The research of an automatic object reconstruction method based on
limit visible region of the laser-scanning vision system

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ABSTRACT
This paper presents an automatic multi-view selection approach for 3D reconstruction by means of a
laser scanning vision system. It is realized based on the visual region of the laser scanning vision
system. The candidate next best view (NBV) position is obtained by computing the unknown space area
according to the limit visual region of the vision system. The final NBV position, which can acquire the
maximal visual area, is obtained by comparing the above candidate’s view positions. Experimental
results show successful implantation of the proposed view planning method for digitization and
reconstruction of freeform objects.

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1. Introduction

At present, active vision systems such as laser scanning vision
system or the structured light system is widely used for acquiring
the 3D data from the surface of the object, particularly in reverse
engineering (RE) of geometry [1]. However, the above system
with substantive manually-selected viewpoint and controlled
mechanism is used for this purpose. We are interested to 3D
geometry data of the unknown objects rather than the process of
acquiring them. The vision system, which is able to manipulate
corresponding parameters and performs active behaviors, such as
active sensing, active placement and active reconstruction, will be
very widely used in RE field.

A kind of this system is achieved by combining a fixed scanner
with a turntable in [2]. A more flexible solution is described in [3]
where a coordinate measuring machine is used in combination
with a laser scanner. A recent development by the same author is
presented in [4]. Another autonomous system is described in [5],
where the authors use a range camera, a turntable and an
industrial robot. The active sensor system could be capable of
actively placing the sensor at several viewpoints through a
planning strategy. So their path planning is a next best view
(NBV) problem. The viewpoints are selected by reasoning about
the state of knowledge of imaging work space [4]. Callieri et al. [5]
focuses on the use of a vision sensor to perform the volumetric
modeling of an unknown object in an entirely autonomous
fashion. An entropy-based objective function is used to model
the object or scene in areas of occluding contours. A local maximal
of this function is found to determine the best next view position.
Reed [6] determines the visibility volume, which is the volume of
space within which a sensor has an unobstructed view of a
particular target. Massions and Fisher [7] built on previous voxel
occupancy methods by using the weighted sum of visibility and
“quality” factors as an NBV objective function. Bottino and
Laurentini [8] presented a general approach to interactive,
object-specific volumetric algorithm, based on a necessary
condition for the best possible reconstruction to have been
performed. The outlined approach can be applied to any class of
objects. The proposed algorithms just only suit to simple convex
polyhedra but not for general polyhedra, especially nonconvex
polyhedra. Mauer and Bajcsy [9] describe an automated occlu-
sion-guided view determining system for scene reconstruction.
The system is given a priori knowledge of the environment and
sensor geometry and returns the next view position from which a
complete range image of the surface visible to the camera may be
obtained. The system then computes new scanning planes for
further 3-D data acquisition based on the discontinuities (occlu-
sions) in the most recent range image. Mauer et al. [10] propose
an NBV system using the max–min principle as a heuristic. The
system selects from all possible viewing orientates the one that
maximizes the amount of new information. However this solution
is limited to a particular sensor configuration. Another method is
contour following [11,12]. It works best with objects’ large size
relative to sensor coverage and for relatively simple shapes with
smoothly flowing lines. Sablatnig et al. [13] present an approach
to next view planning for shape from silhouette for 3D shape reconstruction with minimal different views. Pito [14] presents a system that automatically acquires a surface model of an arbitrary part. The concept of positional space is introduced as a basis for view representation. Two types of information are recovered from range data: the visible surface and the void surface. The criterion for determining the next view position is set by using the void surface area visible from a point. Yuan [15] describe the view planning based on mass vector chain (MVC) mechanism. With MVC, only the viewpoint orientate is obtained, but the position of viewpoint is difficult to decide by this method. Scott [16] presents a model-based view planning algorithm based on the modified measurability matrix (3M), which is an enhancement and extension of Tarbox’s [17] measurability matrix (2M) concept. The multi-stage model-based view planning approach separates scene exploration from precise measurement. Other view planning methods are proposed in [18,19].

The aim of active vision system is to determine the pose and settings of a vision sensor or the object for undertaking a vision task. It is an important problem in view planning that how to automatically effective and efficiently acquire 3D model of object with a priori object knowledge and sensor placement. To an arbitrary vision system the field of view (FOV) and depth of field (DOF) are limited. To deal with this issue, an easy method to acquire the DOF and FOV is developed. And based on it, the visual region of the laser scanning vision system is obtained. The method is developed for automatic 3D reconstruction with the arbitrary partial object surface information already acquired.

After the first view was obtained at arbitrary pose, a series of next view pose $P_{i+1}$ ($i=0,1,...,n$) must be determined so that the reconstruction process can continue until the whole object is obtained. During such a process, three spaces need to be discussed. Surface space $S$ is the set of sampled 3D object surface points, and visual space $V$ is the region of the vision system defined by the effective measurement space with the satisfied measurement precision. The system work space $I$ is defined as the system running space position avoiding the collision. For a given vision system and the target object, it is important to find a suitably short view plan satisfying the reconstruction goals and within an acceptable computation time. That is, if $V \cap S = \text{max}$ is happened at next view with the reasonable system work space, a view planning algorithm would be accepted.

The purpose of this paper is to present a new planning approach of generating 3D models automatically to the arbitrary initial position. The new view planning method is proposed based on the limit visual region of the laser scanning vision system. Firstly, the limit visual region is obtained according to the surface reconstruction precision, and then it is modeled based on the known object knowledge obtained from the initial view. Second, the visibility of the limit visual region at next viewpoint is analysis. Finally, the position that can make $V \cap S = \text{max}$ is defined as the next best view position.

### 2. The visual region of the vision system

#### 2.1. The 3D vision system for measurement

The developed laser line scanning vision system is shown in Fig. 1: it consists of a CCD camera, laser line generator and stepper motor driven linear slide providing X and Z scanning motions for the camera and laser. The length of the laser line is 500 mm and the laser line is moved up $s$ steps (which is 50 mm) along Z-axis. The object is placed on a rotary table that can rotate 360° with Z and linear slide providing Y motions. The range of the X scanning motion is $[-400 \text{ mm}, 400 \text{ mm}]$, Y is $[−300 \text{ mm}, 300 \text{ mm}]$ and Z is $[0, 400 \text{ mm}]$. The camera and laser line generator are mounted on the stepper motor driven linear slide, with the laser line perpendicular to the rotary table and an angle between the camera’s optical axis and the laser line. The objective of the vision system is to provide a 3D profile of the object based on a coordinate system defined on the rotary table.

$A(B)$ denotes the point in the nearest (farthest) planar objects. $\|O_A\|$ $(\|O_B\|)$ denotes the nearest (farthest) measurement distance. The line $O_AO_B$ is represented the laser plane, and $O_I$ is represented the visual plane of the camera. $O_I$ is the center of the laser sensor and the camera, respectively.

While planning a viewpoint to observe the object, the parameters of the sensor position, orientation and other settings need to be determined. Particularly, the viewpoint should be planned at an optimal place where it is feasible in the practical environment. In this paper, the rotary angle and translation distance of the table are determined with the obtained surface data of the unknown object.

#### 2.2. Determining the depth of field (DOF) of the vision system

An object surface is measured by the vision system with a certain distance between them, and the surface is fitted simultaneously. If the fitting accuracy belongs to the allowable errors range, then this distance is regarded as effective measurement distance, or else as invalid. According to this method, it is easy to obtain the effective DOF range of the vision systems by moving the rotary table along the Y direction (as shown in Fig. 1). In order to simplify the fitting process of the acquired data, here the planar object is used.

#### 2.3. Determining the field of view (FOV)

If a point on the surface is outside the FOV, it will not be detectable to the CCD camera. But in this vision system, the FOV has satisfied two constraints: (1) the laser line is not occluded by the object surface in order to be detected by the CCD camera; (2) the CCD camera is not occluded by the object surface. So, the FOV is determined by elimination of these two blocks.

Firstly, one planar object is fixed on the rotary table and passed the rotary table center. The initial position of the plane is shown in Fig. 2. The angle between the normal vectors of the plane and the plane of the laser is defined as the left (right) visual angle, and is equal to the rotary angle of the table driven with stepper motor. So, the left (right) visual limit angle is defined as the maximum rotary angle, which the projection of the laser on the plane could be detected by the CCD camera at certain measurement position.

In order to find the effective visual region of the vision system, we need to get the function relationship between left (right) visual limit angle and the arbitrary measurement position under
the effective DOF range. So, let the planar object move from the nearest measurement distance along \( Y \) direction step-by-step (there step length is \( |AB|/n \), which is set to 8 mm). The process figure of acquiring data is shown in Fig. 3. In there, the fitting accurate of planar object is set to 0.02 mm. From the experiments, the nearest and farthest measurement distances are 64 and 186 mm respectively. So the effective measurement distance is 122 mm.

In Fig. 3, \( P_{\text{min}}, P_{\text{arb}} \) and \( P_{\text{max}} \), respectively, denote the nearest, random and farthest plane position of the vision system at the effective measurement range. \( A, B \) and \( C \) denote the projection point of laser plane at the above three plane positions, respectively. \( y_{\text{il}}(y_{\text{ir}}) \) is the left limit visual angle (the right limit visual angle) of the vision system, which denotes the angle between the normal vector \( n_{\text{il}}(n_{\text{ir}}) \) of the tangent plane at the object surface point and the laser plane. Namely, if the angles between the normal vector of the tangent plane at the arbitrary object surface and the laser plane belongs to the area \( y_{\text{il}}(y_{\text{ir}}) \), the surface point is a visible point to the vision system.

The corresponding left (right) visual limit angle is obtained at each position by the above method. Finally, \( n+1 \) pairs of the left (right) visual limit angle \( \theta_{\text{il}}(\theta_{\text{ir}}) \) and the measurement distance \( d_i \) (\( i=0,1, \ldots, n \)) are acquired, is given in Fig. 4.

From Fig. 4, we know the limit visual angle is increased with the increase in measurement distance until it is close to a fixed value. According to \( n+1 \) pairs of data, \( c_i \) (\( i=0,1,2 \)) is obtained in Eq. (1) by the least square method. The fitting figure is given in Fig. 5. So, the relation equations of them to this vision system are written as Eqs. (2) and (3):

\[
\theta_{\text{il}} = \begin{cases} 
0.0053d + 0.2731, & 64 \leq d \leq 160 \\
1.1211, & 160 < d \leq 186 
\end{cases} \tag{2}
\]

\[
\theta_{\text{ir}} = \begin{cases} 
0.0103d + 0.2278, & 64 \leq d \leq 116 \\
1.4226, & 116 < d \leq 186 
\end{cases} \tag{3}
\]

where \( d \) denotes the measurement distance (64 mm \( \leq d \leq 186 \) mm), \( \theta_{\text{il}}(\theta_{\text{ir}}) \) is the left (right) limit visual angle and the units are in radians.

Assume the edge point of the object surface is known, a limited visual curve on \( XY \) plane is obtained according to the above presentation. When \( z \) is varied, the surface is constructed from a series of the limit visual curves, which is denoted as the limit visual surface. It represents the limit visual position of the object surface, which is visible to the vision system.

The coordinate system is constructed with the rotary table center \( O_T \). \( P(x,y) \) is an arbitrary point at the visible visual range. Assume the initial distance between the vision system and the rotary table is \( d_0 \). So, \( |O_TP| \) is equal to \( d_0 + y_{\text{mm}} \). The figure of the limit visual curve is shown in Fig. 5.

Assume the edge point of the object surface is known, a limited visual curve on \( XY \) plane is obtained according to the above presentation. When \( z \) is varied, the surface is constructed from a series of the limit visual curves, which is denoted as the limit visual surface. It represents the limit visual position of the object surface, which is visible to the vision system.

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From Fig. 6, we know the limit visual curve is the critical curve between the seen region and the unseen region. In Fig. 6, if the edge point \( P_0(x_0,y_0) \) of the object surface is known, the limit visual curve (shown as \( P_0P_2P_2 \), \( P_1P_2 \) in Fig. 6) of the vision system will be acquired in the effective measurement distance; there \( P_1P_2 \) is
the line. According to Eqs. (2) and (3), we know
\begin{equation}
\theta_b = \begin{cases} 
0.0053(d_0 + y) + 0.2731, & y_0 \leq y \leq 160 - d_0 \\
1.1211, & 160 - d_0 < y \leq 186 - d_0 
\end{cases} 
\end{equation}
\begin{equation}
\theta_a = \begin{cases} 
0.0103(d_0 + y) + 0.2278, & y_0 \leq y \leq 116 - d_0 \\
1.4226, & 116 - d_0 < y \leq 186 - d_0 
\end{cases} 
\end{equation}
where \( y_0 \leq y \leq 186 - d_0 \) and \( \theta_a(\theta_b) \) is the left (right) limit visual angle.

From Fig. 6(a) and combining with Eq. (4), the left limit visual curve is written as
\begin{equation}
y = y(\theta_b) = \begin{cases} 
188.7 \theta_b - 51.5 - d_0, & \theta_b < 1.1211 \\
160 - d_0, & \theta_b = 1.1211 
\end{cases} 
\end{equation}

According to the geometric relation in Fig. 6(a), we know
\[ \frac{\partial y}{\partial \theta_a} = -188.7 \cot \theta_a, \quad \theta_a < 1.1211 \]
\[ \frac{\partial y}{\partial \theta_a} = -\tan(1.1211) - 2.1, \quad \theta_a = 1.1211 \]
Solving Eq. (7) we have
\begin{equation}
\begin{cases} 
x = x(\theta_a) = -188.7 \ln(\sin \theta_a) + C_1, & \theta_a < 1.1211 \\
y = -2.1x + C_2, & \theta_a = 1.1211 
\end{cases} 
\end{equation}
Combining (6) with (8), the left limit visual curve of the known area is obtained:
\begin{equation}
\begin{cases} 
x = -188.7 \ln \left( \frac{y + d_0 + 51.5}{188.7} \right) + C_1, & y_0 \leq y \leq 160 - d_0 \\
y = -2.1x + C_2, & y > 160 - d_0 
\end{cases} 
\end{equation}

According to the same principle, to the right limit visual surface, we have
\begin{equation}
\begin{cases} 
x = 97.1 \ln \left( \frac{y + d_0 + 22.1}{97.1} \right) + C_3, & y_0 \leq y \leq 116 - d_0 \\
y = 6.7x + C_4, & y > 116 - d_0 
\end{cases} 
\end{equation}

3. The view planning strategy

To an unknown object with complex surface shape, it is difficult to predict its 3D shape only according to partial obtained surface information. However as the above section, the limit visual area model of the vision system would be constructed according to the partial obtained object surface information. By this model, a sequence of viewing poses and corresponding sensor configurations for acquiring the 3D surface data of the object would be generated. Finally, the position, where obtained the maximal visual area, is determined as the next best view pose.

During the process of the automatic 3D reconstruction, the first view is obtained at an arbitrary pose, a 3D depth map is created and the initial object model is modeled. Fig. 7 shows the relation between the partial object surfaces obtained from the initial viewpoint and the limit visual surfaces at this viewpoint.

Suppose the initial knowledge of the model has been got at viewpoint 1 (shown as the solid line \( A_1 B_1 C_1 D_1 \) in Fig. 7). Then the view planning strategy is presented: (1) with the partially acquired object surface, the left (right) limit visual surfaces \( D_1 F_x(A_1 H_1) \) (shown as the dashed line in Fig. 7) is constructed to predict the unknown area information. (2) The corresponding configuration parameters of the vision system about the rotation angle and the translation distance are computed according to the limit visual surface. (3) The corresponding visible surface area is obtained and compared the value of them to select the final viewpoint position as the NBV position where visible surface area is the largest.

3.1. Analyzing the visibility of next viewpoint

Based on the prediction model of the unknown area in Fig. 7, the visibility of next viewpoint would be obtained. Without loss of generality, the left limit visual surface is selected as an illustration to explain how to analyze its visibility. The limit visual surface
consists of the curve surface $D_1E_1$ and the plane $E_1F_1$. The details are analyzed as follows:

(1) Visual angle analysis of the curve surface $D_1E_1$: when the object is rotated $\theta$ around the $z$ axis to obtain viewpoint 2, the initial limit visual surface $D_1E_1$ would reach the new position $D_2E_2$ (shown as the solid line in Fig. 8(a)). And then, the new left limit visual curve $D_2E_2L$ is obtained at viewpoint 2 as above mentioned (shown as the dashed line in Fig. 8(a)). Here we give the conclusion: the whole curve $D_2E_2$ would be visible if both the start point $D_2$ and end point $E_2$ on the curve are visible.

The proof is:
Assume that there exists an invisible point $I_2$ (whose corresponding point is $I_{2L}$ on the curve $D_2E_{2L}$) on the curve $D_2E_2$, so the corresponding visual angle $\theta_{2L} > \theta_{2L}$. Because the point $D_2$ is visible, the equation is satisfied: $\theta_{2L} \leq \theta_{2L}$. According to above mentioned, the function relation between them is linear. So, the relative variety of the angle between the normal vector of the point on the curve $D_2E_2(L_{2L})$ and the laser plane is obtained, respectively, as

$$\frac{\Delta \theta}{\Delta y} = \frac{\theta_{2L} - \theta_{2L}}{y_{2L} - y_{2L}}$$

and

$$\frac{\Delta \theta_{2L}}{\Delta y} = \frac{\theta_{2L} - \theta_{2L}}{y_{2L} - y_{2L}}\quad \Delta \theta_{2L} / \Delta y > \Delta \theta_{2L} / \Delta y$$

satisfied according to the assumption. And then, the following equations are obtained:

$$\theta_{2L} = \theta_{2L} + \frac{\Delta \theta}{\Delta y}(y_{2L} - y_{2L}) \quad (11)$$

$$\theta_{2L} = \theta_{2L} + \frac{\Delta \theta_{2L}}{\Delta y}(y_{2L} - y_{2L}) \quad (12)$$

From (11) and (12), the conclusion ($\theta_{2L} > \theta_{2L}$) is obtained. It shows the point $E_2$ is invisible, which is contradictory to the condition that the point $E_2$ is visible. So the above conclusion is accurate.

When the limit visual curve rotates to viewpoint 3 (shown as $D_3E_3$ in Fig. 8(b)), there definitely exists a point $P_3$ on the curve $D_3E_3$ whose angle between its normal vector and laser plane is zero. The left and right limit visual curves $P_3E_{3L}$ and $P_3D_{3R}$ are obtained from the point $P_3$ (shown as the dashed line in Fig. 8(b)). The curves $P_3E_{3L}$ and $P_3D_{3R}$ are visible if the start point $D_3$ and end point $E_3$ are visible based on the above conclusion. Thus the curve $D_3E_3$ is visible.

(2) Visibility analysis of plane $E_1F_1$: because all the angles between normal vectors of points on the plane and the laser plane are equal, the limit visual angle of the point on the object surface would be increased with the increase in measurement distance. Accordingly we could get the conclusion: the plane within the visual region of the vision system should be visible if the points of the plane closest to the vision system are visible.

It is known that the visual angle of plane $E_1F_1$ is 1.1211 rad (shown in Fig. 9), and the visibility of the plane $E_1F_1$ in the next viewpoint is relevant to the rotary angle $\theta$. The plane $E_2L_2$ would be visible if the point $E_2$ was visible when $\theta \leq 1.1211$ rad. If the point $L_2$ was visible, the plane $E_3L_3$ would be visible when $\theta > 1.1211$ rad.

$L_2(L_3)$ are the points that the plane $F_2E_2(F_3E_3)$ intersects with the limit measurement position of the vision system, and is defined as the limit measurement point.

The visibility of the right limit visual surface would be obtained based on the same principle.

As discussed in the previous section, the visual criteria of the limit visual surface are:

(1) As rotary angle $\theta < 1.1211$ rad, it is visible when the start and end points of all limit curves are visible;
(2) As $\theta > 1.1211$ rad, it is visible when the start point and the limit intersection point of all limit curves are visible.

3.2. NBV algorithm

Based on the visibility of the limit surface, the pose of NBV would obtain the maximum visual region, which depends on the length of the limit curve. So, the NBV algorithm is to find with the best parameters of the vision system (such as best rotary angle and translation distance) to obtain the maximum of the limit curves at the visual region of the vision system.

3.2.1. Acquiring the parameters of the vision system based on the visibility of the start and end points of the limit curve

When the table rotates counterclockwise by $\theta$ to the next position, the relationship of the coordinates between the two positions is

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

(13)

where $(x_1, y_1)$ and $(x_2, y_2)$ are the point coordinates at the viewpoint 1 and viewpoint 2, respectively.

The limit visual angle $\theta_{L}$ of the local boundary point $D_{l}(x_{dL}, y_{dL})$ is obtained by Eq. (4). The visibility of the start and end points of the limit visual curve is relevant to the rotary angle $\theta_{L}$ and translation distance $d_4$ during the process of the view planning. Here $d_4$ is positive when moving along $+Y$ direction, else is negative.

Based on the visible conclusion of the rotation, the visibility of the limit visual curve is analyzed as following:

(1) After rotating angle $\theta$, the left boundary point $D_2$ is still at the left plane (shown in Fig. 10(a)):
If the start point $D_2$ is visible and the $\theta_{d_1} < \theta_{d_1 \text{, limit}}$, where $\theta_{d_1}$ is the angle between the normal vector of the point $D_2$ and the laser plane at viewpoint 2, then the curve $D_2F_2$ is translated along the negative $Y$ axis to the limit position $D_2F_{2 \text{, limit}}$, where the visual angle of the point $D_2$ is equal to the left limit visual angle. The corresponding $y$ coordinate ($y_{d_1}$) of the point $D_2$ is computed by Eq. (6). From Eq. (13) the position of the point $D_2$ at viewpoint 2 is:

$$y_{d_1} = y_d \sin \theta_d + y_d \cos \theta_d$$

So the translation distance $d_d$ of the vision system is

$$y_{d_1} - y_{d_1} \leq d_d \tag{14}$$

If $D_2$ is invisible or $\theta_{d_1} < \theta_{d_1 \text{, limit}}$, the curve $D_2F_2$ is translated along the positive $Y$ axis until $D_2$ is visible. So, the translation distance $d_d$ is

$$d_d \leq 186 - d_0 - y_{d_1} \tag{15}$$

where 186 mm is the maximal measurement distance of the vision system, $d_0$ is the initial distance between the vision system and the table.

Combining Eqs. (14) and (15), the translation distance is

$$y_{d_1} - y_{d_1} \leq d_d \leq 186 - d_0 - y_{d_1} \tag{16}$$

If the end point $E_2$ is visible and the $\theta_{d_2} < \theta_{d_2 \text{, limit}}$ (shown in Fig. 10(b)), where $\theta_{d_2}$ is the angle between normal vector of the point $E_2$ and the laser plane at viewpoint 2, then the curve $D_2F_2$ is translated along the negative $Y$ axis to the limit position $D_2F_{2 \text{, limit}}$, where the visual angle of the point $E_2$ is equal to the left limit visual angle. And the corresponding $y$ coordinate ($y_{d_2}$) of the point $E_2$ is computed by Eq. (6). From Eq. (13) the position of the point $E_2$ at viewpoint 2 is:

$$y_{d_2} = C_2 \sin \theta_e + (280 - d_0) \cos \theta_e$$

So the translation distance $d_e$ of the vision system is

$$y_{d_2} - y_{d_2} \leq d_e \tag{17}$$

If $E_2$ is invisible or $\theta_{d_2} < \theta_{d_2 \text{, limit}}$, the curve $D_2F_2$ is translated along the positive $Y$ axis until $E_2$ is visible. So, the translation distance $d_e$ is

$$d_e \leq 186 - d_0 - y_{d_2}$$

Combining Eqs. (17) and (18), the translation distance is

$$y_{d_1} - y_{d_1} \leq d_e \leq 186 - d_0 - y_{d_1}$$

(2) Similarly, the left boundary point $D_2$ is located the right plane after rotating angle $\theta$, visibility analysis of the start point ($D_2$) is obtained by the same method (shown in Fig. 10(c)). The translation distance is

$$y_{d_1} - y_{d_1} \leq d_e \leq 186 - d_0 - y_{d_1}$$

And the visibility analysis of the limit intersection point ($M_2$) is obtained with the same method as shown in Fig. 10(d). The translation distance is

$$y_{m_1} - y_{m_1} \leq d_e \leq 186 - d_0 - y_{m_1}$$

In conclusion, the potential rotation range ($\theta_a \leq \theta \leq \theta_0$) is obtained according to the visibility of start point and end point. After the rotary angle is certain, the visible translation distance of the start point is $d_{\text{min}} \leq d_e \leq d_{\text{max}}$, and to the end point the translation distance is $d_{\text{min}} \leq d_e \leq d_{\text{max}}$. So, the final corresponding translation range is $d_{\text{min}} \leq d_e \leq d_{\text{max}}$, where $d_1(\theta) = \max(d_{\text{min}}, d_{\text{min}})$ and $d_2(\theta) = \min(d_{\text{max}}, d_{\text{max}})$. In the same way, the rotary angle and translation distance which make the right limit visual curve visible at the next viewpoint can be obtained.

### 3.2.2 Determining the pose of NBV

As mentioned at the outset, the next viewpoint position can be represented by the rotary angle $[\theta_a, \theta_b]$ and translation distance $\delta[d_1(\theta), d_2(\theta)]$. After these parameters are determined, the next viewpoint can be obtained by computing the surface area of the limit visual surface. This can be illustrated by analyzing view planning strategy to the left limit visual surface (shown in Fig. 11).

Assume that the predict limit visual curve $D_1F_1$ at the viewpoint 1 is rotated $\theta$ around $Z$ axis to get the new line shown as the dotted dashed line $D_2F_2$ in Fig. 11(a). $D_2F_2$ is the position with the negative direction translation distance $d_1(\theta)$ and $D_2F_2$ with the positive translation distance $d_2(\theta)$, which are shown...
as the dashed lines. The shaded area is the visual translation area of the curve at the next viewpoint. In Fig. 11, $M_1(-400, y_m)$ (named as the left limit point) denotes the intersection point of the predicted curve with the left limit motion position of the vision system. From Fig. 11(b), it is known that $y_m$ would be decreased with the increase in rotary angle. So the position of the left limit visual curve in the visual region of the vision system can be achieved by estimating the position of $M_1$. And the position that could obtain the maximal length of the limit visual curve $D_{i}F_{i}$ is defined as the next best view position. Details are presented as following:

(1) The limit visual curve is still located at the left plane after it is rotated $\theta(\theta \geq \theta_b)$. And the position of $M_2$ is relevant to the rotary angle $\theta$. So, in order to obtain the minimal length of the limit curve $D_{i}F_{i}$, the best translation distance is $d=d_1(\theta)$ (shown as $D_{i}F_{i}$ in Fig. 11(a), corresponding to the visual part is $D_{i}M_{2}$).

(2) The limit curve enters into the right plane after it is rotated $\theta$ ($\theta \leq \theta_b$). And the position of $M_2$ is relevant to the rotary angle $\theta_b$. So, the best translation distance is $d=d_0(\theta)$ for obtaining the maximal length of the limit curve $D_{i}F_{i}$, which is shown as $D_{i}M_{3}$ in Fig. 11(b), and the corresponding visual part is $D_{i}M_{3}$.

So, by analyzing the position of the left limit point with the rotary angle $[\theta_{a}, \theta_{b}]$ and translation distance $[d_1(\theta), d_2(\theta)]$, the position of the next best view is defined as obtaining the maximal length of the limit curve during the process of the left view planning. The corresponding limit surface area $S_1$ is obtained. And then, the maximal right limit visual surface area $S_2$ is obtained with a similar method. The final NBV position is determined by comparing $S_1$ and $S_2$.

4. Experiments

The experiments are carried in our laboratory for reconstruction of the object models. The range data are obtained by the proposed vision system. To obtain the limit visual curves, the acquired “data cloud” is sliced with a given interval distance in a certain direction (e.g. the z direction) and projected in the neighborhood of each cross section curve onto the plane on which the cross section curve lies. Here the interval between two cross section curves was set at 1.5 mm and the neighborhood of each cross section curve is set at 0.3 mm.

The experimental model 1: the test model (shown in Fig. 12). The initial information of the test model is obtained from the first view of the vision system in random position. The experiment results in the incremental construction of the first object are shown in Table 1 illustrate the computation at each step.

In Table 1, $\theta_1$($\theta_2$) and $d_1$($d_2$) denote the optimal position that can obtain the maximum unknown space area $S_1$($S_2$) in the left (right) view planning according to the proposed algorithm. For the $S_1 > S_2$, so the position of NBV is selected of the left view planning. The corresponding parameters are: $\theta_1 = 1.56$ rad; $d_1 = 58.1$ mm. And then, the viewpoint 2 is obtained as Fig. 13(c). The left limit visual surface at viewpoint 2 is shown as Fig. 13(d). The corresponding parameters are seen in Table 2. By the same way, the next viewpoint is selected at $\theta_2 = 1.62$ rad and $d_2 = 59$ mm. The final model is reconstructed by four steps.

The experimental model 2: cup. The initial information of the cup model is obtained from the first view of the vision system in random position (shown as Fig. 14(a)). Then the limit visual surfaces are obtained according to the boundary region information of known object based on the proposed method (shown in Fig. 14(b)). And the experimental data of viewpoint 1 to plan the next best view is shown in Table 3.

In Table 3, $\theta_1$($\theta_2$) and $d_1$($d_2$) denote the optimal position that can obtain the maximal unknown space area $S_1$($S_2$) in the left (right) view planning according to the proposed algorithm. For $S_1 > S_2$, so the next best view is the position that rotary table counterclockwise rotates 1.77 rad and translates 71.6 mm. Then we can get the viewpoint 2, which is shown in Fig. 14(c).
Similarly, the limit visual surface of viewpoint 2 is obtained, and the experimental data are shown in Table 4.

For $S_{2r} > S_{1r}$, viewpoint 3 is obtained by counterclockwise rotating 1.72 rad and translating 69.8 mm (shown as Fig. 14(d)). The residual process of the reconstruction object in the same way is shown as Fig. 14(e–f). The final reconstructed model is shown as Fig. 14(g).

At the same time, the process of reconstruction of the cup model is taken with Sablatnig’s approach [13]. And the comparative results of the two methods are shown in Table 5.

Table 2

<table>
<thead>
<tr>
<th>$d_0$ (mm)</th>
<th>$\theta_{2l}$ (rad)</th>
<th>$d_{2l}$ (mm)</th>
<th>$S_{2l}$ (mm²)</th>
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<tr>
<td>160</td>
<td>1.62</td>
<td>59</td>
<td>$2.28 \times 10^4$</td>
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</table>

Similarly, the limit visual surface of viewpoint 2 is obtained, and the experimental data are shown in Table 4.

The comparative results show that the error of reconstruction volume of our method is lower than Sablatnig’s one but a little more of reconstruction times. The reason of more times is that Sablatnig used two cameras but only one is used in this paper.
The results also show that the proposed method is effective in practical implementation.

5. Conclusions

In this paper, a planning strategy was proposed to determine where to find the next view for a laser vision system. The limit measurement region was obtained based on the measurement precision of the vision system. According to it, the limit visual region of the vision system was obtained based on the boundary data of the partially obtained object surface. The range of the rotary angle and the translation distance of the vision system were computed according to the limit visual surface. So, the corresponding visible surface area was obtained. The final NBV was the position that obtained the largest visible space area among the viewing points. And the experimental results show that the method is effective in practical implementation.

Moreover, the methods proposed in this paper cannot deal well with the concave models or self-occlusion of the object, which will be improved in the future work by integrating the trend surface of the object and the limit visual surface of the vision system.

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<table>
<thead>
<tr>
<th>Approach</th>
<th>CPU times (min:s)</th>
<th>Reconstruction volume (mm³)</th>
<th>Actual volume (mm³)</th>
<th>Error (%)</th>
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<tr>
<td>Sablatnig's</td>
<td>8:22</td>
<td>305,046</td>
<td>317,925</td>
<td>4.1</td>
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<tr>
<td>This paper's</td>
<td>9:01</td>
<td>310,883</td>
<td>317,925</td>
<td>2.2</td>
</tr>
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Table 5
Comparison of Sablatnig’s method and this paper’s of reconstruction of the cup model.

References