A Novel 2-DOF Parallel Mechanism Based Design of a New 5-Axis Hybrid Machine Tool
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Abstract

In this paper, the concept of a new 5-axis hybrid machine tool is proposed. The machine is a tool being with both parallel and serial structures, which is based on a novel 2-DOF parallel platform and serial orientations. The machine tool has advantages in terms of high stiffness, high dexterity, high speed, and being capable of long components manufacturing. The kinematics characteristics, such as inverse and forward kinematics, conditioning indices, of the novel 2-DOF parallel platform are studied. The dimensional synthesis based on the workspace and conditioning indices is presented. The results of the paper are very useful for the design of the hybrid machine tool.

1. Introduction

The parallel kinematics machine (PKM) is a new type of machine tool which was firstly showed at the 1994 International Manufacturing Technology Show in Chicago by two American machine tool companies, Giddings & Lewis and Ingersoll. These machine tools, named Hexpod, were based on the paradigm of the spatial 6-DOF parallel mechanism. The parallel kinematics machine technology promises to offer manufacturers a number of advantages relative to conventional machine tools, such as a higher stiffness-to-mass ratio, higher speeds, higher accuracy, reduced installation requirements, mechanical simplicity, and high flexibility.

The six-DOF Stewart platform [1] is one PKM configuration that has been used in a number of new machine tool designs at the beginning of the born of PKMs. For machining applications, disadvantages of the Stewart platform include a complex workspace, limited orientation range of motion and a requirement of six actuators for a five degree-of-freedom task (milling, drilling, and similar operations). Moreover, there are some disadvantages for the parallel kinematics itself, e.g., the forward kinematics can not be described in closed-form, and the calibration is difficult, and so on. For such reasons, many researchers begin to pay their attentions to PKMs with less than 6 DOFs [2,3], especially, hybrid PKMs [4,5,6], such as the Tricept HPI (Neos), Hexam (Toyota), PA35 (Hitachi Seiki), and Georg V (IFW-University of Hannover). PKMs with hybrid kinematics are always built as Tripod structures, for which all points within the workspace are reachable with high dynamics and high accuracy [5] through the used parallel mechanism. By means of the two-axis wrist joint the end-effector gets the desired orientation in the workspace. By this arrangement of the kinematics the dexterity of the system can be increased compared to fully parallel kinematics (Hexapod systems). Another advantage to design a machine tool as hybrid structure based on a 3- or 4-DOF parallel mechanism, to the author's knowledge, there is no hybrid machine tool is based on a 2-DOF parallel mechanism, is that the stiffness can be improved by increasing redundant constraints.

This paper proposes another design concept for the hybrid machine tool, which is a 5-axis hybrid gantry structure based on a novel 2-DOF parallel mechanism, a two-axis wrist joint and a long movable worktable. The machine tool has following advantages: (a) high stiffness, (b) high dexterity, (c) high speed, and (d) being capable of the manufacture for long components. The machine tool has been developed by Tsinghua University and the Second Machine Tool Works of QiQiHaer in China.

Firstly, a novel 2-DOF parallel mechanism is proposed. The output of the moving platform is planar translations of a rigid body but not a point. The kinematics design, such as the workspace, inverse and forward kinematics problems, the conditioning indices, and dimensional synthesis, is discussed. The results are very useful for the design of the machine tool.

2. Description of the novel 2-DOF parallel mechanism

Two-DOF parallel mechanisms are very important systems in the family of the parallel mechanism. The existing planar two-DOF parallel mechanisms [7,8] can only position a point not a rigid body in a plane.

The novel 2-DOF parallel mechanism proposed in this paper is shown in Fig.1. A schematic of the mechanism is shown in Fig.2, where the base is labeled 1 and the moving platform is labeled 2. The moving platform is connected to the base by two identical legs. Each leg consists of a planar four-bar parallelogram: links 2, 3, 4, and 5 for the first leg; 2, 6, 7, and 8 for the
second leg. In each planar four-bar parallelogram, the joints are all revolute pairs. Links 3 and 8 are actuated by prismatic actuators, respectively. Motions of the moving platform are achieved by the combination of movements of the links 3 and 8 that can be transmitted to the platform by the system of the two parallelograms. Due to the structure, one can see that the moving platform or the rigid body 2 has two pure translational degrees of freedom with respect to the base because of the planar four-bar parallelograms. What we should notice is that the system is an over-constraint one. To obtain two DOFs of a rigid body in this system, only one planar four-bar parallelogram is enough. The reason to use two planar four-bar parallelograms is to increase the system’s stiffness and make the system symmetry.

![Figure 1: A novel planar 2-DOF parallel mechanism](image)

3. Kinematics Analysis

As illustrated in Fig.2, a reference frame $\mathbb{R}:O-xy$ is fixed to the base and a moving reference frame $\mathbb{R}' : O'-x'y'$ is attached to the moving platform, where $O'$ is the reference point on the moving platform. For the characteristic of a planar four-bar parallelogram, we can consider chains $P_iB_1$ and $P_2B_2$, as shown in Fig.2, to resolve the kinematics of the mechanism. And vectors $p_{iR}$ and $p_{jR}$ ($i=1,2$) will be defined as the position vectors of points $P_i$ in frames $\mathbb{R}'$ and $\mathbb{R}$, respectively, vectors $b_{iR}$ ($i=1,2$) as the position vectors of points $B_i$ in frame $\mathbb{R}$. The geometric parameters of the mechanism are $P_iB_j = L$ ($i=1,2$), the moving platform parameter $r$, and the distance between two guideways $2R$. And the position of point $O'$ in the fixed frame $\mathbb{R}$ is denoted as vector

$$e_{R} = (x, y)^T$$

(1)

Vectors of $b_{iR}$ in the fixed frame $\mathbb{R}$ can be written as

$$b_{iR} = (R, y_i)^T, \ b_{2R} = (-R, y_2)^T$$

(2)

where $y_i$ are actuated inputs. And vectors $p_{iR}$ in the fixed frame $\mathbb{R}$ can be written as

$$p_{iR} = p_{jR} + e_{R}, \ i = 1,2$$

(3)

Then the inverse kinematics problem of the mechanism can be solved by writing following constraint equation

$$\begin{eqnarray}
\| p_{iR} - b_{jR} \| = L \quad i = 1,2
\end{eqnarray}$$

(5)

that is

$$\begin{eqnarray}
y_1 = \pm \sqrt{L^2 - (r + x - R)^2} + y
y_2 = \pm \sqrt{L^2 - (x - r + R)^2} + y
\end{eqnarray}$$

(6)

(7)

from which we can see that there are four solutions for the inverse kinematics of the mechanism. Hence, for a given mechanism and for prescribed values of the position of the moving platform, the required actuated inputs can be directly computed from Eqs. (6) and (7). To obtain the configuration as shown in Fig.1, the "±" in Eqs.(6) and (7) should be "+".

The objective of the direct kinematics solution is to define a mapping from the known set of the actuated inputs to the unknown pose of the output platform. From Eqs.(6) and (7), the direct kinematics of the mechanism can be described as

$$x = ay + b$$

(8)

where

$$a = \frac{y_2 - y_1}{2(R - r)}, \ b = \frac{y_1^2 - y_2^2}{4(R - r)}$$

(9)

and

$$y = \frac{-f \pm \sqrt{f^2 - 4eg}}{2e}$$

(10)

in which

$$e = a^2 + 1, \ f = 2ar + b - R - 2y_1, \ g = (r + b - R)^2 + y_1^2 - L^2$$

(11)

To obtain the forward configuration as shown in Fig.1, the "±" in Eq.(10) should be "-".

From above equations, we can see that the inverse and direct kinematics problems of the mechanism are very easy and can be described as closed-forms.

![Figure 2: A schematic of the mechanism](image)
4. Jacobian matrix and conditioning indices

Equation (5) can be differentiated with respect to time to obtain the velocity equations. This leads to an equation of the form
\[ A\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = B\begin{bmatrix} \dot{x} \\ y \end{bmatrix} \]  

where \( A \) and \( B \) are, respectively, the \( 2 \times 2 \) inverse and forward Jacobian matrices of the mechanism and can be expressed as
\[ A = \begin{bmatrix} y - y_1 & 0 \\ 0 & y - y_2 \end{bmatrix}, \quad B = \begin{bmatrix} r + x - R & y - y_1 \\ x - r + R & y - y_2 \end{bmatrix} \]  

If the matrix \( A \) is not singular, the Jacobian matrix of the mechanism can be obtained as
\[ J = A^{-1}B = \begin{bmatrix} (r + x - R)/y - y_1 \\ (x - r + R)/y - y_2 \end{bmatrix} \]  

Let
\[ |E \lambda - J| = 0 \]  
where
\[ E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]  

and there is
\[ \lambda^2 + s \lambda + t = 0 \]  

where
\[ s = \left( \frac{r + x - R}{y - y_1} + 1 \right), \quad t = \frac{r + x - R}{y - y_1} \cdot \frac{x - r + R}{y - y_2} \]  

Then the eigenvalue of Jacobian matrix \( J \) can be obtained,
\[ \lambda = \frac{-s \pm \sqrt{s^2 - 4t}}{2} \]  

As well known, the dexterity of a parallel mechanism can be evaluated by the conditioning index (CI) \( 1/\kappa \) [9], which is written as
\[ 1/\kappa = \lambda_2/\lambda_1 \]  

where \( \lambda_1 \) and \( \lambda_2 \) are the maximum and minimum eigenvalues of the Jacobian matrix, which can be obtained from Eq.(18), respectively. The corresponding global conditioning index (GCI) will be
\[ \eta = \frac{\int_w 1/\kappa dW}{\int_w dW} \]  

where \( W \) is the reachable workspace of the mechanism.

In the conditioning indices, the singular configurations should be avoided. From above analysis, one can see that the mechanism is very simple. And the singularity analysis will also be simple, which can be reached from Jacobian matrices \( A \) and \( B \). When \( |A| = 0 \) and \( |B| \neq 0 \), from Eq.(13), there is \( y = y_1 \) or \( y = y_2 \), which means that the first or second leg is parallel to \( x \) axis. This corresponds to the first kind of singularity. If \( |B| = 0 \) and \( |A| \neq 0 \), the second kind of singularity occurs, i.e., \( r + x = R \) for the first leg when \( x \) is positive and \( R + x = r \) for the second leg when \( x \) is negative. In such case, the mechanism is in the configuration that four bars of the parallelogram in one of the two legs are parallel to each other. \( |B| = 0 \) and \( |A| = 0 \) lead to the third kind of singularity, in which the two legs are both parallel to \( x \) axis. The geometric parameter condition for this singularity is \( r + R = L \).

5. Workspace of the mechanism

One of the most important issues in the process of design of the mechanism is its workspace. For parallel mechanisms, this issue may be more critical since parallel mechanisms will sometimes have a rather limited workspace.

The workspace of the planar 2-DOF parallel mechanism is often represented as a region of the plane. And the determination of the workspace is more simply, which can be obtained geometrically from the inverse kinematics equation. From Eq.(5), one obtains
\[ (r + x - R)^2 + (y - y_1)^2 = L^2 \]  
\[ (x - r + R)^2 + (y - y_2)^2 = L^2 \]  

which means that if \( y_i \) are specified, Eqs.(21) and (22) represent two circles centered at \((R - r, y_1)\) and \((r - R, y_2)\), respectively. Their radii are \( L \). If \( y_i \in [y_{i_{\min}}, y_{i_{\max}}] \), Eqs. (21) and (22) represent two enveloping surfaces, each of which is the locus of a circle (the radius is \( L \)), when the center is rolling on line segments \( x = R - r \) and \( x = r - R \), respectively. The intersection of the two enveloping surfaces is the workspace of the mechanism.

For example, the workspace of such a mechanism with \( R = 4, r = 1, L = 6 \) and \( y_i \in [-2, 2] \) is shown in Fig.3.

![Figure 3: A workspace example](image)

6. The dimensional synthesis

6.1 The dimensional synthesis based on the workspace
The objective of the dimensional synthesis is to determine geometric parameters of the mechanism for a desired workspace. The parameters are $R$, $r$, $L$ and the input $|y_{\text{min}} - y_{\text{max}}|$. In this paper the desired workspace is assumed to be given by a rectangle $x_w \times y_w$.

Because the moving platform only has translational DOFs, the parameter $r$ will not bring any effect to performances of the mechanism. The determination of parameter $r$ is depended on the designer’s demand, in this paper $r = 75$mm. One of advantages of the mechanism is that it gives a total freedom in the choice of the workspace in $y$ direction, as shown in Fig.2. The workspace volume along $y$ - axis, denoted as $y_w$, will not effect the parameters $R$ and $L$ but the input $|y_{\text{min}} - y_{\text{max}}|$, for which we can consider firstly the workspace volume along $x$ - axis, denoted as $x_w$, to determine parameters $R$ and $L$. When the moving platform reaches the boundary of the desired workspace, as shown in Fig.4, there are

$$\cos \alpha = \frac{d + x_w - r}{L}, \quad \sin \beta = \frac{d - r}{L}$$

(23)

Figure 4: Dimensional synthesis of the mechanism

When the moving platform travels $x_w$ along $x$ - axis, the input, denoted as $B_{12}$, of each leg should be

$$B_{12} = L(\cos \beta - \sin \alpha)$$

(24)

from which we can see that if $\alpha = \beta$ there is $B_{12} = x_w$, which means that the ratio between the input and the output is

$$B_{12} : x_w = 1 : 1$$

(25)

Actually, we hope that the effect of the input to output should be small, i.e., $B_{12} : x_w > 1 : 1$. If the condition of Eq.(25) is considered in the design of the mechanism, which means $\alpha = \beta$, Eq.(23) can be rewritten as

$$d = \frac{(x_w - r) \tan \alpha + r}{1 - \tan \alpha}$$

(26)

from which one can obtain $d = 417.5$mm if $\alpha = 10^\circ$. The parameter $\alpha$ can get other value, which will depend on the designer’s demand. But if $\alpha$ is very small the configuration of the mechanism as shown in Fig.4 will be near the singularity. What is more, if $\alpha = 0$, it is not difficult to find out that the 2-DOF parallel mechanism proposed in this paper reaches singularity. Other parameters $R$, $L$, and $|y_{\text{min}} - y_{\text{max}}|$ will be obtained as

$$R = 2d + x_w, \quad L = d - \frac{r}{\sin \beta}$$

(27)

and

$$|y_{\text{max}} - y_{\text{min}}| = x_w + y_w$$

(28)

Therefore, if $x_w = 1.6$m, $y_w = 1.0$m, $\alpha = 10^\circ$, and $r = 75$mm, we will obtain

$$R = 1217.5$$mm, $L = 1972.5$mm,

$$|y_{\text{max}} - y_{\text{min}}| = 2600.0$mm

(29)

6.2 The dimensional synthesis based on the CI

As shown by Strang [10], the condition number of a matrix is used in numerical analysis to estimate the error generated in the solution of a linear system of equations by the error on the data. When applied to the Jacobian matrix, the condition number will give a measure of the accuracy of the Cartesian velocity of the end effector and the static load acting on the end effector. It can also be used to evaluate the dexterity and stiffness of a manipulator [9,11]. After we reach the results of dimensional design based on workspace, we should consider the stiffness of the mechanism, which will make our design better.

In this Section, what we should do is to find out relationship between the conditioning indices and parameters of the mechanism. So that we can obtain the optimal parameters of the mechanism for the given workspace $1.6$m$\times$1$\times(x_w \times y_w)$. Firstly, the distribution of the conditioning index $1/\kappa$ in $x_w$ of the workspace is plotted as shown in Fig.5, in which the parameter $L = 1972.5$mm and $R$ is specified as $R \in [x_w/2, L + r - x_w/2]$. From Fig.5, we can see that

- The conditioning index is symmetric with respect to $x = 0$;
- The index reaches its maximum value when $x = 0$, for any value $R$.

In the process of dimensional synthesis, only $x_w$ is considered. The GCI, Eq.(20), will be rewritten as

$$\eta = \int_{x_w} 1/\kappa dW / \int_{x_w} dW$$

(30)

The relationship between GCI and parameter $R$ is illustrated as Fig.6, from which one obtains that if $L = 1972.5$mm, the GCI reaches its maximum value $\eta = 0.5616$ in $R \in [1203, 1210]$. And the
The dimensional synthesis result will be:

\[ R = 1203.0 \text{mm} , \quad L = 1972.5 \text{mm} , \quad |y_{i,\text{max}} - y_{i,\text{min}}| = 2528.04 \text{mm} \quad (31) \]

The index to evaluate the volume of the mechanism is defined roughly as

\[ V = R \times |y_{i,\text{max}} - y_{i,\text{min}}| + L \times 4 \quad (32) \]

Then, there is \( V = 3.05 \text{m}^3 \) for the design.

**Figure 5:** Distribution of the CI in workspace when \( L \) is specified

**Figure 6:** The relationship between the GCI and \( R \)

Similarly, when \( R = 1217.5 \text{mm} \) is specified, the corresponding maps are shown in Fig.7 and Fig.8, respectively, from which we can reach another dimensional synthesis result:

\[ R = 1217.5 \text{mm} , \quad L = 2061.5 , \quad |y_{i,\text{max}} - y_{i,\text{min}}| = 2342.6 \text{mm} \quad (33) \]

The maximum value of GCI will be \( \eta = 0.5710 \) in \( L \in [2061.5 , 2068.5] \), and there is \( V = 2.86 \text{m}^3 \). It is clear that the workspace/volume ratio of the later design is less than that of the former design.

### 7. The design of a 5-axis hybrid machine tool

Recently, more and more hybrid machine tools [4,5,6] based on parallel mechanisms are proposed because of their high speed, high mobility, and high flexibility. According to demands of the Second Machine Tool Works of QiQiHaer in China, which are high speed, high mobility, high stiffness, and being capable of long components manufacturing, e.g., the vane, a 5-axis hybrid machine tool with gantry structure is proposed, as shown in Fig.9. The machine tool is based on the planar 2-DOF parallel mechanism proposed in this paper, which provides the machine tool with high stiffness and high speed. Especially, the upper and lower links of each of the two planar four-bar parallelograms are substituted by two plates, which can improve the system’s stiffness. By means of the two-axis wrist joint the end-effector gets the desired orientation in the workspace, which provides it with high mobility. Only single-DOF joints are used in the machine tool, which can increase the accuracy. The worktable can move freely along the \( z \)-axis, which endows the machine tool with the capability of manufacturing long components.
upper plate and the lower plate, the moving platform is
design as trapezoidal profile, as shown in Fig.9.

One can see that, in this parallel mechanism design,
all joints are single-DOF ones, which means that it is not
difficult to fabricate all joints with high accuracy and
fine tolerance.

The new hybrid machine tool is developed in
associate with the Second Machine Tool Works of
QiQiHaer as shown in Fig.10. Now the device is under
test to machine a kind of vane. Other properties about
this machine tool will be reported in the future work.

Figure 9: A new 5-axis hybrid machine tool

Figure 10: The developed 5-axis hybrid machine tool

8. Conclusion

Recently, the parallel kinematics machine with less
than 6 DOFs becomes more attractive. In this paper, a
novel planar 2-DOF parallel mechanism is proposed, the
design theory is studied in detail. And a new 5-axis
hybrid machine tool is presented based on the 2-DOF
parallel mechanism, motivated by the application in the
Second Machine Tool Works of QiQiHaer in China. The
machine tool has many advantages, such as high speed,
high stiffness, high mobility, high accuracy, and being
capable of long components manufacturing. We believe
that the machine tool will be more interesting in the
manufacturing.

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