Parameter-dependent Lyapunov function approach to robust stability analysis for discrete-time descriptor polytopic systems

Xiangyu Gao¹,², Guang-Ren Duan¹*, Xian Zhang²

Abstract—This paper investigates robust stability of discrete-time descriptor polytopic systems (DTDPSs for short). The concept of affine generalized quadratic stability, which has less conservatism than generalized quadratic stability, of DTDPSs is proposed. It not only investigates affine generalized quadratic stability of DTDPSs but also presents criteria in terms of linear matrix inequalities to test the (affine) generalized quadratic stability based on time-varying parameter-dependent Lyapunov function. Finally, a numerical example presents the effectiveness of the proposed method.

I. INTRODUCTION

The Lyapunov theory has been one of the most appealing methods for investigating robust stability of linear polytopic systems (LPSs for short) during the last decade. In particular, some results are based on the concept of quadratic stability (see, e.g. [1]–[3]). Unfortunately, these results may be very conservative due to the use of a common Lyapunov function for the entire uncertainty set. In order to reduce the conservatism, several kinds of parameter-dependent Lyapunov functions have been proposed in literature to investigate robust stability of LPSs. For example, Lyapunov functions with linear dependence (see [4]–[9] and references therein), Lyapunov functions with polynomial dependence [10], [11], homogeneous polynomially parameter-dependent Lyapunov function of arbitrary degree [12], [13], homogeneous polynomially parameter-dependent quadratic Lyapunov functions [14], [15], Lyapunov functions which are parameter-dependent in negative as well as positive power series of parameters [16].

Besides, Lavaei and Aghdam [17] deal with the robust stability of discrete-time linear time-invariant systems with parametric uncertainties that can be expressed by a semialgebraic set, which can solve wider problem than the ones given in [13] and [14] for discrete-time systems. By introducing extra variables and using additional LMIs, Leite and Peres [18] proposed a sufficient condition for robust D-stability of LPSs, which contains the condition for robust D-stability from [9] as a particular case and encompasses the robust stability results of [6] (discrete-time systems) and [7] (continuous-time systems) when these special cases of D-stability are investigated. In addition, some robust stability results of linear parameter-varying systems with parametric uncertainties are obtained [19]–[21].

On the other hand, descriptor systems have been extensively studied during the past four decades due to their wide applications in circuits, economic, large scale systems, and other areas [22]. Many notions and results in state-space systems have been extended to descriptor systems [23], since the latter can represent a much wider class of systems than state-space systems can. However, few results on robust stability analysis for descriptor polytopic systems have been reported so far [24]–[26]. So the study of such problems is of both practical and theoretical importance.

In this paper, we will study robust stability of discrete-time descriptor polytopic systems (DTDPSs for short) by constructing Lyapunov functions with linear dependence, which is a generalization of the results in [4]. The concept of affine generalized quadratic stability of DTDPSs, which has less conservatism than generalized quadratic stability, is proposed in Section II. Based on some preliminary results introduced in Section III, it is shown in Section IV that affine generalized quadratic stability of DTDPSs implies robust stability, and thereby, sufficient conditions for (affine) generalized quadratic stability of DTDPSs are investigated in terms of linear matrix inequalities (LMIs for short). At last, the effectiveness of the proposed method is demonstrated by a numerical example in Section V.

It should be pointed out that the robust stability problem for descriptor polytopic systems is much complicated than that for LPSs because it requires to consider not only stability and robustness, but also regularity and impulse-free for continuous-time (causality for discrete-time) descriptor systems simultaneously.

The notations occurred in this paper are as follows: Let ℜ be the real number set, and $Z^+$ the nonnegative integer set. Denote by $I[k_1, k_2]$ the set $\{k_1, k_1+1, \cdots, k_2\}$ for $k_1, k_2 \in Z^+$ with $k_1 < k_2$. Let

$$\Lambda_N := \{(\lambda_1, \cdots, \lambda_N) : \sum_{i=1}^{N} \lambda_i = 1, \lambda_i \geq 0, \forall i \in I[1, N]\}. $$

For $\lambda = (\lambda_1, \cdots, \lambda_N) \in \Lambda_N$ and a group of matrices $M_1, M_2, \cdots, M_N$, we denote by $M(\lambda)$ the matrix $\sum_{i=1}^{N} \lambda_i M_i$.

For real symmetric matrices $P$ and $Q$, $P > Q (P < Q)$ means that the matrix $P - Q$ is positive (negative) definite, and $P \geq Q (P \leq Q)$ means that the matrix $P - Q$ is...
positive (negative) semi-definite. Denote by \( \| \cdot \|_2 \) either the Euclidean vector norm or its induced matrix 2-norm. The transpose, inverse, determinant, adjoint matrix and the minimum eigenvalue of the matrix \( A \) are represented by \( A^T \), \( A^{-1} \), \( \det A \), \( \text{adj} A \) and \( \lambda_{\text{min}}(A) \) respectively. Let \( A^{-T} = (A^{-1})^T \).

II. PROBLEM FORMULATION AND DEFINITIONS

Consider the DTDPs

\[
Ex_{k+1} = A(\alpha_k)x_k, \quad k \in \mathbb{Z}^+,
\]
where \( x_k \) is the state variable, \( E \in \mathbb{R}^{n \times n} \) is constant and singular, \( \alpha_k = (\tilde{\alpha}_1(k), \cdots, \tilde{\alpha}_N(k)) \in \Lambda_N \), and \( A(\alpha_k) \), the time-varying system matrix, belongs to the polytope of matrices \( \mathcal{A}(k) \) defined by

\[
\mathcal{A}(k) := \left\{ \sum_{j=1}^N \tilde{\alpha}_j(k) A_j : \forall \alpha_k \in \Lambda_N, k \in \mathbb{Z}^+ \right\}
\]
with given vertex set \( \{ A_j \in \mathbb{R}^{n \times n}, j \in [1,N] \} \).

Denote parameter increment by

\[
\Delta \tilde{\alpha}_i(k) = \tilde{\alpha}_i(k+1) - \tilde{\alpha}_i(k), \quad i = 1, 2, \cdots, N.
\]

Obviously, \( \max_{k \in \mathbb{Z}^+} |\Delta \tilde{\alpha}_i(k)| \leq 1, \forall i \in [1,N] \) owing to \( \tilde{\alpha}_i(k) \in [0,1] \).\( \tilde{\alpha}_i(k+1) \in [0,1]. \) Thus we suppose in the following that

\[
|\Delta \tilde{\alpha}_i(k)| \leq \rho_i, \forall i \in [1,N],
\]
where \( \rho_i \in [0,1] \). It follows from \( |\Delta \tilde{\alpha}_i(k)| \leq 1 \) and

\[
\sum_{i=1}^N \Delta \tilde{\alpha}_i(k) = \sum_{i=1}^N \tilde{\alpha}_i(k+1) - \sum_{i=1}^N \tilde{\alpha}_i(k) = 0
\]
that

\[
|\Delta \tilde{\alpha}_N(k)| = \sum_{i=1}^{N-1} |\Delta \tilde{\alpha}_i(k)| \leq \min \left\{ 1, \sum_{i=1}^{N-1} \rho_i \right\},
\]
Therefore, we can assume that \( \rho_N = \min \left\{ 1, \sum_{i=1}^{N-1} \rho_i \right\}, \) but not necessary (In other words, \( \rho_N \) can be less than \( \min \left\{ 1, \sum_{i=1}^{N-1} \rho_j \right\} \)).

Definition 1: The DTDPs (1) is said to be Robust stable (RS for short), if it is admissible (regular, causal and stable) \([22, 23]\) for all \( \alpha_k \in \Lambda_N \).

Definition 2: The DTDPs (1) is said to be generalized quadratically stable (GQS for short), if there exists a symmetric matrix \( X \) such that

\[
E^T X E \geq 0,
\]
\[
A^T(\alpha_k) X A(\alpha_k) - E^T X E < 0, \forall \alpha_k \in \Lambda_N.
\]

Remark 1: The term "generalized quadratic stability" only means that the descriptor system is different from the linear system, the concept is analogous to the classic quadratic stability of the linear system.

Definition 3: The DTDPs (1) is said to be affinely generalized quadratically stable (AGQS for short), if there exist symmetric matrices \( X_1, X_2, \cdots, X_N \) such that

\[
E^T X(\alpha_k) E \geq 0, \forall \alpha_k \in \Lambda_N,
\]
\[
A^T(\alpha_k) X(\alpha_k) A(\alpha_k) - E^T X(\alpha_k) E < 0, \forall \alpha_k \in \Lambda_N.
\]

Obviously, GQS must be AGQS, and the converse is not true.

Inspired by \([4]\), the following two questions will be considered in this paper:
(i) Discuss the relationship between RS and AGQS of the DTDPs (1);
(ii) Establish sufficient conditions which guarantee the DTDPs (1) to AGQS and GQS, respectively.

III. PRELIMINARY RESULTS

In order to prove the main results (i.e., Theorems 1 and 2 in next section), we first introduce the following lemmas.

Lemma 1: \([4]\) Let \( R \in \mathbb{R}^{n \times n} \) is a constant matrix. Then

\[
\begin{pmatrix} P(\alpha_k) & R^T \\ R & Q(\alpha_k) \end{pmatrix} > 0, \forall \alpha_k \in \Lambda_N
\]
is equivalent to

\[
\begin{pmatrix} P_i & R^T \\ R & Q_i \end{pmatrix} > 0, \forall i \in [1,N].
\]

Lemma 2: \([27]\) Let \( H \in \mathbb{R}^{n \times n} \) is a constant matrix such that \( H + HT < 0 \). Then \( H \) is invertible.

Lemma 3: \([28]\) Let \( M = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \in \mathbb{R}^{n \times n} \), where \( M_1 \in \mathbb{R}^{m \times m} \). If \( M + MT < 0 \), then

\[
M_1 + M_1^T - M_2 M_3^{-1} M_3 - M_3^T M_3^{-1} M_1 M_2 < 0.
\]

Lemma 4: \([29]\) For real \( n \times n \) symmetric matrices \( X_1, X_2, \cdots, X_n \), there exist real \( n \times n \) positive definite matrices \( P_1, P_2, \cdots, P_n, Q_1, Q_2, \cdots, Q_n \) such that \( X(\alpha_k) = P(\alpha_k) - Q(\alpha_k) \).

Lemma 5: \([30]\) Consider the following nonlinear system

\[
y_{k+1} = f(y_k, k), y(0) = y_0, y_k \in \mathbb{R}^n
\]
Suppose \( f(0, k) = 0, \forall k \in \mathbb{Z}^+ \). Then the equilibrium point \( 0 \) is globally uniformly asymptotically stable, if there exist a scalar function \( V(y_k, k) \) and class-K functions \( \phi_1, \phi_2, \phi_3 \) such that, for any \( y_k \) and \( k \in \mathbb{Z}^+ \):
(i) \( 0 < \phi_1(\|y_k\|) \leq V(y_k, k) \leq \phi_2(\|y_k\|) \);
(ii) \( \Delta V(y_k, k) = V(y_{k+1}, k+1) - V(y_k, k) \leq -\phi_3(\|y_k\|) \);
(iii) \( \lim_{y_k \to -\infty} \phi_1(\|y_k\|) = \infty \).

IV. MAIN RESULTS

Based on the previous preparation, now we can present our main results, that is, Theorems 1 and 2 below.

Theorem 1: If the DTDPs (1) is AGQS, then it is RS.
Proof: Since the DTDPs (1) is AGQS, there exists symmetric matrices \( X_1, X_2, \cdots, X_N \) such that (7) holds.
Let \( \text{rank}(E) = r < n \). Then there exist invertible matrices \( P_0 \) and \( Q_0 \) such that
\[
P_0EQ_0 = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}.
\] (8)

Correspondingly, let
\[
P_0^T X(\alpha_k) P_0^{-1} = \begin{pmatrix} \tilde{X}_1(\alpha_k) \\ \tilde{X}_2(\alpha_k) \end{pmatrix}, \tag{9a}
\]
\[
P_0 A(\alpha_k) Q_0 = \begin{pmatrix} \tilde{A}_1(\alpha_k) \\ \tilde{A}_2(\alpha_k) \\ \tilde{A}_3(\alpha_k) \\ \tilde{A}_4(\alpha_k) \end{pmatrix}, \tag{9b}
\]
Using (7) we have \( \tilde{X}_1(\alpha_k) > 0 \) for any \( \alpha_k \in \Lambda_N \).

For the sake of simplicity, \( \tilde{A}_i(\alpha_k) \) will be abbreviated as \( \tilde{A}_i \) below for \( i = 1, 2, 3, 4 \).

From (7b),(8) and (9), we have
\[
\begin{pmatrix} \tilde{F}_1 \\ \tilde{F}_2 \\ \tilde{F}_3 \end{pmatrix} = 0, \tag{10}
\]
where
\[
\tilde{F}_1 = A_1^T \tilde{X}_1(\alpha_k+1) \tilde{A}_1 + A_2^T \tilde{X}_2^T(\alpha_k+1) \tilde{A}_2 - X_1(\alpha_k),
\]
\[
\tilde{F}_2 = A_1^T \tilde{X}_1(\alpha_k+1) \tilde{A}_3 + A_3^T \tilde{X}_3(\alpha_k+1) \tilde{A}_3 - \tilde{X}_1(\alpha_k),
\]
\[
\tilde{F}_3 = A_2^T \tilde{X}_2(\alpha_k+1) \tilde{A}_2 + A_4^T \tilde{X}_4^T(\alpha_k+1) \tilde{A}_2 - \tilde{X}_2(\alpha_k).
\]

Since \( \tilde{X}_1(\alpha_k) > 0 \) for any \( \alpha_k \in \Lambda_N \) and \( \tilde{F}_3 < 0 \), so
\[
\tilde{X}_1^T(\alpha_k+1) \tilde{A}_2 + \tilde{X}_2(\alpha_k+1) \tilde{A}_4 + \tilde{X}_1^T \tilde{X}_3(\alpha_k+1) \tilde{A}_4 < 0.
\]

Let \( H = \tilde{X}_2^T(\alpha_k+1) \tilde{A}_4 + \tilde{X}_3(\alpha_k+1) \tilde{A}_4 \) is a diagonal matrix, and so \( \tilde{A}_4 \). Hence the DTDPS (1) is causual and regular for all \( \alpha_k \in \Lambda_N \).

Next, we show that the DTDPS (1) is stable.

Let \( x_k = Q_0 y_k + z_k \) where \( y_k \in \mathbb{R}^r, z_k \in \mathbb{R}^{n-r} \). Then the DTDPS (1) is restricted equivalent to the following system
\[
y_{k+1} = \hat{A} y_k, \tag{11a}
z_k = -\hat{A}_3^{-1} \hat{A}_3 y_k, \tag{11b}
\]
where \( \hat{A} = \hat{A}_1 - \hat{A}_2 \hat{A}_3^{-1} \hat{A}_3 \).

Let \( M = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \), where
\[
M_1 = \tilde{A}_1^T \tilde{X}_1(\alpha_k+1) \tilde{A}_1 + \tilde{A}_2^T \tilde{X}_2^T(\alpha_k+1) \tilde{A}_2 + \tilde{A}_3^T \tilde{X}_3(\alpha_k+1) \tilde{A}_3 - \tilde{X}_1(\alpha_k),
\]
\[
M_2 = \tilde{A}_2^T \tilde{X}_2(\alpha_k+1) \tilde{A}_2 + \tilde{A}_3^T \tilde{X}_3(\alpha_k+1) \tilde{A}_3 - \tilde{X}_2(\alpha_k),
\]
\[
M_3 = -\tilde{A}_2^T \tilde{X}_2^T(\alpha_k+1) \tilde{A}_2 + \tilde{A}_3^T \tilde{X}_3(\alpha_k+1) \tilde{A}_3,
\]
\[
M_4 = \tilde{A}_3^T \tilde{X}_3(\alpha_k+1) \tilde{A}_3.
\]

Then \( M + M^T = (\frac{\tilde{F}_1}{\tilde{F}_2}) < 0 \) from (10). From Lemma 3, we can obtain
\[
\tilde{A}_1^T X_k(\alpha_k+1) \tilde{A} - \tilde{X}_1(\alpha_k) < 0. \tag{12}
\]

Assume that all eigenvalues of matrix \( \tilde{X}_1(\alpha_k) \) are \( \lambda_1(\alpha_k), \ldots, \lambda_r(\alpha_k) \). Since eigenvalues of \( X_k(\alpha_k) \) are continuous functions in its elements, it follows that \( \lambda_i(\alpha_k), i = 1, 2, \ldots, r \) are continuous functions in \( \alpha_k \). Because \( \Lambda_N \) is a bounded closed set and \( \tilde{X}_1(\alpha_k) > 0 \) for any \( \alpha_k \in \Lambda_N \), there exist \( \alpha_k \) and \( \sigma_k \in \Lambda_N \) such that
\[
0 < \lambda_i(\alpha_k) \leq \lambda_i(\sigma_k), i = 1, \ldots, r, \forall \alpha_k \in \Lambda_N.
\]

Let \( \lambda = \min_{1 \leq i \leq r} \{ \lambda_i(\alpha_k) \} \) and \( \sigma = \max_{1 \leq i \leq r} \{ \lambda_i(\sigma_k) \}. \)

Consequently, \( 0 < \lambda \leq \lambda_i(\alpha_k) \leq \sigma, \forall \alpha_k \in \Lambda_N, i = 1, 2, \cdots, N. \)

Similarly, from (12), there exists a positive number \( \lambda \) such that
\[
\tilde{A}_1^T X_k(\alpha_k+1) \tilde{A} - \tilde{X}_1(\alpha_k) \leq -\lambda M.
\]

Let \( V(y_k, k) = y_k^T \tilde{X}_1(\alpha_k) y_k \). Then
\[
0 < \lambda \| y_k \|^2 \leq V(y_k, k) \leq \sigma \| y_k \|^2
\]
and
\[
\Delta V(y_k, k) = y_k^T \tilde{X}_1(\alpha_k+1) y_k - y_k^T \tilde{X}_1(\alpha_k) y_k \leq -\lambda \| y_k \|^2.
\]

Therefore, from Lemma 5, it follows that the system (11a) is globally uniformly asymptotically stable.

Using properties of continuous functions on a bounded closed set and norm continuity, it results that there exist positive numbers \( \delta_1, \delta_2 \) and \( \delta_3 \) such that \( \delta_1 \leq \| \hat{A}_Z \| \leq \delta_2 \) and \( \| \hat{A}_Z \| \leq \delta_3 \). Then
\[
\| \hat{A}_Z \|^2 \leq \| \hat{A}_Z \| \leq \delta_3 \delta_3 \tag{12a}
\]
Similarly, there exists a real number \( \xi > 0 \) such that \( \| \hat{A}_Z \| \leq \xi \). Thus, it follows from (11b) that
\[
\| z_k \|^2 \leq \| \hat{A}_Z \| \| \hat{A}_Z \| \| y_k \|^2 \leq \delta_3 \delta_3 \delta_3 \tag{12b}
\]
So the system (1) is asymptotically stable for all \( \alpha_k \in \Lambda_N \).

Since the parameter \( \alpha_k \) is not known a priori, the condition (7) is not numerically verifiable.

The DTDPS (1) is AGQS if there exist matrices
\[
X(\alpha_k) = \tilde{A}_1(\alpha_k) X_1 + \cdots + \tilde{A}_N(\alpha_k) X_N, \alpha_k \in \Lambda_N, \tag{13}
\]
i.e., that \( X(\alpha_k) \) is affine in the parameters, the following sufficient condition can be obtained.

**Theorem 2**: Given the vertex set \( \{ A_j, j \in I[1, N] \} \) and the bounds of the parameter increments \( 0 \leq \rho_i \leq 1, i \in I[1, N] \). The DTDPS (1) is AGQS if there exist matrices
Given the vertex set $X$.

**Proof:** From Lemma 1, we have that the inequality (14b) is equivalent to the

$$\begin{pmatrix}
Q(\alpha_k) & R & 0 \\
R^T & A(\alpha_k) & A^T(\alpha_k)Y^T \\
0 & YA(\alpha_k) & \mathbb{P}(\alpha_k)
\end{pmatrix} > 0, \forall \alpha_k \in \Lambda_N,$$

(16)

where

$$\begin{align*}
Q(\alpha_k) &= Q(\alpha_k) - \sum_{j=1}^{N} \rho_j Q_j, \\
A(\alpha_k) &= E^T(P(\alpha_k) - Q(\alpha_k))E + R^T A(\alpha_k) + A^T(\alpha_k)R, \\
\mathbb{P}(\alpha_k) &= Y^T + Y - P(\alpha_k) - \sum_{j=1}^{N} \rho_j P_j.
\end{align*}$$

It follows from (3), (4), (5) and $\sum_{i=1}^{N} \bar{\alpha}(k) = 1$ that

$$\begin{align*}
Y^T + Y - P(\alpha_{k+1}) &= \sum_{i=1}^{N} \bar{\alpha}_i(k)(Y^T + Y) - \sum_{i=1}^{N} \bar{\alpha}_i(k)P_i - \sum_{i=1}^{N} \Delta \bar{\alpha}_i(k)P_i \\
&= \sum_{i=1}^{N} \bar{\alpha}_i(k)(Y^T + Y - P_i) - \sum_{i=1}^{N} \Delta \bar{\alpha}_i(k)P_i + \Delta \bar{\alpha}_N P_N \\
&\geq Y^T + Y - P(\alpha_k) - \sum_{j=1}^{N} \rho_j P_j \\
&= \mathbb{P}(\alpha_k) > 0.
\end{align*}$$

(17)

This, together with (17), implies that

$$\begin{pmatrix}
Q(\alpha_{k+1}) & R & 0 \\
R^T & A(\alpha_k) & A^T(\alpha_k)Y^T \\
0 & YA(\alpha_k) & \mathbb{P}(\alpha_k)
\end{pmatrix} > 0,$$

(19)

and hence

$$\begin{pmatrix}
A(\alpha_k) - R^T Q(\alpha_{k+1})^{-1}R \\
A^T(\alpha_k)Y^T \\
YA(\alpha_k)
\end{pmatrix} > 0.$$

(20)

It follows from

$$\begin{pmatrix}
Y - P(\alpha_{k+1}) \\
Y - P(\alpha_{k+1}) - Y + P(\alpha_{k+1})
\end{pmatrix} \geq 0$$

that

$$YP^{-1}(\alpha_{k+1})Y^T \geq Y^T + Y - P(\alpha_{k+1}).$$

This, together with (20), implies that

$$\begin{pmatrix}
A(\alpha_k) - R^T Q(\alpha_{k+1})^{-1}R \\
A^T(\alpha_k)Y^T \\
YA(\alpha_k)
\end{pmatrix} > 0.$$
where
\[ \mathcal{X}_i = P_i - X - \sum_{j=1}^{N} \rho_j (P_j - X), \]
\[ \mathcal{A}_i = E^T X E + A_i^T R + R^T A_i, \]
\[ \Xi_i = Y^T + Y - P_i - \sum_{j=1}^{N} \rho_j P_j, \]
then DTDPS (1) is GQS.

If parameter \( \alpha_k \in \Lambda_N \) of system (1) is constant (i.e., \( \alpha_k = \alpha_0 \) for any \( k \in Z^+ \)), which implies that bounds of parameter increment \( \rho_i = 0 \) for all \( i \in [1,N] \), then the next two corollaries can be immediately obtained from Theorem 2 and Corollary 1, respectively. This offers sufficient conditions under which the system
\[ E x_{k+1} = A(\alpha_0) x_k \]  
(24)
is AGQS and GQS, respectively.

**Corollary 2:** Given the vertex set \( \{ A_j, j \in [1,N] \} \) and the bounds of the parameter increments \( 0 \leq \rho_i \leq 1, i \in [1,N] \). If there exist matrices \( R, Y \in \mathbb{R}^{n \times n}, \det Y \neq 0 \) and positive definition matrices \( P_i, Q_i \) such that
\[ E^T (P_i - Q_i) E \geq 0, \forall i \in [1,N], \]
(25a)
\[ \begin{pmatrix} Q_i & R \\ R^T & \mathcal{A}_i \\ 0 & Y A_i & Y^T + Y - P_i \end{pmatrix} > 0, \forall i \in [1,N], \]
(25b)
where \( \mathcal{A}_i = E^T (P_i - Q_i) E + A_i^T R + R^T A_i \), then the system (24) is AGQS.

**Corollary 3:** Given the vertex set \( \{ A_j, j \in [1,N] \} \) and the bounds of the parameter increments \( 0 \leq \rho_i \leq 1, i \in [1,N] \). If there exist matrices \( R, X, Y \in \mathbb{R}^{n \times n}, \det Y \neq 0 \) and \( P_i > 0, i \in [1,N] \) such that
\[ E^T X E \geq 0, \forall i \in [1,N], \]
(26a)
\[ \begin{pmatrix} P_i - X & R \\ R^T & \mathcal{A}_i \\ 0 & Y A_i & Y^T + Y - P_i \end{pmatrix} > 0, \forall i \in [1,N], \]
(26b)
where \( \mathcal{A}_i = E^T X E + A_i^T R + R^T A_i \), then the system (24) is GQS.

**Remark 3:** The results analogous to that of corollary 1 and 3 can be obtained in the literature [4] for linear system, since the paper considers time-varying descriptor systems, the results of corollary 1 and 3 have never been seen in the literatures which we have found.

**V. NUMERICAL EXAMPLES**

In this section, we provide an example to demonstrate the applicability of the proposed method.

Consider the DTDPS (1) with
\[ E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \]
\[ A_1 = \begin{pmatrix} -0.4 & 0.5 & 0.3 \\ -0.08 & -1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}, \]
\[ A_2 = \begin{pmatrix} 1 & 1 & 0.1 \\ -0.906 & -1 & 0 \\ 0 & 0.2 & 1 \end{pmatrix}. \]
Assume that \( \rho_1 = 0.1 \) and \( \rho_2 = 0.08 \). Using the LMI Control Toolbox in MATLAB, we obtain solutions of LMIs in (14) as follows:
\[ R = \begin{pmatrix} -0.1106 & -0.0461 & 0.3773 \\ -0.1674 & -0.2574 & 0.1606 \\ 0.2697 & -0.1751 & 1.5078 \end{pmatrix}, \]
\[ Y = \begin{pmatrix} 2.9311 & 2.2789 & -0.0819 \\ 3.0400 & 5.7271 & -0.4469 \\ -0.1039 & -0.2922 & 1.4624 \end{pmatrix}, \]
\[ P_1 = \begin{pmatrix} 2.5059 & 2.7603 & -0.1589 \\ 2.7603 & 6.1193 & -0.5031 \\ -0.1589 & -0.5031 & 0.8926 \end{pmatrix}, \]
\[ P_2 = \begin{pmatrix} 3.7941 & 3.6328 & -0.0872 \\ 3.6328 & 5.1798 & -0.2328 \\ -0.0872 & -0.2328 & 0.8008 \end{pmatrix}, \]
\[ Q_1 = \begin{pmatrix} 0.9537 & 0.8220 & 0.4714 \\ 0.8220 & 1.0119 & 0.1542 \\ 0.4714 & 0.1542 & 4.3302 \end{pmatrix}, \]
\[ Q_2 = \begin{pmatrix} 1.0307 & 1.0038 & 0.2068 \\ 1.0038 & 1.5623 & 0.0933 \\ 0.2068 & 0.0933 & 4.1798 \end{pmatrix}. \]
Therefore, by Theorem 2, we can conclude that the considered DTDPS (1) is AGQS.

On the other hand, for the considered system, there is no solutions to the inequality (23) by LMI Toolbox in MATLAB, and hence the considered system is not GQS from Corollary 1. This shows that AGQS has less conservatism than GQS again.

**VI. CONCLUSIONS**

The robust stability analysis for DTDPS (1) is done by using the concept of affine generalized quadratic stability proposed here, which has less conservatism than generalized quadratic stability. LMI conditions for (affine) generalized quadratic stability have been given for this class of systems by using Lyapunov theory. The applicability of the proposed method is illustrated by an example.

**REFERENCES**


