L_3$ give the height, width and spectral channels of HSI, respectively. Then, the compressed tensor $\mathcal{C}$ (also known as the core tensor of $\mathcal{X}$) can be acquired by the following multi-linear projection:

$$\mathcal{C} = \mathcal{X} \times_1 U_1 \times_2 U_2 \times_3 U_3 \tag{1}$$

in which $U_1 \in \mathbb{R}^{J_1 \times L_1}$, $U_2 \in \mathbb{R}^{J_2 \times L_2}$, and $U_3 \in \mathbb{R}^{J_3 \times L_3}$ are series of projection matrices and $J_i \leq L_i$, $i \in \{1, 2, 3\}$. By this way, $\mathcal{X}$ is compressed to $\mathcal{C} \in \mathbb{R}^{J_1 \times J_2 \times J_3}$ with the rate of $\prod_{i=1}^3 J_i / \prod_{i=1}^3 L_i$, and the reconstructed tensor could be acquired by the following multi-linear projection:

$$\hat{\mathcal{X}} = \mathcal{C} \times_1 U_1^T \times_2 U_2^T \times_3 U_3^T \tag{2}$$

The reconstructed tensor $\hat{\mathcal{X}}$ given in (2) is in fact a low-rank tensor, thus the reconstruction error $\mathcal{E}$ could be computed by:

$$\mathcal{E} = \mathcal{X} - \hat{\mathcal{X}} \tag{3}$$

As an effective HSI compression algorithm, we expect the reconstructed tensor $\hat{\mathcal{X}}$ should be close to the original tensor data $\mathcal{X}$ as much as possible. According to this aspect, the required projection matrices $U_i$, $i \in \{1, 2, 3\}$ should be optimized by minimize the Euclidean distance between $\mathcal{X}$ and $\hat{\mathcal{X}}$, which also
could be written as the Frobenius norm of $\mathcal{E}$:

$$\arg \min_{U_1, U_2, U_3} \|X - \hat{X}\|^2 = \arg \min_{U_1, U_2, U_3} \|\mathcal{E}\|^2$$

(4)

By combining (1), (2) into (4), we have the following optimization of the proposed HSI compression algorithm:

$$\arg \min_{U_1, U_2, U_3} \|X - X \times_1 U_1^T U_1 \times_2 U_2^T U_2 \times_3 U_3^T U_3\|^2$$

(5)

Eq. (5) presents the same form with a tensor decomposition technology, i.e., the Tucker decomposition [30, 31], which is a form of higher-order PCA and aims to decomposes a tensor into a core tensor transformed by a factor matrix along each mode. Thus we abbreviate the proposed method as "TenD" in the rest of this paper. The objective function of (5) could be locally optimized by alternating optimization. The basic idea of this solution comes from the fact that any one of the projection matrix could be simply acquired by an eigenvalue decomposition problem when the remaining two matrices are fixed. So, after initializing $U_i$, $i \in \{1, 2, 3\}$, the optimal projection matrices along all modes can be acquired iteratively.

Specifically, the projection matrices could be initialized as either identity matrices or arbitrary columnly-orthogonal matrices. In this paper, we suggest to use the higher-order SVD (HOSVD) [30] to find a good starting point for an alternating optimization. Then, the higher-order orthogonal iteration (HOOI) [31] is used to optimize $U_i$, $i \in \{1, 2, 3\}$ in an iterative way. The detailed procedure for solve Eq. (5) is given below.

It is worth noting that some representative HSI spectral dimension reduction (DR) algorithms, e.g., PCA and maximum noise fraction (MNF) [32, 33], could also perform HSI compression and reconstruction but only in the spectral domain. This branch of approaches consider the HSI data as a set of spectral feature vectors $x_i \in \mathbb{R}^{L_3}$ $i = [1, \cdots, L_1 L_2]$ in which $L_3$ gives the spectral channels and $L_1 L_2$ is the number of pixels in HSI. Then, the DR algorithm output the linear projection matrix $U \in \mathbb{R}^{d \times L_3}$ ($d \leq L_3$) by some certain criterions, e.g., P-
Algorithm 1 Procedure for solve TenD

Input: Input HSI data $\mathcal{X} \in \mathbb{R}^{L_1 \times L_2 \times L_3}$ and compressed dimensionality in each mode $J_1$, $J_2$ and $J_3$;

Initialize $U_i$, $i \in (1, 2, 3)$ using HOSVD;

repeat
  • $C = \mathcal{X} \times_2 U_2 \times_3 U_3$, let $U_1$ to be the $J_1$ leading left singular vectors of $C(1)$;
  • $C = \mathcal{X} \times_1 U_1 \times_3 U_3$, let $U_2$ to be the $J_2$ leading left singular vectors of $C(2)$;
  • $C = \mathcal{X} \times_1 U_1 \times_2 U_2$, let $U_3$ to be the $J_3$ leading left singular vectors of $C(3)$;
until Convergence

Output: Projection matrices $U_1$, $U_2$ and $U_3$ for HSI compression.

CA finds the principal components in accordance with the maximum variance of the data and MNF transforms the principal components which ranked by SNR. Similar to tensor compression (1), the low-dimensional feature representation $y_i \in \mathbb{R}^d$ (here the compression rate is $d/L_3$) is obtained by:

$$y_i = U \times x_i, \quad i = [1, \cdots, L_1 L_2] \tag{6}$$

and the reconstructed feature vector could be recovered by the backward projection:

$$\hat{x}_i = U^T \times y_i, \quad i = [1, \cdots, L_1 L_2] \tag{7}$$

Obviously, Eqs. (6) and (7) consider the feature redundancy in the spectral domain while ignore the cross-domain redundancy of the input HSI data. Correspondingly, the proposed TenD algorithm deals with the HSI data by simultaneously considering both the spatial and spectral domains of the data cube, which can make sure that the significant data quality in HSI could be preserved.
as complete as possible as shown in the following experimental reports.

3. Experiments and Analysis

In this section, two public benchmark HSIs are used to demonstrate the superiority of the proposed TenD algorithm in several aspects. Since the particular advantage of the TenD algorithm lies in that it considers the hyperspectral image as a whole 3-order-tensor data rather than the series of vectors, therefore, we compare it to PCA and MNF which consider the HSI data as a set of spectral feature vectors and then perform the HSI compression and reconstruction only in the spectral domain. The first dataset is the Airborne Visible Infrared Imaging Spectrometer (AVIRIS) Cuprite hyperspectral image, which had been extensively investigated by researchers and actually serves as the standard data for HSI endmember extraction and spectral unmixing. The second one is a HyMap image provided by the Rochester Institute of Technology (RIT) self-test project [34]. This dataset was particularly designed for target detection and equipped with the exact locations and Spectral Library (SPL) files of all the desired targets. Therefore, it is also one of the standard datasets for hyperspectral target detection algorithms.

3.1. Performance on AVIRIS Data Set

The AVIRIS data cube is shown in Fig. 1(a), this data cube includes 190 lines, 250 rows, and 182 spectral channels. In order to comprehensively evaluate the HSI compression performance, we show the reconstructed HSI quality respect to various of compression rates. In detail, we select $d = [3, 6, 9, 12, 15]$ in PCA and MNF to plot Fig. 1(b) and the compression parameters $J_1$, $J_2$ and $J_3$ in TenD are set in line with the certain compression rate. The Mean Peak Signal to Noise Ratio (MPSNR) value is introduced to measure the quality of the reconstructed HSI by comparing it with the original HSI data. Fig. 1(b) show the MPSNR values regarding the various of compression rates by PCA, MNF and proposed TenD algorithms, respectively. It is obvious that TenD algorithm achieves the best HSI reconstruct quality in all of the compression rates.
Figure 1: (a) AVIRIS data cube (bands 28, 19, and 10 for red, green, and blue, respectively); (b) MPSNR values respect to various of compression rates in AVIRIS data cube.

We hereby use the AVIRIS data set to show the effect of the HSI compression algorithms on endmember extraction and spectral unmixing. In this experiment, the compression rate of all algorithms is fixed at 0.049. Firstly, the endmembers extracted from the original HSI data as well as their locations are recorded as the reference. In this step, the number of endmembers is estimated as 18 by the HySime [35] algorithm while the endmembers are extracted by the Vertex Component Analysis [36] algorithm. Then, we compare the endmember pixels at the recorded locations of the reconstructed HSIs with the reference, the detailed spectral curves of 18 endmember pixels are plotted in Fig. 2. It is obvious that the curves provided by TenD algorithm have the similar shape to the reference curves in all of the sub-figures. The quantitative comparison of the endmember extraction results are measured by the spectral angle mapper (SAM), as given in TABLE 1 the proposed TenD algorithm outperforms PCA and MNF in 14 endmembers of the all 18 ones in the reference.

Finally, we use the endmembers in reference to perform spectral unmixing of the original HSI cube and the reconstructed cubes, respectively, by the Un-Constrained Least Squares (UCLS) abundance estimation method. The Root Mean Squared Error (RMSE) is adopted as the metric to compare the performance. As indicated in TABLE 2, the proposed TenD algorithm performs
Table 1: SAM Values Between Extracted Endmembers in the Original HSI and Reconstructed Data Cubes.

<table>
<thead>
<tr>
<th>ID</th>
<th>1#</th>
<th>2#</th>
<th>3#</th>
<th>4#</th>
<th>5#</th>
<th>6#</th>
<th>7#</th>
<th>8#</th>
<th>9#</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCA</td>
<td>0.0113</td>
<td>0.0162</td>
<td>0.0135</td>
<td>0.0261</td>
<td>0.0214</td>
<td>0.0122</td>
<td>0.0245</td>
<td>0.0158</td>
<td>0.0193</td>
</tr>
<tr>
<td>MNF</td>
<td>0.0174</td>
<td>0.0204</td>
<td>0.0142</td>
<td>0.0353</td>
<td>0.0245</td>
<td>0.0137</td>
<td>0.0286</td>
<td>0.0175</td>
<td>0.0209</td>
</tr>
<tr>
<td>TenD</td>
<td>0.0455</td>
<td><strong>0.0126</strong></td>
<td><strong>0.0075</strong></td>
<td><strong>0.0156</strong></td>
<td><strong>0.0171</strong></td>
<td><strong>0.0080</strong></td>
<td><strong>0.0146</strong></td>
<td><strong>0.0126</strong></td>
<td><strong>0.0165</strong></td>
</tr>
</tbody>
</table>

Table 2: RMSE Values of All Algorithms in AVIRIS Data Cube.

<table>
<thead>
<tr>
<th></th>
<th>PCA</th>
<th>MNF</th>
<th>TenD</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>276.04</td>
<td>313.36</td>
<td>194.45</td>
</tr>
</tbody>
</table>

better than PCA and MNF.

3.2. Performance on HyMap Data Set

Fig. 3 shows the HyMap data cube of the RIT project, with a size of 280 × 800 × 126. In the HyMap experiment, we also firstly show MPSNR values of the reconstruct HSIs respect to various of compression rates (see Fig. 4(a)). Since we experimentally observe that this data cube can be compressed to some lower rates than the AVIRIS data cube, we select \( d = [3, 6, 9, 12, 15] \) in PCA and MNF to plot Fig. 4(a) and the compression parameters \( J_1, J_2 \) and \( J_3 \) in TenD are set in line with the certain compression rate. Similar to the technical indices reported in the previous subsection, we observe that the proposed algorithm outperforms the comparison approaches at all of the compression rates.

The target detection is performed on both the original and reconstructed HSIs by a famous hyperspectral detector, i.e., the adaptive cosine estimator (ACE) [37]. The compressed rate is fixed to even a little lower than the AVIRIS
Table 3: FARS Values of All Algorithms in HyMap Data Cube.

<table>
<thead>
<tr>
<th>Target</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>2.90×10^{-4}</td>
<td>3.25×10^{-4}</td>
<td>1.15×10^{-2}</td>
<td>5.06×10^{-2}</td>
<td>2.83×10^{-2}</td>
<td>8.76×10^{-3}</td>
<td>5.93×10^{-2}</td>
</tr>
<tr>
<td>PCA</td>
<td>2.14×10^{-4}</td>
<td>2.26×10^{-4}</td>
<td>2.24×10^{-1}</td>
<td>2.91×10^{-1}</td>
<td>1.57×10^{-1}</td>
<td>1.05×10^{-1}</td>
<td>2.07×10^{-1}</td>
</tr>
<tr>
<td>MNF</td>
<td>2.31×10^{-4}</td>
<td>3.92×10^{-4}</td>
<td>2.63×10^{-1}</td>
<td>3.62×10^{-1}</td>
<td>1.21×10^{-1}</td>
<td>3.21×10^{-2}</td>
<td>1.74×10^{-1}</td>
</tr>
<tr>
<td>TenD</td>
<td>3.13×10^{-3}</td>
<td>9.67×10^{-3}</td>
<td>1.65×10^{-2}</td>
<td>7.82×10^{-2}</td>
<td>5.62×10^{-2}</td>
<td>2.46×10^{-2}</td>
<td>9.01×10^{-2}</td>
</tr>
</tbody>
</table>

experiment (0.015). According to the RIT project [34, 38], there are 7 target of interests in this HSI scene, including 4 fabrics and 3 vehicles, the prior spectra of which are obtained and preprocessed by the SPL files. In detail, the SPL spectra are rescaled according to its reflectance factor of 100 and resampled according to the HSI wavelength, the result spectra are shown in Fig. 4(b), and feed as the input of ACE. Since the true locations of all targets are known, the detection performance could be evaluated by the False Alarm Rate (FAR), which is defined as the number of non-target pixels that have an ACE output value equal to or higher than the true target pixel value, divided by the total number of pixels in the HSI. It is evident from Table 3 that the proposed algorithm gives a superior performance for all the targets in the HyMap experiment, but only a little higher than the original HSI by the FAR value.

4. Conclusion

This paper proposes an HSI compression and reconstruction algorithm by the tensor data processing approach. Since the HSI data is treated as a 3-order-tensor by which the spatial-spectral structure could be preserved as much as possible, we introduce a tensor decomposition technology to simultaneously project the original tensor into a core tensor with much lower dimensionality in each mode, by using the factor matrices, the HSI can be reconstructed by a simply multi-linear backward projection. Compared to the spectral DR based methods, the proposed TenD algorithm can significantly preserves the HSI data.
quality and presents good performance in the following applications.

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References


Figure 2: 18 Endmembers extracted from the original AVIRIS data cube and the reconstructed data cube by three algorithms.
Figure 3: HyMap data cube of the RIT project (bands 16, 8, and 1 for red, green, and blue, respectively).

Figure 4: (a) MPSNR values respect to various of compression rates on HyMap data cube; (b) All target spectral curves equipped in SPL file.