A Gauss-Markov Model for Hyperspectral Texture Analysis of Urban Areas

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Abstract

In this paper we tackle the problem of texture segmentation using a joint spectral and spatial analysis of pixel distribution. Hyperspectral images are considered and using a Markovian model we develop a vectorial approach for this image type. A classification algorithm using this model has been implemented for extracting and classifying urban areas. Results obtained from AVIRIS images are shown.

1 Introduction

Texture analysis is frequently used for disambiguating the classification process of remote sensing images. In particular, urban areas are not well classified by using only pixel radiometry. So, in order to decrease the classification error a segmentation of urban areas is performed using texture arguments and the classification of the other areas is based on radiometry analyses.

Usually texture classification of urban areas is carried out using a panchromatic high resolution image, whereas classification of natural elements uses multi/super/hyperspectral images that have a better spectral resolution. The output of this process is a thematic map obtained by merging the result of the two classifications.

In this framework, it is likely that urban areas can be better modelled by working jointly in spatial (contrast) and spectral (color) domains. So, in this paper, we focus on the determination and test of a texture model able to jointly characterize urban areas from spatial and spectral variations of hyperspectral remote sensing images.

In the next section, we describe a Markovian texture model able to perform such an analysis. In section 3, we show how to insert it into a hyperspectral image classification algorithm. We present some results in section 4 and conclude in section 5.

2 Multivariate Markovian texture model

In the case of a joint spectral and spatial texture analysis, classical texture features (gray level cooccurence matrices [2], Gabor transform [5], ...) can not easily be used. Indeed, algorithmic complexity, due to the huge amount of hyperspectral data, is high. In our case, we prefer to build one single compact operator capable of analyzing textures both spatially and spectrally, which is also well suited to classification.

The proposed model, within a multivariate data analysis framework, is based on a Markov Random Field (MRF). The usual 2-D MRF model can be adapted in two ways for multispectral images : consider it as a 2-D spatial field of vectorial (spectral) measures (see e.g. [3]) or as a 3-D spatial-spectral field of scalar variables (see e.g. [8]). What follows id a description of a model of the first type.

2.1 Multivariate Gauss-Markov model

We consider an hyperspectral image as a realization of a field $X = \{X_s\}$, where each site $s = (i,j)$ is defined by two spatial coordinates on a lattice of size $N_x \times N_c$. Each $X_s$ is a random vector of dimension $N_b$, the number of bands.

The conditional probability of a Multivariate Gauss-Markov Random field (MGMRF) is given by :

$$P(x_s|x_t, t \in N_s) = \frac{1}{Z} \exp \left\{ -\frac{1}{2} \left\| x_s - \sum_{t \in N_s} \theta_{t-s} x_t \right\|^2_{\Sigma} \right\}$$

(1)

Where $\|a\|^2_{\Sigma} = a^T \Sigma^{-1} a$, the $\theta_t$ is some transformation matrices, $\Sigma$ is the conditional covariance matrix, $N_s$ is the neighborhood of pixel $s$ and $Z$ is the partition function.

This formula can also be written in terms of a conditional probability with respect to the means of neighborhood:
where \( k \) represents the number of different neighborhood systems \( \mathcal{N}^k \) (a neighborhood system is a set of neighboring pixels interacting in the same way with a given pixel, and which are thus weighted by the same \( \theta \)). \( m_{k}^{*} \) is the mean of the \( k \)-type neighbors of pixel \( x_{s} \), and finally \( \theta_{k}^{*} = \frac{\theta_{k}}{m_{k}^{*}} \), where \( t \in \mathcal{N}_{s}^{k} \) and \( n_{k} \) is the number of pixels in \( \mathcal{N}_{s}^{k} \).

This formulation is also equivalent to :

\[
P(x) = \frac{1}{Z(S)} \exp \left( -\frac{1}{2} x^t S^{-1} x \right)
\]

where \( S = (I_{N_k} \otimes I_{N_k} \otimes \Sigma^{-1}) \), \( A \otimes \Sigma \) symbolizing Kronecker product, and \( A \) is a matrix defined by the interactions \( \theta \) and the given neighborhood system.

2.2 Adaptation to hyperspectral images

Applying this model to hyperpectral data without any change is risky. Indeed, these images are characterized by a large number of spectral bands (for instance, the AVIRIS sensor provides 224 bands), thus the vectors \( X_{s} \) have a high dimension, and so the curse of dimensionality makes our statistical analysis extremely difficult (it is also known as the Hughes phenomenon [4]).

One solution consists in reducing the dimension through projecting the data onto a subspace (as shown later in this paper). Another solution is to reduce the number of parameters of the model as follows.

The model can be simplified by making an assumption with respect to the transformation matrices \( \theta_{i} \). We consider each matrix as being diagonal (for the sake of simplicity we omit the subscript \( i \) corresponding to the type of neighbor)

\[
\theta = \text{diag}(a_0, \ldots, a_j, \ldots, a_{N_k-1})
\]

This is equivalent to assuming that the spatial interactions in the different bands are independent.

2.3 Parameter estimation

The parameters to be estimated for each texture are the set of \( a_j \) and the conditional covariance matrix \( \Sigma \).

2.3.1 Maximum Likelihood

A very popular criterion in parameter estimation is the Maximum Likelihood (ML). Its principle is to choose the parameter vector \( \phi \) (in the present case \( \phi = (\{a_k\}, \Sigma) \)), such that :

\[
\hat{\phi} = \arg \max_{\phi} P(x / \phi)
\]

Usually the loglikelihood \( LL(\phi) = \log P(x / \phi) \) is used. From equation (3), and since we can compute partition function \( Z(S) \), we can rewrite \( LL(\phi) \) as :

\[
LL(\phi) = C - \frac{1}{2} \ln (\det(S)) - \frac{1}{2} x^t S^{-1} x
\]

This function can be maximized using numerical algorithms (e.g. conjugate gradient), but the process is rather slow.

2.3.2 Maximum Pseudo-Likelihood

A criterion which provides a faster estimation is the Maximum Pseudo-Likelihood (MPL, see [1]). It assumes a condition of independence so that :

\[
P(x) \approx \prod_{s \in \Omega} P(x_s | \{x_i : t \in V_s \})
\]

This function is maximized by solving the following system (see [7]) :

\[
\begin{pmatrix}
m_0 j m_0 j & \cdots & m_0 j m_k-1 j \\
\vdots & \ddots & \vdots \\
m_k-1 j m_0 j & \cdots & m_k-1 j m_k-1 j
\end{pmatrix}
\begin{pmatrix}
a_0^0 \\
\vdots \\
a_k-1^0
\end{pmatrix}
= \begin{pmatrix}
x_j m_0^0 \\
\vdots \\
x_j m_k-1^0
\end{pmatrix}
\]

where \( \overline{m} \) is the mean of \( m \). This matrix system is solved for each \( j = 0 \ldots N_b \).

3 Application to classification

3.1 Classification

Let us consider a problem with \( L \) classes \( C_l \), \( l = 0 \ldots L - 1 \). The ML decision rule consists in assigning the label \( \lambda_s \) to pixel \( s \) as follows :

\[
\lambda_s = \arg \max_{l \in \{0, L-1\}} P_l(x_s / \{m_k\})
\]

Unfortunately, this rule is very localized and therefore yields irregular classifications. It can be regularized by including a Potts prior.

3.2 Dimension reduction

As explained in section 2.2, statistical parameters of high dimensional models are not easy to estimate. We have already simplified the model, but the problem of estimating the conditional covariance matrix still remains. To solve
this problem, a dimension reduction by projecting the data in a low dimensional subspace is necessary before any other processing. Usual methods include Principal Component Analysis (PCA) and discriminant analysis. But PCA is not a classification oriented algorithm, and discriminant analysis gives poor results when class centroids are not homogeneously distributed.

We carry out this reduction using a parametric projection pursuit algorithm [6] adapted to the model described above [7]. This algorithm consists in finding by numerical optimization a subspace in which a projection index is optimized. In our case, as in [6], this index is linked to the Bhattacharyya distance between classes (defined by the operator), in the subspace. In the proposed model, the conditional probability with respect to its neighbors is Gaussian. We can thus easily compute the distance as a function of the neighborhood. In the case of only one neighbor type, we have:

\[
d_B(m_s) = \frac{1}{2} \|\mu_0(m_s) - \mu_1(m_s)\|_\Sigma_0 + \frac{1}{2} \ln \left( \frac{|\Sigma_0|}{|\Sigma_1|} \right)
\]

where \(\mu_i(m_s) = a'_i m_s + b_i\) and \(\Sigma_{01} = \Sigma_{0} + \Sigma_{1}\). The texture mean \(b_i\) has been added to deal with textures of different means.

For each pair of classes, we define a partial projection index \(I_{i,j}\) as:

\[
I_{i,j} = E\{d_B(m_s)\} = \int d_B(m_s)f(m_s) dm_s
\]

The global projection index is then:

\[
I = \min_{i,j} I_{i,j}
\]

4 Results

Classification tests are performed on data extracted from an AVIRIS image of Moffett Field, California.

4.1 Texture mosaic

We first test our algorithm on an image made of 4 different texture patches (see fig. 1) extracted from the Moffett Field AVIRIS scene.

We compare the classification results given by different methods in table 1, for two different subspace dimensions : 10 and 20 bands. In this case, it is possible to evaluate the methods by examining the classification accuracy. PCA is followed by a non-contextual ML classification. Parametric projection pursuit based on the same modeling is denoted by PP-G, and is followed by a classification using the same criterion. Projection pursuit based on the Gauss-Markov model is denoted by PP-MG, and the classification criterion is the ML criterion associated with this model.

We remark that for low variability textures (as is the case here since the patches are rather homogeneous, except for the lower right one) non-contextual models are able to catch most of the information.

4.2 Real image

We then test the classification algorithm on a sub-image extracted from another part of the Moffet field AVIRIS image. In this test, we compare the classification results obtained by PP-G and PP-MG methods.

Figure 2 shows the result of a PP-MG classification, with one type of neighbor (i.e. first order isotropic MRF). The result shows urban area classification, with the other classes being pooled together into a “non-urban” class. We define two kinds of urban areas: the first one consists of large separated buildings, and the second one is a dense area consisting of small buildings and residential areas.

Some ground truth has been established using U.S. Geological Survey maps of the area. The final subspaces are of dimension 40. Using the non-contextual model we obtain a classification accuracy of 80.3% for large buildings, and one of 97.6% for the residential zones. For the method presented in this paper, we obtain accuracies of 82.2% and 96.5%, respectively. Therefore, the proposed method yields better results for highly textured areas made of large buildings and slightly lower results for residential areas.

5 Conclusion

We have proposed a Markovian texture model for hyperspectral image analysis. This model allows a texture analysis which is both spatial, through spatial interactions, and spectral, through vectorial modeling. The model is used within a ML classification algorithm. The tests we have performed consist of a dimension reduction, in order to avoid
Table 1. Classification accuracy (%) of the 4 textures (see fig. 1) with different models and dimension reduction (the final subspace dimensions are 10 and 20 respectively)

<table>
<thead>
<tr>
<th>Dimension reduction</th>
<th>Classification accuracy (10 bands)</th>
<th>Classification accuracy (20 bands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCA</td>
<td>85.1</td>
<td>88.3</td>
</tr>
<tr>
<td>PP-G</td>
<td>86.7</td>
<td>88.7</td>
</tr>
<tr>
<td>PP-MG</td>
<td>84.5</td>
<td>88.0</td>
</tr>
</tbody>
</table>

statistical estimation problems due to dimensionality (also known as the curse of dimensionality), followed by a ML classification.

The results show that taking texture into account can be useful in the case of non-homogeneous textures. In the opposite case, a non-contextual model is able to catch the variability of the data, and the results are nearly similar.

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References