Precision IPACS in the Presence of Dynamic Uncertainty

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Abstract—An adaptive robust integrated power and attitude control system (IPACS) is presented for a variable speed control moment gyroscopes (VSCMG)-actuated satellite. The developed IPACS method is capable of achieving precision attitude control while simultaneously achieving asymptotic power tracking for a rigid-body satellite in the presence of dynamic uncertainty in the VSCMG gimbals and wheels. In addition, the developed controller compensates for the effects of uncertain, time-varying satellite inertia properties. Some challenges encountered in the control design are that the control input is premultiplied by a non-square, time-varying, nonlinear, and uncertain matrix and is embedded in a discontinuous nonlinearity. Globally uniformly ultimately bounded attitude tracking and asymptotic power tracking results are proven via Lyapunov stability analysis.

I. INTRODUCTION

Due to the inherent limitations of using chemical batteries to store excess energy for satellites, flywheels have been proposed as a viable substitute for energy storage. By combining the attitude control capability of control moment gyroscopes (CMG) with the energy storage capability of variable-speed flywheels, variable speed control moment gyroscopes (VSCMGs) offer the potential to combine energy storage and attitude control functions in a single device. This integration of attitude control and energy storage functions can reduce the satellite bus mass, volume, and cost. In light of the exorbitant costs of launching large and heavy payloads into space, the advantage of integrated functionality is apparent.

The variable wheel spin rates of VSCMGs endow them with additional degrees of freedom, which can be used to achieve multiple objectives such as simultaneous attitude control and energy storage. For this reason, VSCMGs are often utilized in the design of integrated power and attitude control systems (IPACS) [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. However, the energy storage and attitude control capabilities of VSCMGs can deteriorate over time due to changes in the dynamics such as bearing degradation and increased friction in the gimbals or wheels [13]. For example, the ramifications of friction buildup include degraded power transfer capabilities and potential destabilizing disturbances. For example, friction buildup in the constant speed CMGs on the Skylab space station and the Magellan satellite [14] resulted in catastrophic failures of those systems. The potential for a failure due to friction uncertainties/changes in the gimbal and flywheel dynamics necessitates special consideration in designing IPACS for VSCMG-actuated satellites.

Motivated by the virtues of using VSCMGs, the problem of designing IPACS in the presence of uncertainties has been investigated by several researchers. In [10], a feedback control law is designed for a VSCMG-actuated satellite, which achieves asymptotic attitude regulation for a satellite with known inertia properties. Model-based and adaptive control strategies are presented in [15], which achieve asymptotic tracking for a spacecraft in the presence of constant uncertainty in the spacecraft inertia, while simultaneously tracking a desired energy/power profile. In [12], model-based and indirect adaptive controllers are developed for a spacecraft with uncertain inertia properties. An adaptive control algorithm is developed in [16], which achieves attitude control for a VSCMG-actuated satellite in the presence of unknown misalignments of the axis directions of the VSCMG actuators. The control developments in [10], [15], [12], [16] assume that the satellite inertia properties are constant. While this assumption may be valid for larger satellites, significant fluctuations in the overall satellite inertia can occur in smaller satellites (small-sats) due to the motion of the VSCMGs. Further, the controllers in [10], [15], [12] assume no dynamic uncertainty in the VSCMG actuators. While the aforementioned controllers perform well for applications involving large satellites, they may not be well suited for IPACS for VSCMG-actuated small-sats.

The control development in this paper is motivated by the desire to include the uncertain dynamics of the gimbals and flywheels in the control design for improved robustness to these disturbances. Specifically, an adaptive robust attitude controller is developed, which compensate for uncertain, time-varying inertia and unknown friction in the VSCMG gimbals and wheels while simultaneously providing asymptotic power tracking. The inclusion of friction effects in the VSCMG gimbals and wheels creates significant complications in the control development. The dynamic friction effects manifest themselves as nonsquare, time-varying, input-multiplicative uncertainty in the tracking error dynamics. The static friction effects in the dynamic model result in the control input gimbal angular rate being embedded inside of a discontinuous nonlinearity (i.e., the standard signum function). A robust control method is used to mitigate the disturbance resulting from the static friction, and an adaptive control law is used to compensate for the dynamic friction. 

1This research is supported in part by the NSF CAREER award number 0547448, NSF award number 0901491, support from ASTREC an NSF I/UCRC, and the Department of Energy grant number DE-FG04-86NE37967 as part of the DOE University Research Program in Robotics (URPR).
and inertia uncertainties. Lyapunov-based stability analyses are provided, which prove globally uniformly bounded (GUUB) attitude tracking and asymptotic power tracking in the presence of the aforementioned VSCMG anomalies and satellite inertia uncertainty.

II. DYNAMIC MODEL

The dynamic model for a rigid body VSCMG-actuated satellite can be expressed as [12]

$$
\dot{J}\omega + J\dot{\omega} + A_g I_{cg}\delta + A_1 I_{us} \Gamma^d \delta + A_\tau f = T
$$

$$
+ \omega \times \left( J \omega + A_g I_{cg} \dot{\delta} + A_1 I_{us} \Omega \right) = \tau_f
$$

$$
I_{us} \dot{\Omega} = \tau_f.
$$

In (1), $\omega(t)$, $\dot{\omega}(t) \in \mathbb{R}^3$ denote the angular velocity and acceleration of the satellite body-fixed frame $F$ with respect to $\mathcal{I}$ expressed in $F$, $\Omega(t)$, $\dot{\Omega}(t) \in \mathbb{R}^3$ denote the angular rate and acceleration of the VSCMG wheels, and $\delta(t), \dot{\delta}(t), \ddot{\delta}(t) \in \mathbb{R}^4$ denote the angular rate, velocity, and acceleration of the VSCMG gimbals. Also in (1), $J(\delta) \in \mathbb{R}^{3 \times 3}$ is the total spacecraft inertia matrix approximated as

$$
J = B I + A_g I_{cg} A_g^T + A_1 I_{cg} I_{cg}^T + A_\tau f
$$

which is positive definite and symmetric such that

$$
\frac{1}{2} \lambda_{\min} \{ J \} \| \xi \|^2 \leq \xi^T J \xi \leq \frac{1}{2} \lambda_{\max} \{ J \} \| \xi \|^2 \quad \forall \xi \in \mathbb{R}^n
$$

where $\lambda_{\min} \{ J \}, \lambda_{\max} \{ J \} \in \mathbb{R}$ are the minimum and maximum eigenvalues of $J(\delta)$, respectively. In (1) and (3), the matrices $A_g, A_\delta(\delta), A_\tau(\delta) \in \mathbb{R}^{3 \times 4}$ transform the gimbal, spin, and transverse inertia components, respectively, of each flywheel/gimbal assembly to the satellite body-fixed frame; $\tau_f(t) \in \mathbb{R}^4$ denotes the torque generated by the flywheels; $B I \in \mathbb{R}^{3 \times 3}$ is a matrix containing the combined inertia of the satellite platform and the point masses of the VSCMGs; and $I_c, I_s, I_d \in \mathbb{R}^{4 \times 4}$ are constant diagonal matrices whose elements are the centroidal inertia components of each flywheel/gimbal assembly about its gimbal, spin, and transverse direction, where $I_c = I_g + I_w$, where $I_g$ and $I_w \in \mathbb{R}^{4 \times 4}$ denote the gimbal and inertia of the VSCMG gimbals, respectively, expressed in the $\cdot$ axes (i.e., $\cdot$ is $g$, $s$, or $t$ for the gimbal, spin, or transverse directional axes). The torque vector $T(\delta, \dot{\delta}, \Omega) \in \mathbb{R}^3$ in (1) is defined as

$$
T = -A_\tau(\delta) \left( F_{dg}\delta + F_{sg} \text{sgn}(\delta) \right) - A_g(\delta) \left( F_{dw}\Omega + F_{sw} \text{sgn}(\Omega) \right).
$$

In (5), $F_{dg}, F_{sg} \in \mathbb{R}^{4 \times 4}$ and $F_{dw}, F_{sw} \in \mathbb{R}^{4 \times 4}$ are diagonal matrices containing the uncertain dynamic and static friction coefficients for the gimbals and wheels, respectively, and $\text{sgn}(\cdot) \in \mathbb{R}^4$ denotes a vector form of the standard $\text{sgn}(\cdot)$ function where the $\text{sgn}(\cdot)$ is applied to each element of $\delta(t)$. The notation $\Gamma^d(\Omega)$ in (1) denotes the diagonal matrix $\text{diag}\{ \Omega_1, \Omega_2, \Omega_3, \Omega_4 \} \in \mathbb{R}^{4 \times 4}$, and $\zeta^\times \in \mathbb{R}^4$ is the following skew-symmetric matrix:

$$
\zeta^\times = \begin{bmatrix}
0 & -\zeta_3 & \zeta_2 \\
\zeta_3 & 0 & -\zeta_1 \\
-\zeta_2 & \zeta_1 & 0
\end{bmatrix}.
$$

The transformation matrices $A_\tau(\delta)$ and $A_g(\delta)$ in (1), (3), and (5) are defined as

$$
A_\tau = \begin{bmatrix}
-s_\gamma c_\delta & s_\gamma s_\delta & -s_\delta & -s_\gamma c_\delta \\
-s_\delta & -s_\gamma c_\delta & s_\gamma s_\delta & s_\delta \\
c_\gamma c_\delta & c_\gamma s_\delta & c_\gamma c_\delta & c_\gamma s_\delta
\end{bmatrix}
$$

and

$$
A_g = \begin{bmatrix}
-s_\gamma s_\delta & s_\gamma c_\delta & c_\delta & -s_\gamma s_\delta \\
-c_\delta & -s_\gamma c_\delta & s_\gamma s_\delta & s_\delta \\
c_\gamma s_\delta & c_\gamma c_\delta & c_\gamma s_\delta & c_\gamma c_\delta
\end{bmatrix},
$$

where $\gamma \in \mathbb{R}$ is a constant angle $(54.74\text{ deg})$ of each wall of the pyramid-shaped CMG cluster, and the shorthands $s_x$ and $c_x$ denote $\sin(x)$ and $\cos(x)$, respectively. Since the elements of $A_\tau(\delta)$ and $A_g(\delta)$ contain bounded trigonometric terms, the following inequalities can be developed:

$$
\| A_\tau(\delta) \|_{\infty} \leq \zeta_1
$$

$$
\| A_g(\delta) \|_{\infty} \leq \zeta_0
$$

where $\zeta_0, \zeta_1 \in \mathbb{R}$ are positive bounding constants, and $\| \cdot \|_{\infty}$ denotes the induced infinity norm of a matrix.

III. KINEMATIC MODEL

The rotational kinematics of the rigid-body satellite can be expressed as

$$
\dot{q}_v = \frac{1}{2} (q_v^T \omega + q_0 \omega)
$$

$$
\dot{q}_0 = -\frac{1}{2} q_v^T \omega.
$$

In (11) and (12), $q(t) \triangleq \{q_0(t), q_v(t)\} \in \mathbb{R} \times \mathbb{R}^3$ represents the unit quaternion describing the orientation of the body-fixed frame $F$ with respect to $\mathcal{I}$, subject to the constraint

$$
q_v^T q_v + q_0^T q_0 = 1.
$$

Rotation matrices that bring $\mathcal{I}$ onto $F$ and $\mathcal{I}$ onto $F_d$ (desired body-fixed orientation), denoted by $R(q_v, q_0) \in SO(3)$ and $R_d(q_v, q_0) \in SO(3)$, respectively, are defined as

$$
R \triangleq \left( q_0^T - q_v^T q_v \right) I_3 + 2 q_v q_v^T - 2 q_v q_v^T q_v
$$

$$
R_d \triangleq \left( q_0^T - q_v^T q_v \right) I_3 + 2 q_v q_v^T - 2 q_v q_v^T q_v
$$

where $I_3$ denotes the $3 \times 3$ identity matrix, and $q_d(t) \triangleq \{q_0d(t), q_vd(t)\} \in \mathbb{R} \times \mathbb{R}^3$ represents the desired unit quaternion that describes the orientation of the body-fixed frame $F_d$ with respect to $\mathcal{I}$. Using (11) and (12), $\omega(t)$ can be expressed in terms of the quaternion as

$$
\omega = 2 (q_0 \dot{q}_v - q_v \dot{q}_0) - 2 q_v^T \dot{q}_v.
$$

The desired angular velocity of the body-fixed frame $F_d$ with respect to $\mathcal{I}$ expressed in $F_d$ can also be determined as

$$
\omega_d = 2 (q_0d \dot{q}_v - q_vd \dot{q}_0) - 2 q_vd^T \dot{q}_v.
$$

The subsequent analysis is based on the assumption that the desired quaternion $q_0d(t)$, $q_vd(t)$, and their first three
time derivatives are bounded for all time. This assumption ensures that $\omega_d(t)$ of (17) and its first two time derivatives are bounded for all time.

IV. CONTROL OBJECTIVE

A. Attitude Control Objective

The attitude control objective is to develop a flywheel acceleration and gimbal rate control law to enable the attitude of $\mathcal{F}$ to track the attitude of $\mathcal{F}_d$. To quantify the objective, an attitude tracking error denoted by $\hat{R}(e_v, e_0) \in \mathbb{R}^{3 \times 3}$ is defined that brings $\mathcal{F}_d$ onto $\mathcal{F}$ as

$$\hat{R} \triangleq RR^T_d = (e_0^T - e_v^T e_v) I_3 + 2 e_v e_0^T + 2 e_0 e_v^T \quad (18)$$

where $R(q_v, q_0)$ and $R_d(q_{vd}, q_{0d})$ were defined in (14) and (15), respectively, and the quaternion tracking error $e(t) \triangleq \{e_0(t), e_v(t)\} \in \mathbb{R} \times \mathbb{R}^3$ is defined as

$$e_0 \triangleq q_0 d q_0d - q_0 d q_v - q_v d q_0 + q_0 d q_v. \quad (19)$$

$$e_v \triangleq q_0 d q_v - q_0 d q_0d + q_v d q_0 - q_0 d q_0d. \quad (20)$$

Based on (18), the attitude control objective can be stated as

$$\hat{R}(e_v(t), e_0(t)) \rightarrow I_3 \quad \text{as} \quad t \rightarrow \infty. \quad (21)$$

Based on the tracking error formulation, the angular velocity of $\mathcal{F}$ with respect to $\mathcal{F}_d$ expressed in $\mathcal{F}$, denoted by $\hat{\omega}(t)$, is defined as

$$\hat{\omega} \triangleq \omega - \hat{R} \dot{\omega}_d. \quad (22)$$

From the definitions of the quaternion tracking error variables, the following constraint can be developed [17]:

$$e_v^T e_v + e_0^2 = 1 \quad (23)$$

where $\|\cdot\|$ represents the standard Euclidean norm. From (23),

$$\|e_v(t)\| \rightarrow 0 \Rightarrow |e_0(t)| \rightarrow 1 \quad (25)$$

and hence, (18) can be used to conclude that if (25) is satisfied, then the control objective in (21) will be achieved.

B. Power Tracking Objective

The kinetic energy $E(g) \in \mathbb{R}$ stored in the flywheels of a VSCMG can be expressed as [18]

$$E(g) = \frac{1}{2} \Omega^T f_1 I_{ws} \Omega(t). \quad (26)$$

The power tracking control objective is to develop a flywheel acceleration control law to enable the actual VSCMG power to track a desired power profile $P_d(t) \in \mathbb{R}$ while simultaneously tracking a desired time-varying attitude. The desired power profile can be related to a desired kinetic energy profile $E_d(t) \in \mathbb{R}$ as

$$E_d(t) = \int_0^t P_d(\sigma) d\sigma \quad (27)$$

where the desired kinetic energy and power profiles are assumed to be bounded. To quantify the energy tracking objective, a kinetic energy tracking error $\eta_E(t) \in \mathbb{R}$ is defined as

$$\eta_E = E_d - E. \quad (28)$$

Based on (28), the power tracking control objective can be stated as

$$\eta_E \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty. \quad (29)$$

V. ADAPTIVE IPACS

A. Adaptive Attitude Control Development

To facilitate the controller design, an auxiliary signal $r(t) \in \mathbb{R}^3$ is defined as [18]

$$r \triangleq \omega - \hat{R} \omega_d + \alpha e_v. \quad (30)$$

where $\alpha \in \mathbb{R}^{3 \times 3}$ is a constant, positive definite, diagonal control gain matrix. After substituting (30) into (22), the angular velocity tracking error can be expressed as

$$\hat{\omega} = r - \alpha e_v. \quad (31)$$

Motivation for the design of $r(t)$ is obtained from the subsequent Lyapunov-based stability analysis and the fact that (16) - (20) can be used to determine the open-loop quaternion tracking error as

$$\dot{e}_v = \frac{1}{2} \left( e_v^\times + e_0 I \right) \hat{\omega} \quad \dot{e}_0 = -\frac{1}{2} e_v^\times \hat{\omega}. \quad (32)$$

After taking the time derivative of (30) and multiplying both sides of the resulting expression by $J(\delta)$, the following expression can be obtained:

$$J \dot{r} = J \dot{\omega} + J \omega^\times \hat{R} \omega_d - J \hat{R} \dot{\omega}_d + \frac{1}{2} J \alpha (e_v^\times + e_0 I) \hat{\omega}. \quad (33)$$

where the fact that

$$\dot{\hat{R}} = -\omega^\times \hat{R}$$

was utilized. Under the standard assumption that the gimbal acceleration term $A_g I_{cg} \delta(t)$ is negligible [10], [19], [20], (1), (2), (5), (30), and (32) can be used to express (33) as

$$J \dot{r} = \Upsilon_1 \dot{\delta} + \Upsilon_2 \Omega + \Upsilon_3 \dot{\theta}_1 - \frac{1}{2} J \dot{r}$$

$$-A_t F_{sg} sgn(\hat{\delta}) - A_s F_{sw} sgn(\Omega) \quad (34)$$

where the uncertain matrix $\Upsilon_1 (e_v, e_0, r, \delta, \Omega, t) \in \mathbb{R}^{3 \times 4}$ is defined via the parameterization

$$\Upsilon_1 (\delta, t) \triangleq -\frac{\partial J}{\partial \delta} \left( \frac{1}{2} r^\times + \hat{R} \omega_d - \alpha e_v \right) - A_t F_{sg} \hat{\delta} - A_s F_{sw} \delta \quad (35)$$

and the uncertain matrix $\Upsilon_2 (\delta, t) \in \mathbb{R}^{3 \times 4}$ is defined as

$$\Upsilon_2 (\delta, t) \triangleq -A_s F_{sw} I_{ws} \quad (36)$$

Also in (34), $\Upsilon_1 (e_v, e_0, r, \omega, \omega_d, \delta, \Omega, t) \dot{\theta}_1$ is defined via the parameterization

$$\Upsilon_1 (\delta, t) \triangleq -A_t F_{dwc} \omega_d - \omega^\times J \omega - \omega^\times A_s I_{ws} \Omega \quad (37)$$

$$+J \omega^\times \hat{R} \omega_d - J \hat{R} \dot{\omega}_d + \frac{1}{2} J \alpha (e_v^\times + e_0 I) \hat{\omega}.$$
In (37), $Y_1(\cdot) \in \mathbb{R}^{3 \times p_1}$ is a measurable regression matrix, and $	heta_1 \in \mathbb{R}^{p_1}$ is a vector of unknown constants. In (34), the auxiliary matrices $\bar{Y}_1(\cdot)$ and $Y_2(\cdot)$ contain only linearly parameterizable uncertainty, so the terms are grouped as

$$\bar{Y}_1 \dot{\delta} + Y_2 \dot{\Omega} \triangleq Y_2 \theta_2$$ (38)

where $Y_2 \left[ e_v, e_0, r, \omega, \Omega, t \right]^T \in \mathbb{R}^{3 \times p_2}$ is a measurable regression matrix, and $\theta_2 \in \mathbb{R}^{p_2}$ is a vector of unknown constants. Some of the control design challenges for the open-loop system in (34) are that the control input $\delta(t)$ is premultiplied by a nonsquare, unknown time-varying matrix $\bar{Y}_1(\cdot)$, and the gimbal rate control input $\dot{\theta}(t)$ is embedded inside of a discontinuous nonlinearity (i.e., $A_t F_{sgn}(\delta)$).

To address the fact that the control input is premultiplied by a nonsquare, unknown time-varying matrix, estimates of the uncertainty in (38), denoted by $\tilde{Y}_1(t) \in \mathbb{R}^{3 \times 4}$ and $\tilde{Y}_2(t) \in \mathbb{R}^{3 \times 4}$, are defined as

$$\dot{\tilde{Y}}_1 \sim Y_2 \dot{\Omega} \triangleq Y_2 \tilde{\theta}_2$$ (39)

where $\tilde{\theta}_2(t) \in \mathbb{R}^{p_2}$ is a subsequently designed estimate for the parametric uncertainty in $\bar{Y}_1(\cdot)$ and $Y_2(\cdot)$. Based on (38) and (39), (34) can be rewritten as

$$J \dot{r} = \dot{\tilde{Y}}_1 \dot{\delta} + \dot{\tilde{Y}}_2 \dot{\Omega} + Y_1 \theta_1 + Y_2 \tilde{\theta}_2 - \frac{1}{2} J r$$ (40)

where the notation $\tilde{\theta}_2(t) \in \mathbb{R}^{p_2}$ is defined as

$$\tilde{\theta}_2 = \theta_2 - \tilde{\theta}_2.$$ (41)

Based on the expression in (40) and the subsequent stability analysis, the flywheel acceleration control input is designed as

$$\dot{\Omega} = -\tilde{Y}_2^T u_c - \left( I_n - \tilde{Y}_2^T \tilde{Y}_2 \right) g$$ (42)

where $g(t)$ is an auxiliary control signal designed to achieve the subsequent power tracking objective [18]. In (42), the auxiliary control input $u_c(t)$ is designed as

$$u_c = Y_1 \hat{\theta}_1 + e_v$$ (43)

and the gimbal rate control input is designed as

$$\dot{\delta} = -\tilde{Y}_1^+ (k + k_n) r$$ (44)

where $k, k_n \in \mathbb{R}$ denote positive control gains. Since the matrices $\tilde{Y}_1(t)$ and $\tilde{Y}_2(t)$ are nonsquare, the pseudo-inverses $\tilde{Y}_1^T \in \mathbb{R}^{n \times 3}$ and $\tilde{Y}_2^T \in \mathbb{R}^{n \times 4}$ are defined so that $\tilde{Y}_1 \tilde{Y}_1^T = I_3$, and the matrix $I_n - \tilde{Y}_1^T \tilde{Y}_1$, which projects vectors onto the null space of $\tilde{Y}_1$, satisfies the following properties:

$$\begin{align*}
(I_n - \tilde{Y}_1^+ \tilde{Y}_1) (I_n - \tilde{Y}_1^+ \tilde{Y}_1) & = I_n - \tilde{Y}_1^+ \tilde{Y}_1 \\
\tilde{Y}_1 (I_n - \tilde{Y}_1^+ \tilde{Y}_1) & = 0 \\
(I_n - \tilde{Y}_1^+ \tilde{Y}_1) (I_n - \tilde{Y}_1^+ \tilde{Y}_1) & = \left(I_n - \tilde{Y}_1^+ \tilde{Y}_1 \right)^T = \left(I_n - \tilde{Y}_1^+ \tilde{Y}_1 \right) = 0.
\end{align*}$$ (45)

After substituting (42)-(44) into (40), the following closed-loop dynamics for $r(t)$ can be obtained:

$$J \ddot{r} = -\frac{1}{2} J r + Y_1 \ddot{\theta}_1 + Y_2 \ddot{\tilde{\theta}}_2 - kr - k_n r$$ (46)

$$-A_t F_{sgn}(\dot{\delta}) - A_s F_{sw}(\dot{\Omega}) - e_v$$

where the notation $\tilde{\theta}_2(t) \in \mathbb{R}^{p_2}$ is defined as

$$\tilde{\theta}_2 = \theta_2 - \theta_2.$$ (47)

Based on (40) and the subsequent stability analysis, the parameter estimates $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ are designed as

$$\dot{\hat{\theta}}_1 = \hat{\theta}_1 - \hat{\theta}_1$$ (48)

where $\hat{\theta}_1, \tilde{\theta}_1, \theta_2, \tilde{\theta}_2 \in \mathbb{R}$ denote known, constant lower and upper bounds for each element of $\theta_1(t)$ and $\theta_2(t)$, respectively.

B. Adaptive Power Tracking Control Development

Based on (27) and (28), the power tracking error can be quantified as

$$\hat{\eta}_E = P_a - \hat{E}.$$ (50)

To develop the closed-loop dynamics for the power tracking error, the time derivative of (26) is substituted into (50) for $\dot{E}(t)$ as

$$\dot{\hat{\eta}}_E = P_a - \bar{Y}_3 \hat{\Omega}$$ (51)

where (2) was utilized, and the uncertain vector $\bar{Y}_3(\Omega, t) \in \mathbb{R}^{1 \times 4}$ is defined as

$$\bar{Y}_3 \triangleq \Omega^T I_{ws}.$$ (52)

Since the uncertainty in (51) is linearly parameterizable, the following parameterization can be developed:

$$\bar{Y}_3 \hat{\Omega} \triangleq Y_3 \theta_3$$ (53)

where $Y_3(\Omega, \hat{\Omega}, t) \in \mathbb{R}^{1 \times p_3}$ is a measurable regression matrix, and $\theta_3 \in \mathbb{R}^{p_3}$ is a vector of unknown constants. To address the fact that the control input $\Omega(t)$ is premultiplied by an unknown time-varying matrix, an estimate of the uncertainty in (53), denoted by $\hat{Y}_3(t) \in \mathbb{R}^{1 \times 4}$ is defined as

$$\hat{Y}_3 \hat{\Omega} \triangleq \hat{Y}_3 \hat{\theta}_3$$ (54)
where $\dot{\hat{\theta}}_3(t) \in \mathbb{R}^{p_3}$ is a subsequently designed estimate for the parametric uncertainty in $\Upsilon_3(\Omega, t)$. Based on (53) and (54), (51) can be rewritten as

$$\dot{\hat{\theta}}_3 = P_d - Y_3 \dot{\hat{\theta}}_3 - \dot{\hat{\Upsilon}}_3 \Omega \tag{55}$$

where the notation $\dot{\hat{\theta}}_3(t)$ is defined as

$$\hat{\theta}_3 \triangleq \hat{\theta}_3 - \bar{\theta}_3. \tag{56}$$

Based on (55) and the subsequent stability analysis, the parameter estimate $\hat{\theta}_3(t)$ is designed as

$$\dot{\hat{\theta}}_3 = \text{proj} \left(-\Gamma_3 \dot{\hat{\Upsilon}}_3 \eta_E\right) \tag{57}$$

where $\Gamma_3 \in \mathbb{R}^{p_3 \times p_3}$ denotes a constant, positive-definite, diagonal adaptation gain matrix, and $\text{proj}(\cdot)$ denotes a projection algorithm utilized to guarantee that the $i^{th}$ element of $\hat{\theta}_3(t)$ can be bounded as

$$\hat{\theta}_3^i \leq \hat{\theta}_{3i} \leq \bar{\theta}_3 \tag{58}$$

where $\hat{\theta}_3^i, \bar{\theta}_3^i \in \mathbb{R}$ denote known, constant lower and upper bounds for each element of $\hat{\theta}_3(t)$, respectively. After substituting (42) into (55), the following expression can be obtained

$$\dot{\hat{\theta}}_3 = P_d - Y_3 \dot{\hat{\theta}}_3 + \dot{\Upsilon}_3 \tilde{T}_3^T u_c + \dot{\Upsilon}_3 \left( I_n - \tilde{T}_3^T \tilde{T}_2 \right) g. \tag{59}$$

Based on the structure of (59), the signal $g(t)$ is designed to satisfy the following relationship

$$\hat{\Upsilon}_3 \left( I_n - \tilde{T}_2^T \tilde{T}_2 \right) g = -P_d + \hat{\Upsilon}_3 \tilde{T}_2^T u_c - k_E \eta_E \tag{60}$$

where $k_E \in \mathbb{R}$ is a positive constant control gain. Based on the Moore-Penrose pseudo-inverse properties introduced in (45), the minimum norm solution of (60) is given as

$$g = \left( I_n - \tilde{T}_2^T \tilde{T}_2 \right) \hat{\Upsilon}_3^T \left( I_n - \tilde{T}_2^T \tilde{T}_2 \right) \hat{\Upsilon}_3^T \eta_E \tag{61}$$

Based on (61), simultaneous attitude and power tracking is possible anytime $\left( I_n - \tilde{T}_2^T \tilde{T}_2 \right) \neq 0$. Since $\left( I_n - \tilde{T}_2^T \tilde{T}_2 \right) \neq 0 \forall \tilde{T}_2(t)$, the simultaneous attitude and power tracking objective can be achieved as long as $\tilde{T}_2(t)$ and $\tilde{T}_3(t)$ are not included in the null space of $I_n - \tilde{T}_2^T \tilde{T}_2$. Since $\hat{\Upsilon}_3(t)$ contains the adaptive elements of $\hat{\theta}_3(t)$, the projection function in (57) can be selected to expand the domain within which the simultaneous objective is possible. After substituting (61) into (59) for $g(t)$, the following closed-loop error system can be obtained:

$$\dot{\hat{\theta}}_3 = -k_E \eta_E - Y_3 \hat{\theta}_3. \tag{62}$$

\section*{C. Stability Analysis}

\textbf{Theorem 1}: The flywheel control input of (42), (43) and (61) along with the adaptive update laws given in (48) and the gimbal rate control input of (44) ensure global uniformly bounded (GUUB) attitude tracking such that

$$\|e_v(t)\| \to e_0 \exp(-\varepsilon_1 t) + e_2 \quad \text{as} \quad t \to \infty \tag{63}$$

where $e_0, e_1, e_2 \in \mathbb{R}$ denote positive bounding constants and asymptotic energy/power tracking in the sense that

$$\eta_E(t) \to 0 \quad \text{as} \quad t \to \infty. \tag{64}$$

\textbf{Proof}: To prove the asymptotic power tracking result, let $V_E(\eta_E, \theta_3, t) \in \mathbb{R}$ be a nonnegative function defined as

$$V_E = \frac{1}{2} \|e_v\|^2 + \frac{1}{2} \dot{\theta}_3^T \Gamma_3^{-1} \dot{\theta}_3. \tag{65}$$

After using (57) and (62), the time derivative of $V_E(t)$ can be expressed as

$$\dot{V}_E = -k_E \eta_E^2. \tag{66}$$

Based on (65) and (66), $\eta_E(t) \in L_\infty \cap L_2$. The assumption that $\theta_3(t) \in L_\infty$ can be used along with (58), to show that $\dot{\theta}_3(t) \in L_\infty$. Given that $\eta_E(t) \in L_\infty$, (24), (42), (43), (49), and (61) can be used to show that $\Omega(t) \in L_\infty$. Since $\Omega(t) \in L_\infty$, (54) can be used along with (58) to conclude that $Y_3(t) \in L_\infty$. Given that $\eta_E(t), \dot{\theta}_3(t), Y_3(t) \in L_\infty$, (62) can be used to conclude that $\dot{\eta}_E(t) \in L_\infty$ (i.e., $\eta_E(t)$ is uniformly continuous). Barbala’s Lemma can now be used to show that $\eta_E(t) \to 0$ as $t \to \infty$.

To prove the GUUB attitude tracking result, consider the nonnegative function $V(v_e, e_0, r, \theta_1, \theta_2, t) \in \mathbb{R}$ defined as

$$V \triangleq e_v^T e_v + (1 - e_0)^2 + \frac{1}{2} r^T J_r \tag{67}$$

$$+ \frac{1}{2} \theta_3^T \Gamma_3^{-1} \theta_3 + \frac{1}{2} \dot{\theta}_3^T \Gamma_2^{-1} \dot{\theta}_3.$$

By using the bounds given in (4), (24), and (49), $V(t)$ can be upper and lower bounded as

$$\lambda_1\|z\|^2 + c_1 \leq V(t) \leq \lambda_2\|z\|^2 + c_2 \tag{68}$$

where $\lambda_1, \lambda_2, c_1, c_2 \in \mathbb{R}$ are known positive bounding constants, and $z(t) \in \mathbb{R}^6$ is defined as

$$z \triangleq \left[ e_v^T \quad r^T \right]^T. \tag{69}$$

After using (32), (41), (46), and (47), the time derivative of $V(t)$ can be expressed as

$$\dot{V} = e_v^T \left( e_v + e_0 I \right) \omega + (1 - e_0) e_v^T \omega$$

$$+ r^T \left( Y_1 \dot{\theta}_1 + Y_2 \dot{\theta}_2 - kr - k_n \right) - e_v$$

$$- A_1 F_{sg} \text{sgn} \left( \delta \right) - A_1 F_{sw} \text{sgn} \left( \Omega \right)$$

$$- \dot{\theta}_3^T \Gamma_3^{-1} \dot{\theta}_3 - \dot{\theta}_3^T \Gamma_2^{-1} \dot{\theta}_3. \tag{70}$$

By using (9), (10), (31), (48), and exploiting the fact that $e_v^T e_v \omega = 0$ the expression in (70) can be upper bounded as

$$\dot{V} \leq -\lambda_3\|z\|^2 - k_n\|r\|^2$$

$$+ (\zeta_0\|F_{sg}\|_{\infty} + \zeta_1\|F_{sw}\|_{\infty})\|r\| \tag{71}$$

where $\lambda_3 = \min \{ \alpha, k \} \in \mathbb{R}$. After completing the squares, (71) can be written as

$$\dot{V}(t) \leq -\lambda_3\|z\|^2 + \frac{\beta^2}{4k_n} \tag{72}$$

where $\beta \triangleq \zeta_0\|F_{sg}\|_{\infty} + \zeta_1\|F_{sw}\|_{\infty}$. Since the inequality in (68) can be utilized to lower bound $\|z(t)\|^2$ as
\[ \|z\|^2 \geq \frac{1}{\lambda_2} V(t) - c_2 \lambda_2^{-1} \] \quad (73)

The inequality in (72) can be expressed as

\[ \dot{V}(t) \leq -\frac{\lambda_1}{\lambda_2} V(t) + \varepsilon \] \quad (74)

where \( \varepsilon \in \mathbb{R} \) is a positive constant that is defined as

\[ \varepsilon = \frac{\beta^2}{4k_1n} + \frac{\lambda_2c_2}{\lambda_2}. \] \quad (75)

The linear differential inequality in (74) can be solved as

\[ V(t) \leq V(0) \exp \left( -\frac{\lambda_1}{\lambda_2} t + \frac{\lambda_2}{\lambda_3} \right) - \exp \left( -\frac{\lambda_2}{\lambda_2} t \right). \] \quad (76)

The expressions in (67) and (76) can be used to conclude that \( r(t) \in \mathcal{L}_\infty \). Thus, from (24), (31), and (69), \( \dot{\omega}(t), z(t) \in \mathcal{L}_\infty \), and (30) can be used to conclude that \( \omega(t) \in \mathcal{L}_\infty \). Equation (32) then shows that \( \dot{c}_0(t), \dot{\omega}_0(t) \in \mathcal{L}_\infty \). Hence, (9), (10), (37), (39), (42), (43), (44), (49), (61), and (64) can be used along with the assumption that \( P_0(t) \in \mathcal{L}_\infty \) to prove that the control inputs \( \delta(t), \Omega(t) \in \mathcal{L}_\infty \). Standard signal chasing arguments can then be utilized to prove that all remaining signals remain bounded during closed-loop operation. The inequalities in (68) can now be used along with (75) and (76) to conclude that

\[ \|z\|^2 \leq \left( \frac{\lambda_2}{\lambda_1} \|z(0)\|^2 + c_2 \right) \exp \left\{ -\frac{\lambda_3}{\lambda_2} t \right\} \]

\[ + \left( \frac{\lambda_2 \beta^2}{4k_1n \lambda_3} + c_2 - c_1 \right). \]

The result in (63) can now be directly obtained from (77).

VI. CONCLUSIONS

In this paper, an IPACS for a VSCMG-actuated satellite is presented. In the presence of uncertain dynamic and static friction in the VSCMG gimbals and wheels, the controller is capable of achieving globally uniformly ultimately bounded (GUUB) attitude tracking while simultaneously tracking a desired power profile asymptotically. In addition, the controller compensates for the effects of uncertain, time-varying satellite inertia properties. The difficulties arising from dynamic friction and uncertain satellite inertia are mitigated through innovative development of the error system along with a Lyapunov-based adaptive law. In the presence of static friction, the control design is complicated due to the control input being embedded in a discontinuous nonlinearity. This difficulty is overcome with the use of a robust tracking control law. The attitude and power tracking results are proven via Lyapunov stability analysis.

REFERENCES


