Robust Convergence of Low Data Rate Distributed Controllers

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Abstract
This paper is focused on control systems in which the observer is not co-located with the controller and the communication channel between them is of limited bandwidth. Due to the low data rate requirement, the control law must be of extremely simple form. In this regard, a tri-state distributed controller is examined. The control algorithm is proven to converge if the system structure satisfies certain technical conditions. The convergence is robust in the sense that the system parameters and even the structure of the underlying system dynamics need not to be completely known.

1. Introduction

Many challenging control problems in networking and communication systems can be cast in the framework of controlling a complex system with a large number of decision-makers distributed over a wide area. Examples that readily come to mind include the power control problem in wireless communication (see for example [7], [16], [22]) and the congestion control problem in the Internet (see for example [6], [12]).

Although the application nature of these problems may vary, they all have three quintessential aspects, which in our view make them also interesting from a theoretical point of view. First of all, the distributed nature of the system implies that observation information is highly distributed and the decision-makers have to determine their control in a distributed manner, based on limited, partial information. Secondly, since the observer, decision-maker, and physical controller are not necessarily co-located, measurement data and control information need to be communicated over a data network. In order not to impose a heavy overhead demand, it is desirable to design algorithms that require low communication data rate. Finally, in many of these systems, system parameters and even the structure of the underlying dynamics may not be known or are at best only partially known. As a result, the controlling algorithms are required to be robust so that errors in knowledge about system parameters or structures do not seriously affect algorithm performance.

In this paper, we consider a class of problems that deals with performance-target-tracking. Our motivation comes from power control problems in wireless communication. However, these models also occur in other application contexts involving multiple distributed users such as a peer-to-peer network. A
class of distributed feedback control algorithms, called tri-state algorithms, is proposed. These algorithms are closely related to relay feedback (see for example [10]) and require only one ternary symbol for coding the control signal. It is shown here that a tri-state algorithm converges for a large class of systems with different system structures. In this sense, such an algorithm is robust. This is an important property for many complex network models in wireless and Internet control problems.

The tracking problem discussed here is non-classical. However, there is a close relation to the output regulation problem, in particular the robust regulation problem, (see for example [2], [5], [8], [9], [11]) as well as the relay feedback problem. However this work differs from these classical results in two major aspects. First of all, the information structure and the control decision are highly distributed. Each player only sees its individual set of observations. Secondly, the structure of the underlying system considered here is assumed to be largely unknown, precluding the detailed analysis in the classical models. Another interesting connection for this work is with low data rate controllers, see for example [1], [3], and [20].

2. Motivation from the Power Control Problem

The performance-target-tracking problem is modeled after the Quality-of-Service (QoS) tracking problem in wireless communication ([3]). Consider a cellular network consisting of \( M \) mobile units distributed over \( L \) cells as depicted in figure 1. Each mobile unit intends to communicate with the base station controlling the cell to which it belongs. The communication channels are usually duplex, that is, bi-directional. For simplicity we will focus on the uplink channel, that is, the communication channel from mobile units to the base stations. Due to the propagation characteristic of electromagnetic wave, the signal received for each communication channel is corrupted by interference noises coming from other mobile units.

Let \( G_{ij} \) represent the channel power gain between the \( i \)-th receiver and the \( j \)-th transmitter. Note that \( G_{ij} > 0 \). Define the \( M \)-by-\( M \) channel gain matrix by \( G = \begin{pmatrix} G_{ij} \end{pmatrix} \), (we use the convention that bold face denotes vectors or matrices.) Mathematically, we can represent the quality of a channel by the signal-to-noise ratio defined by:
\[\Gamma_i(p) = \frac{G_ip_i}{\sum_j G_j p_j + \eta_i}\] (2.1)

where \( p = (p_1, ..., p_M) \) denotes the power vector and \( \eta = (\eta_1, ..., \eta_M) \) denotes the thermal noise vector and they are both non-negative vectors. (For vector \( x \), \( x \geq 0 \) if and only if all components of \( x \) are non-negative.)

Notice that in this model, the observer, the base station measuring the signal-to-noise ratio and the controller, the mobile unit, are not co-located. Hence, a communication channel is needed between the observer and the controller.

Given a set of QoS targets, defined by signal-to-noise ratio levels, \( \gamma_i \)'s, the objective of the target-tracking problem is to adjust the vector \( p \) to satisfy the goal:

\[\Gamma_i \geq \gamma_i \] (2.2)

for all \( i \). If the channel gain matrix and the noise vector are completely known, the problem can be easily solved by classical theory of non-negative matrices (see for example [15].) However, this classical result cannot be used to provide a robust and decentralized algorithm to the target-tracking problem because it assumes that a central decision-maker knows the channel gain matrix, computes the solutions and conveys to all the mobile units. In fact, it is almost impossible to know the exact value of the gain channel. Moreover, due to physical phenomena known as fading effects, the gain matrix behaves as a stochastic process, so any parameter identification has to be a continuous process.

In [4], Foschini and Miljanic proposed an algorithm that does not require any information on the value of \( G \). Furthermore, the algorithm is distributed in the sense that in order to update its power level, each mobile unit only needs to communicate with its base station independently. The only information needed is its latest signal-to-noise ratio. This algorithm and its asynchronous version [13] provide a more practical approach for the target-tracking problem in regard to our stated criterions. However, a major shortcoming, due to the requirement of updating based on precise signal-to-noise ratios, is that a high data rate communication channel between the base station and the mobile unit is needed. Inevitably, it is necessary to quantize the observation data. The granularity of quantization levels is closely tied with the communication data rate requirement and fundamentally affect the convergence issue. In [17], Sung and Wong proposed a quantized version of the Foschini-Miljanic algorithm that requires only 1 ternary symbol of data per unit cycle per mobile unit. The tri-state algorithm is simple to describe:

\[ P_i^{(n+1)} = \omega(p^{(n)}, \gamma_i) = \begin{cases} \delta p_i^{(n)}, & \text{if } \Gamma_i^{(n)} < \delta^{-1} \gamma_i, \\ \delta^{-1} p_i^{(n)}, & \text{if } \Gamma_i^{(n)} > \delta \gamma_i, \\ p_i^{(n)}, & \text{otherwise} \end{cases} \] (2.3)

Here \( p^{(n)} = (p_1^{(n)}, ..., p_N^{(n)}) \), and \( \delta > 1 \) represents the basic multiplicative quantization unit of the power levels. That is, the power level is of the form \( c \delta^i \), for some integer \( i \). The following convergence property was established:

**Theorem 1** [17]: Suppose a feasible solution exists to the QoS target-tracking problem. Given any positive initial point, the discrete algorithm (2.3) converges to a solution, \( \hat{P} \), with the property:
\[ \delta^{-1} \gamma_i < \Gamma_i(\hat{P}) < \delta \gamma_i \] (2.4)

Schematically, the result is captured in the following figure:

Note that it is possible to construct a similar algorithm with only two states by removing the convergence zone. In this case, the algorithm will not converge, but with the right technical assumption it can be shown to oscillate periodically around the target. In fact, this is basically the power control mechanism adopted in the IS95 CDMA standard (see for example [14].)

The proposed algorithm achieves both the distributive and low data rate objectives. In this paper, we will show that it also possesses strong robustness properties. In this regard, there is a potential connection between this work and the results of Xie and Guo in [21].

3. Performance-Target-Tracking Model

To fix ideas for subsequent discussion, a performance-target-tracking model is defined in this section. Consider a discrete time model described by the equations:

\[
\begin{align*}
\begin{cases}
x_i^{(n)} = x_i^{(n-1)} + u_i^{(n-1)}, & x_i^{(n)} \in \mathbb{R}, \\
y_i^{(n)} = h_i(x_i^{(n)}, \ldots, x_M^{(n)}), \\
u_i^{(n)} = u_i(y_i^{(n)}),
\end{cases}
\end{align*}
\] (3.1)

where \( x_i^{(n)} \) and \( u_i^{(n)} \) represent the state and the control value of player \( i \) at iteration \( n \) respectively. Here, \( y = (y_1, \ldots, y_M) \) denotes the vector of observation measurements that drive the controllers. Given a set of performance-targets \( \gamma = (\gamma_1, \ldots, \gamma_M) \), one for each individual player, the objective of the performance-target-tracking problem is to find feedback controllers that guarantee performance-targets can be achieved and maintained. Note that the power control model is a special case of (3.1) provided that we define \( x_i^{(n)} = \log P_i^{(n)} \). This model also can be applied to some peer-to-peer network control problems. Note that it is possible to generalize the work stated here to cases where the inherent dynamics of the system is non-trivial. Details of that will not be discussed here, however.

Definition: The performance target, \( \gamma = (\gamma_1, \ldots, \gamma_M) \), is feasible if there exists a state \( (x_1^*, \ldots, x_M^*) \) such that
\[ y_i = h_i(x_i^1, \ldots, x_i^M) = \gamma_i. \]  

(3.2)

for all \( i \).

Assuming that a performance target is feasible, a natural question is how to achieve the target. As in the finite communication bandwidth models investigated in [1], [12], and [13], the focus of this paper is on models where the observer is not co-located with the controller. The picture can be schematically described in the following figures:

In the first scheme, the controller is co-located with the dynamical system and requires high data rate transmission from the observable sensor. For systems with low data rate connection between the observable sensor and the controller, the scheme depicted in figure 3b is more desirable. In this scheme, high-level decisions, such as setting the target levels, is separated from the low-level decisions, such as target-tracking for a given performance target.

For systems using discrete-value controllers, it is unreasonable to expect infinite precision in controlling the performance-targets. Instead, a modified concept of convergence is needed. Let \( \delta \) and \( \varepsilon \) be the quantization unit of the controller and the performance-target respectively. Here, the values are quantized linearly. That is, the values of the states \( x_i^{(j)} \) are of the form \( x_i^{(0)} + j\delta \) for some integer \( j \). Let \( \gamma = (\gamma_1, \ldots, \gamma_M) \) be performance-targets. The performance-target space is also quantized into \( \gamma_i + j\varepsilon \).

**Definition:** Given performance targets, \( \gamma = (\gamma_1, \ldots, \gamma_M) \), and quantization unit, \( \varepsilon \), the system \( \Sigma \) is said to be convergent if for any initial state, there exists an \( N \), such that:

\[ |y_i^{(n)} - \gamma_i| \leq \varepsilon \]  

(3.4)
for all $i$ and $n \geq N$.

In this paper, the convergence property of a class of tri-state algorithms is studied. The algorithm is defined with respect to two quantization units. In particular, given quantization levels $\delta$ and $\varepsilon$, we define the positive tri-state feedback control, $\omega^+$,

$$u_i^{(n)} = \omega^+(y_i^{(n)}, \gamma_i) = \begin{cases} 
\delta & y_i^{(n)} < \gamma_i - \varepsilon, \\
-\delta & y_i^{(n)} > \gamma_i + \varepsilon, \\
0 & \text{otherwise}.
\end{cases} \quad (3.5)$$

Similarly, one defines the negative tri-state feedback control, $\omega^-$, by reversing the inequality signs in (3.5.) The focus of this paper is to investigate the convergent issue of these classes of tri-state controllers.

4. A Counter-Example

The convergence result stated in Theorem 1 is robust in the sense that the tri-state controller converges for any channel gain matrix. However, the power control problem has a specific structure, which is essential to the proof. Namely, in the power control problem, whenever a mobile unit raises its power level to improve its performance target the rest of the mobile units suffer a detrimental effect. For problems in which a player may simultaneously improve its own performance-target as well as other players, the convergence property may not hold for the tri-state controller.

Consider a system with 4 players and the utilization functions are of the form:

$$y_1 = x_1 + x_2 - x_3, \quad y_2 = x_1 + x_2 - x_4, \quad y_3 = x_3 + x_4 - x_1, \quad y_4 = x_3 + x_4 - x_2 \quad (4.1)$$

In a sense, players 1 and 2 are in one clique with mutual interests and player 3 and 4 in another. Let $\varepsilon = \delta$ and set the targets for all players to 0. This target is obviously feasible. If the initial starting points are given by:

$$x_1^{(0)} = 2\delta, \quad x_2^{(0)} = -2\delta, \quad x_3^{(0)} = 2\delta, \quad x_4^{(0)} = -2\delta \quad (4.2)$$

Since

$$y_1^{(0)} = -2\delta = -2\varepsilon, \quad y_2^{(0)} = 2\varepsilon, \quad y_3^{(0)} = -2\varepsilon, \quad y_4^{(0)} = 2\varepsilon \quad (4.3)$$

it follows that:

$$x_1^{(1)} = 3\delta, \quad x_2^{(1)} = -3\delta, \quad x_3^{(1)} = 3\delta, \quad x_4^{(1)} = -3\delta \quad (4.4)$$

In general,

$$x_i^{(n)} = (2+n)\delta, \quad x_2^{(n)} = -(2+n)\delta, \quad x_3^{(n)} = (2+n)\delta, \quad x_4^{(n)} = -(2+n)\delta \quad (4.5)$$
Hence, the algorithm is divergent. This counter-example may seem counter-intuitive. In fact, this just illustrates a scenario where the algorithm fails due to players in a clique are confused about their roles due to a lack of direct communication. One may label this as the “too many cooks syndrome.”

5. Convergence Results

In this section, the convergence issue of the tri-state controller is discussed. In particular, a sufficient condition for convergence is presented. The result depends on two technical conditions on the structure of the system parameters.

Technical Assumption 1: The observation functions, $h_i$, admit the following decomposition:

$$h_i(x_1, \ldots, x_M) = c_i x_i + d_i(x_1, \ldots, x_M)$$

where $d_i$ satisfies the Lipschitz condition:

$$|d_i(a_1, \ldots, a_M) - d_i(b_1, \ldots, b_M)| \leq L \|a_1 - b_1, \ldots, a_M - b_M\|_\infty$$

for some $L$ that satisfies the condition:

$$|c_i| \geq L.$$  (5.3)

Note that for linear observation functions, this technical condition translates into a simple diagonal dominant condition. As a result, this assumption holds for a wide class of systems.

Technical Assumption 2: The quantization units satisfy the relation:

$$|c_i| + L \delta \gtrsim \epsilon$$  (5.4)

for all $i$.

As noted earlier, it is reasonable to expect that in order to control the output with a high accuracy, the controller should have a fine quantization level. In particular, note that according to (3.5) there is an output convergence zone with width $2\epsilon$ such that the controller takes a zero value. Suppose the system is in a state with output slightly below this zone and suppose the controller step size, $\delta$, is large enough, one can easily construct an example in which the output of the system oscillates around the convergence zone. As a result, such a system cannot converge. However, if condition (5.4) holds, this possibility is ruled out.

Under these technical assumptions, one can establish the following result:

Proposition 1: Suppose the performance targets, $\gamma = (\gamma_1, \ldots, \gamma_M)$, are feasible. That is, there exists a state, $(x_1^*, \ldots, x_M^*)$, such that
\[ y_i(x_1^*, \ldots, x_M^*) = \gamma_i, \quad \text{for } i = 1, 2, \ldots, M. \] (5.5)

If technical assumptions 1 and 2 hold, then for any initial state, there exists a quantized solution, \((\hat{x}_1, \ldots, \hat{x}_M)\), such that

\[ |y_i(\hat{x}_1, \ldots, \hat{x}_M) - \gamma_i| \leq \epsilon. \] (5.6)

**Proof:** Let the initial states be \((x_1^{(0)}, \ldots, x_M^{(0)})\). There exists, \((\hat{x}_1, \ldots, \hat{x}_M)\), of the form \(x_i^{(0)} + j\delta\) with the property

\[ x_i^* - \frac{\delta}{2} \leq \hat{x}_i < x_i^* + \frac{\delta}{2}. \] (5.7)

Using the decomposition stated in Technical Condition 1,

\[ |y_i(\hat{x}_1, \ldots, \hat{x}_M) - \gamma_i| = \left| y_i(\hat{x}_1, \ldots, \hat{x}_M) - y_i(x_1^*, \ldots, x_M^*) \right| \leq |c_i| \delta + \frac{L\delta}{2} \leq \epsilon. \] (5.8)

We are now ready to state the main theorem:

**Theorem 2:** Consider the system \((\Sigma)\) satisfying technical assumptions 1 and 2. For each player, a tri-state controller is assigned according to the rule that if in the decomposition of \(y_i\) defined by assumption 1, the term \(c_i\) is positive, then let \(u_i = \omega^+\), otherwise let \(u_i = \omega^-\). Then the resulting system is convergent to any feasible set of targets.

The proof of this theorem is related to the approach in [17] and [18]. The proof can be broken into two major steps. Namely, under the technical assumptions stated, one can show that the algorithm defines a bounded trajectory. Since the feasible states are discrete, this implies that the algorithm either converges to a fixed point or exhibits a cycle trajectory. However, it can be shown that no cycle can exist in the current model. Before proving these results, note that by mapping \(x_i\) to \(-x_i\) if necessary, there is no loss in generality in assuming that \(c_i\) is positive and that \(u_i = \omega^+\).

**Proposition 2:** Under the assumptions stated in Theorem 2, the trajectory of the algorithm is bounded.

**Proof:** Let \((\hat{x}_1, \ldots, \hat{x}_M)\) be a vector consisting of components of the form \(x_i^{(0)} + j\delta\) that satisfies (5.6). Such a vector exists according to Proposition 1. Define \(a(i,n)\) by:

\[ x_i^{(0)} = \hat{x}_i + a(i,n). \] (5.9)

Due to the nature of the tri-state algorithm,
\[ |a(i,n+1) - a(i,n)| \leq \delta. \] (5.10)

Let \( K(n) = \max_i |a(i,n)| \). If \( K(n) = 0 \), then the system has converged at or before time \( n \) and must have a bounded trajectory. So assume that \( K(n) \neq 0 \). The proposition clearly holds if \( K(n) \) is a non-increasing function of \( n \) whenever \( K(n) \) has non-zero value. To show this, assume player \( i \) achieves the maximum at time \( n \). There are two possibilities, either

1. \( a(i,n) = K(n) > 0 \) or
2. \( a(i,n) = -K(n) < 0 \).

Assume that the first condition holds. Then,

\[
y_i^{(n)}(x_1^{(n)}, \ldots, x_M^{(n)}) = y_i^{(n)}(\hat{x}_i + a(1,n), \ldots, \hat{x}_M + a(M,n)) \\
= c_i(\hat{x}_i + a(i,n)) + d_i(\hat{x}_i + a(1,n), \ldots, \hat{x}_M + a(M,n)) \\
\geq c_i \hat{x}_i + d_i(\hat{x}_i, \ldots, \hat{x}_M) + c_i K(n) - L \max_j |a(j,n)| \\
\geq \gamma_i - \varepsilon + (c_i - L)K(n) \geq \gamma_i - \varepsilon.
\] (5.11)

Hence, \( a(i,n+1) \leq a(i,n) \). Similarly, for the second case,

\[
y_i^{(n)}(x_1^{(n)}, \ldots, x_M^{(n)}) = y_i^{(n)}(\hat{x}_i + a(1,n), \ldots, \hat{x}_M + a(M,n)) \\
= c_i(\hat{x}_i + a(i,n)) + d_i(\hat{x}_i + a(1,n), \ldots, \hat{x}_M + a(M,n)) \\
\leq c_i \hat{x}_i + d_i(\hat{x}_i, \ldots, \hat{x}_M) - c_i K(n) + L \max_j |a(j,n)| \\
\leq \gamma_i + \varepsilon - (c_i - L)K(n) \leq \gamma_i + \varepsilon.
\] (5.12)

Therefore, \( 0 \geq a(i,n+1) \geq a(i,n) \). Therefore, for any player \( i \) that achieves the maximum value at iteration \( n \),

\[ |a(i,n+1)| \leq |a(i,n)| \] (5.13)

For the player \( j \) where \( |a(j,n)| < K(n) \)

\[ |a(j,n+1)| \leq |a(j,n)| + \delta \leq K(n). \] (5.14)

Hence, \( K(n) \) is non-increasing as a function of \( n \). ■

Proposition 2 shows that for given any initial state, if the performance-target is feasible, the trajectory is bounded within a finite state set. Consider a sequence of vectors, \( \mathbf{x}^{(n)} \), \( n = 0, 1, \ldots \), the sequence is said to be asymptotically periodic if there exists integers, \( N > 0 \) and \( T \geq 1 \) such that for all \( n \geq N \),

\[
\mathbf{x}^{(n)} = \mathbf{x}^{(n+T)}.
\] (5.15)
Since the transition of the algorithm depends only on the current state and is deterministic, if the trajectory is finite, then it either converges to a stationary point, or the trajectory is asymptotically periodic with a minimum period larger than 1. We claim that the system satisfying the conditions of Theorem 2 cannot be asymptotically periodic.

An arbitrary player, say $i$, is said to undergo a peak-slide of length $k$ ($k \geq 1$), from time $m$ to $n$, if there is an integer $n > m$ so that:

\begin{align}
\psi_{i}^{(m-1)} &= \psi_{i}^{(m)} - \delta \\
\psi_{i}^{(m)} &= \psi_{i}^{(n)} + k\delta. 
\end{align}

(5.16)

Pictorially, a peak-slide is depicted in figure 4:

![Figure 4](image)

**Lemma 1**: If there is a peak-slide of length $k$ ($k \geq 1$), from time $m$ to $n$ for player $i$, then there is a player $j$, and an integer $t$, $m \leq t < n$, such that

\[ |\psi_{i}^{(t)} - \psi_{j}^{(m-1)}| \geq (k + 1)\delta. \]

(5.17)

**Proof**: Since $\psi_{i}^{(m)} = \psi_{i}^{(n)} + k\delta$, and the algorithm changes by at most one $\delta$ at a time, there exists an integer $t$, $m \leq t < n$, such that $\psi_{i}^{(t)} = \psi_{i}^{(n)} + \delta$ and $\gamma_{i}^{(t)} > \gamma_{i} - \epsilon$. Therefore,

\[ \psi_{i}^{(m-1)} = \psi_{i}^{(m)} - \delta = \psi_{i}^{(t)} + (k - 2)\delta = \psi_{i}^{(n)} + (k - 1)\delta. \]

(5.18)

Note that, $\gamma_{i}^{(m-1)} < \gamma_{i} - \epsilon$. Hence,

\begin{align}
\gamma_{i} + \epsilon &< \gamma_{i}^{(t)} = c_{i}\psi_{i}^{(t)} + d(\psi^{(t)}) \\
&= c_{i}\psi_{i}^{(m-1)} - c_{i}(k - 2)\delta + d(\psi^{(m-1)}) - d(\psi^{(m-1)}) + d(\psi^{(t)}) \\
&\leq \gamma_{i}^{(m-1)} - c_{i}(k - 2)\delta + L \| \psi^{(t)} - \psi^{(m-1)} \|_{\infty} \\
&< \gamma_{i} - \epsilon - c_{i}(k - 2)\delta + L \| \psi^{(t)} - \psi^{(m-1)} \|_{\infty}.
\end{align}

(5.19)
It follows that
\[
2\varepsilon + c_i(k-2)\delta < L \|x^{(i)} - x^{(m-1)}\|_\infty.
\] (5.20)

According to the technical condition 2, \(2\varepsilon \geq (|c_i| + L)\delta\). So,
\[
\|x^{(i)} - x^{(m-1)}\|_\infty > \left[1 + \frac{c_i}{L}(k - 1)\right] \delta \geq k\delta.
\] (5.21)

Hence, there exists an integer \(j\) such that
\[
|x^{(i)}_j - x^{(m-1)}_j| > k\delta.
\] (5.22)

Since the difference of the two states are multiples of \(\delta\), the Proposition holds. ■

**Proposition 3:** Under the assumption stated in Theorem 2, the trajectory of the algorithm cannot be asymptotically periodic with minimum period larger than 1 if the performance targets are feasible.

**Proof:** Suppose that the trajectory is asymptotically periodic and let \(N > 0\) and \(T > 1\) be integers such that for all \(n \geq N\), \(x^{(n)} = x^{(n+T)}\), where \(T\) is the minimum period. It follows that there exists a peak-slide starting after \(N\) with length larger than or equal to 1. Since the trajectory is bounded, there is a peak-slide with maximum length \(k\). According to Lemma 1, there exists an integer \(j\) such that:
\[
|x^{(i)}_j - x^{(m-1)}_j| \geq (k+1)\delta,
\] (5.23)

where \(N \leq s < t\). Since the trajectory is periodic after time \(N\), (5.23) implies that there is a peak-slide with length larger than \(k\), a contradiction. ■

Proof of Theorem 2: Since the trajectory is bounded, it must be asymptotically periodic. According to Proposition 3, the minimum period is equal to 1. That is, the trajectory converges to a stationary state. ■

6. Conclusion

In this paper, we investigated the convergence properties of a class of distributed feedback controls, the tri-state feedback control, that requires very low communication data rate. The circle of ideas presented here is motivated by wireless communication problem and may find application in other networking systems. Although the tri-state algorithm is robust in the sense that it is convergent for a large class of systems, the convergent rate may be slow. More complicated types of controllers, such as those proposed in [3] and [20], may provide faster convergence at the price of requiring a higher communication data rate. This and other issues are open questions waiting to be answered.

**Reference:**