A large amount of effort has been devoted to design better under-actuated robot hands. The most widely adopted approaches include rigid coupled hands and self-adaptive hands. The objective of this research is to design a robot finger which combines advantages of both ways and overcomes their disadvantages. The concept of coupling and self-adaptation (COSA) was introduced. A linkage-based two-DOF (degree of freedom) COSA robot finger was designed, optimized and studied in this paper. The theoretical analysis and the experiments on the finger show that it is able to execute human-like motion and adaptive grasps in multiple patterns. The research exposes a promising novel under-actuated mechanism for hands design with wide applications.

Keywords: Robot hand; under-actuated finger; coupling mechanism; self-adaptation.

1. Introduction

Application of robot hands in both laboratory and industrial environments have increased in the last few decades. Many designs, in terms of motion and grasps, try to mimic human hands, on which research has been conducted long before robotic hands were invented. Taxonomy of human hand grasps was given in Ref. 1, dividing grasps into two groups: power grasp and precision grasp. Aiming to implement the precise grasp of human hands, dexterous hands were proposed. To gain enough dexterity, an actuator is used for every joint in the hand. As a result, active adaptation to the object’s shape is obtained. Computer control and synthesis algorithms are developed to synchronize the motors. As a complicated but key issue in hand design, it has been studied with a great deal of work as well, in Ref. 2 for example.
Dexterous hand is still among the most active research topics nowadays. Some recent works on dexterous hands were presented in Refs. 3–6. The hands exhibit huge progress; however, the distance between robot hands and human hands remains to be eliminated. It is partly attributed to the integration of mechanism, sensors and control systems. Moreover, integration of force information for individual finger and combination of information from different fingers are quite complex issues. Facing all these difficulties, dexterous hands are restricted from simplification in structure, improvement of reliability and drop in cost at present.

Before having crossed the limit of dexterous hands, requirement for simpler, cheaper and more practical universal grasping hands came about. In contrast to dexterous hands, grasping hands emphasize on power grasp more than precise manipulation of objects to reduce the complexity. Under-actuated hands were proposed. The principle of under-actuation was presented in Ref. 8. It defined under-actuated hands as those that have fewer actuators than degrees of freedom (DOFs). Traditional under-actuated hands can be divided into several types, including the coupling ones and the adaptive ones, and both have their own advantages and disadvantages.

The question is: Is it possible to design a hand that can overcome the shortcomings of coupling and adaptive under-actuation, therefore better performance achieving? We believe integrating the two types into one unit (only one proximal phalanx and one distal phalanx) is one solution. Coupling and self-adaptation (COSA) was proposed with this thought. This paper is organized in the following way. Section 2 provides a brief overview of the existing types of traditional under-actuated hands. Section 3 introduces the generic concept of COSA and the mechanical design of the linkage-based COSA finger. Section 4 describes the method to identify the geometric parameters in the design. Section 5 discusses different grasping scenarios of the finger and provides the simulation. Finally, the experiments validating the design are presented in Sec. 6.

2. Traditional Under-Actuated Hands

Before presenting the novel mechanism proposed in this research, a brief overview of existing types of under-actuated hands is given in this section. Recent under-actuated hands include Southampton Hand,9,10 Barrett Hand11 and the hands presented in Refs. 12 and 13. These designs can be divided to several groups. A three-type taxonomy was addressed in Ref. 14 from the view of joint coupling, regarding under-actuated hands as derivation from coupled mechanical hands. According to the author, simplified hands with fixed-motion coupling reduced the overall hand DOFs, but they were not competent for unstructured grasping tasks. Compliance, as the second type, is implemented to allow coupling without fixed-motion between joints. Similar to compliant coupling, under-actuated hands are provided by decoupling mechanism. The latter two types allow changing in coupling proportion and corresponding adaptation to the object geometry. In Ref. 15, under-actuated hands are divided into two groups, namely rigid coupled hands and passive
adaptive or self-adaptive under-actuated hands. Note that the under-actuation in Ref. 15 does not accord to the definition in Ref. 8. Rigid coupling or fixed-motion coupling are not regarded as under-actuation since the number of mechanism DOFs exactly equals to the number of actuators. In this paper, we adopt the definition and taxonomy in Ref. 15. Under-actuation refers to the mechanisms in which the number of movable joints is larger than the number of the actuators.

Rigid coupled hands can be traced back to the early twenty centuries. Most of them are prosthetic hands invented after the First World War.16-18 This approach is simple in mechanical design and reliable in performance. Thus, it is still widely adopted in modern under-actuated hands19-24 or as a mature mechanical platform in dexterous hands.25-28 Rigid coupling can be easily achieved by linking the motion of joints. It imposes a relationship between phalanges, i.e. fixed transmission ratios determined by the mechanical parameters. Before the object is grasped, the hand pre-shapes, or flexes in human-like motion. The pre-shaping of human hands or dexterous hands is based on information of the target, whereas rigid coupled hands do not “know” where the object locates or what the size and shape of the object is. The coupled hand pre-shapes due to its fixed property. No adaptation on the hand is possible. This fatal shortcoming limits the use of rigid coupled hands for universal tasks.

The second type of under-actuated hands is identified with passive adaptation or self-adaptation to the object geometry. Tendon-cable hands29,30 are usually compact in structure but limited in force output. On the contrary, linkage hands31,32 are able to exert large grasping force. Other mechanisms are proposed, for example, gear-rack mechanism presented in Ref. 33. All these mechanisms are designed in order to decouple the relationship between the joints. The resulting advantages of adaptive under-actuated hands are the ability to grasp in unknown environments compared to rigid coupled hands and the simplicity without sensors and complex control compared to dexterous hands.

At the same time, the unique under-actuated grasp pattern brings about problems. One is the ejection phenomenon addressed in Refs. 8 and 34, further discussed in Ref. 35. It refers to that the grasp on the distal phalanx is unstable when the proximal phalanx is not in contact with the object. Another problem is that if the object is not completely enclosed by the envelope of the fingers and is not sufficiently restrained, the effect of the first contact might move the object away.15 Furthermore, the self-adaptive under-actuated hands fail to flex like human hands. The fingers rotate in a straight configuration as a rigid body before touching the object. Inspite of the abundant work put into solutions for the problems, the inherent characters prevent the hands from being used in a wider field of application.

In addition to the main two types, other mechanisms were adopted recently. An under-actuated mechanism for a three-DOF finger was presented in Refs. 36 and 37. In the design, a compliant coupled linkage connects the proximal phalanx to the middle phalanx. A self-adaptive under-actuated slider-link-spring mechanism connects the middle phalanx to the distal phalanx. In a typical grasping process,
the middle and the distal phalanges rotate with respect to the proximal phalanx before they touch the object and the distal phalanx keeps parallel to the middle phalanx during this process. After the proximal phalanx is blocked by the object, the middle and the distal phalanges contact the object in sequence. The object is adaptively grasped. In Ref. 38, another three-DOF finger was presented where a traditional under-actuated and passive adaptive linkage joints the proximal phalanx and the middle phalanx and a rigid coupled linkage joints the middle proximal phalanx and the distal phalanx. The two designs improve the finger performance by combing coupling and adaptive under-actuation. However, they do not improve rigid coupling and adaptive under-actuation themselves. The finger is separated into two parts to implement coupling and adaptive under-actuation, respectively. Therefore, the two designs do not belong to a new under-actuated type, but a simple addition of two traditional types. In order to design a hand overcoming the shortcomings of coupling and adaptive under-actuation in a novel way, the COSA finger is proposed.

3. Coupled and Self-Adaptive Finger

3.1. Coupling and self-adaptation

The COSA finger can be defined as a finger that is designed to execute coupled motion before it contacts the target and then execute self-adaptive motion after it is restrained by the target. COSA is proposed to improve the inherited properties of rigid coupling and self-adaptive under-actuation, integrating the two traditional approaches together into one unit. As a novel approach of under-actuation mechanism, COSA is generated from the two traditional types of under-actuated hands. Figure 1 illustrates the orientation of COSA showing the motion of coupling and self-adaptation in contrast.

Take two-DOF finger as an example. It can be seen that the rigid coupled finger moves through a specific path in space due to the limitation imposed to the angles between the phalanges. The finger, whose upper phalanx is not likely to touch the
object because of the limitation, lacks adaptation to object geometry. As for self-adaptation, the finger keeps straight and rotates as a rigid body before any contact with the object is made.

Figure 2 shows the typical motion of the COSA finger ends with a self-adaptive grasp of the object. The initial configuration is shown in Fig. 2(a). The motion is divided into two stages.

In stage 1, the finger flexes as a rigid coupled finger does. The proximal phalanx rotates by $\theta$ and the distal phalanx rotates by $\varphi$. The ratio of $\theta$ and $\varphi$ is constant, predetermined by the mechanical design. Stage 1 ends when the proximal phalanx touches the object. In stage 2, the distal phalanx continues rotating by $\gamma$ with respect to the proximal phalanx while the proximal phalanx is blocked. The grasp is completed when the phalanges envelop the object. In brief, before the contact the finger behaves in rigid coupled pattern. After that, the finger alters its motion into self-adaptive motion “automatically”. Compared to the fingers presented in Refs. 37 and 38, rigid coupling and self-adaptation are not achieved in different parts of the finger but integrated into one unit.

The COSA demonstrates several advantages. First, comparing to rigid coupled hands, the COSA finger provides self-adaptation to the object geometry. Thus, it can be used in unexpected environments as universal grippers. Second, COSA hand has two main advantages in contrast to pure self-adaptive hand. One is the human-like motion of the finger when the finger flexes. The other is that the finger is able to reduce the possibility of ejecting the object or causing the object to slide away. It is attributed to the pre-shaping in the context of COSA, referred to as coupled motion before contacts. If the proximal phalanx touches the object first, the pre-rotated distal phalanx might prevent the object from sliding away. If the distal phalanx touches the object fist, it is more likely to hold, instead of causing the object to move away. Finally, the simplicity of under-actuated hands is remained in COSA.

3.2. **Finger Architecture of a Novel Contribution**

There are several types of mechanisms which could achieve COSA. Available solutions include tendon cable, linkages, gear-racks and belt-pulley based mechanism. In this research, a novel contribution is proposed with linkage mechanism
being selected, and can be named as linkage COSA mechanism. To achieve COSA, we combined the two mature linkage mechanisms: rigid coupled linkage and self-adaptive under-actuated linkage as shown in Fig. 4(a). The most important issue here is to coordinate the motions exerted by the two mechanisms, respectively. The COSA mechanism should shift one motion to the other when the finger touches the object. Decoupling elements including a changing bar and a decoupling spring are integrated in the architecture to decouple the coupled mechanism and enable the under-actuated mechanism. The finger architecture of linkage COSA mechanism is detailed in Fig. 4(b). The first driving bar, the under-actuation bar, the second driving bar and the proximal phalanx compose the under-actuated four-bar linkage. The changing bar, the coupling bar, the second driving bar and the proximal phalanx compose the coupled four-bar linkage. The decoupling spring connects the changing bar and the base of the finger.

In stage 1, the driving bar pushes the proximal phalanx to rotate. Meanwhile, the decoupling spring keeps rigid. Due to the constraint of the spring, the changing bar acts as fixed to the base. The distal phalanx rotates with respect to the proximal phalanx. In stage 2, the under-actuation bar pushes the coupling bar. The decoupling

\[ \begin{align*}
A & \quad l_1 \\
B & \quad l_2 \\
C & \quad l_3 \\
D & \quad \alpha \\
\end{align*} \]

Fig. 4. Coupling four-bar linkage.
spring deforms because of the motion of the coupling bar and the changing bar. The coupled four-bar linkage does not influence the motion of the finger. Therefore, the finger is pure adaptively under-actuated. The distal phalanx continues rotating until the object is enclosed, the grasp ends. The changing bar and the decoupling spring are the key elements to separate the two stages. Thus, the COSA is achieved by the linkage mechanism.

4. Parameters in the Design

The generic architecture of the linkage COSA mechanism has been presented. However, parameters in the design should be found to make the finger performance well. In this section, the motion, the grasping stability and the appearance personalization are taken into account to dictate the parameters.

4.1. Coupled four-bar linkage

The length of bars are the parameters to be found. The goal is to obtain the transmission ratio in the coupled four-bar linkage. To simplify the design and the analysis in the research, the ratio is chosen as 1: 1. The linkage schematic is shown in Fig. 5, where the following notations are used:

- \( l_1 \): the length of the proximal phalanx,
- \( l_3 \): the length of the coupling bar,
- \( l_4 \): the length of the first driving bar,
- \( c \): the length of the second driving bar,
- \( \alpha \) and \( \beta \): the angles between the bars.

![Fig. 5. Architecture of the two-DOF finger.](image-url)
The relationship among the bar lengths and the angles can be described as:

\[
\alpha = \angle CAD - \angle CA, \quad (1)
\]

\[
\angle CAB = \arcsin\left(c \sin \beta / l_{AC}\right), \quad (2)
\]

\[
\angle CAD = \arccos\left(\frac{l_1^2 + l_{AC}^2 - l_3^2}{2l_4l_{AC}}\right), \quad (3)
\]

\[
l_{AC} = \sqrt{l_1^2 + c^2 - 2l_4c \cos \beta}. \quad (4)
\]

The discrepancy of the two phalanx rotating angles is defined as:

\[
\delta = (\alpha_0 - \alpha) - (\beta - \beta_0), \quad (5)
\]

where \(\alpha_0\) and \(\beta_0\) are the initial angles, both equating to 90°. To reduce the number of the independent parameters, \(l_1\) is given by 40 mm and \(l_4\) is given by 5 mm which makes the finger approximately the size of a human thumb. By approaching \(\delta\) to zero as accurate as possible, \(l_3\) and \(c\) are obtained. Numerical method is used to minimize the mean of \(\delta\) and variance of \(\delta\), i.e.:

\[
E(\delta) = \sum_{i=1}^{n} \delta_i / n, \quad (6)
\]

\[
D(\delta) = \sum_{i=1}^{n} (\delta_i - E(\delta))^2 / n. \quad (7)
\]

The final result is: \(l_1 = 40\) mm, \(c = 7\) mm, \(l_3 = 42\) mm, and \(l_4 = 5\) mm. The mean is proximately two degrees and the standard variance is two degrees, as well.

4.2. Static analysis

Unlike the coupled linkage, the under-actuated four-bar linkage is not designed to ensure the motion. Grasping stability and the size of the finger are taken into account here. Static model of the finger is built to obtain generic grasping forces exerted by the phalanges. Discussion below is set in the coordinate shown in Fig. 5, where some notations are used.

\(l_2\): the length of the distal phalanx,
\(a\): the length of the first driving bar,
\(b\): the length of the under-actuation bar,
\(f_1\): the grasping force on the proximal phalanx,
\(f_2\): the grasping force on the distal phalanx,
\(h_1\): the force arm of \(f_1\) with respect to \(O_1\),
\(h_2\): the force arm of \(f_2\) with respect to \(O_2\),
\(\theta_a\): the rotating angle of the first driving bar with respect to the base,
\(\theta_1\): the rotating angle of the proximal phalanx with respect to the base,
\(\theta_2\): the rotating angle of the distal phalanx with respect to the proximal phalanx,
\(\psi_1\): the angle between the first driving bar and the proximal phalanx,
\(\psi_2\): the angle between the second driving bar and the distal phalanx.
$T_s$ is the torque produced by the decoupling spring, $T_a$ is the torque of actuator, other notations are defined in Fig. 4.

If the friction is ignored, equation based on the principle of virtual work is given by

$$t^T \omega_a = f^T V,$$

where $t$ is the input torque vector, $f$ is the grasping force vector on the two phalanges, $\omega_a$ is the corresponding rotating velocity vector of two driving bars, $V$ is the velocity vector of contact points. Considering the movement of the phalanges, $V$ is described as

$$V = J_v \dot{\theta} = \begin{bmatrix} h_1 \\
 l_1 \cos \theta_2 + h_2 \\
 h_2 \end{bmatrix},$$

(9)

$$J_v = \begin{bmatrix} h_1 \\
 l_1 \cos \theta_2 + h_2 \\
 h_2 \end{bmatrix},$$

(10)

$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\
 \dot{\theta}_2 \end{bmatrix},$$

(11)

where $v_1$ is the velocity of the contact point on the proximal phalanx, and $v_2$ is the velocity of the contact point on the distal phalanx. To solve Eq. (8), $\omega_a$ is expressed by the rotating velocities of the phalanges through the Jacobian matrix derived as

$$\dot{\theta} = J_\omega \omega_a,$$

(12)

$$J_\omega = \begin{bmatrix} 1 & \frac{c[l_1 \sin(\theta_2 - \psi_2) - a \sin(\psi_1 - \psi_2 + \theta_2)]}{a[l_1 \sin(\psi_1) + c \sin(\psi_1 - \psi_2 + \theta_2)]} \\
 0 & 1 \end{bmatrix},$$

(13)

$$\omega_a = \begin{bmatrix} \dot{\theta}_a \\
 \dot{\theta}_2 \end{bmatrix},$$

(14)

Using Eqs. (9)–(14), we can find the grasping forces vector when $|J_v| = h_1 h_2 \neq 0$

$$f^T = t^T J_\omega^{-1} J_v^{-1}.$$  

(15)

Finally, the grasping forces on the phalanges are obtained as

$$f = \begin{bmatrix} (XY + h_2) T_a - XT_s \\
 h_1 h_2 \\
 T_a - YT_a \\
 h_2 \end{bmatrix},$$

(16)

$$X = l_1 \cos \theta_2 + h_2,$$

(17)
\[ Y = \frac{c[l_1 \sin(\theta_2 - \psi_2) - a \sin(\psi_1 - \psi_2 + \theta_2)]}{a[l_1 \sin(\psi_1) + c \sin(\psi_1 - \psi_2 + \theta_2)]}. \]  

(18)

The torque exerted by the decoupling spring is so small comparing to the torque of the motor that it can be ignored. This simplification has been verified by practice. Thus Eq. (16) is rewritten as:

\[
 f = \begin{bmatrix}
 \frac{XY + h_2}{h_1h_2} T_a \\
 -\frac{YT_a}{h_2}
\end{bmatrix}.
\]

(19)

The result reveals that ignoring the contribution of the decoupling spring, the grasping forces are linear functions of actuator torque determined by the finger configuration and the contact locations on the phalanges. Note that \( \theta_1 \) is absent in the final expression of grasping force. The reason is that \( \theta_1 \) describes the rotating of the finger about the base in coupled motion, where no grasping forces have generated yet. In our design, \( \psi_2 \) is set to 90°. \( \psi_1 \) is dictated by the configuration of the under-actuated four-bar linkage, which can be determined by the rotating of the distal phalanx, described by \( \theta_2 \). Consequently, \( \psi_1 \) is single-value function of \( \theta_2 \). The latter is the only geometric configuration factor that influences the forces. The forces are also influenced by \( h_1 \) and \( h_2 \) while only the latter has contribution to the sign of the force. In addition, given a 1:1 transmission ratio, \( \theta_1 \) equals to \( \theta_2 \) in coupled motion. Thus, \( \theta_2 \) in Eqs. (16) and (19) is never less than \( \theta_1 \).

### 4.3. Optimal under-actuated four-bar Linkage

The parameters in the architecture remaining to be found are \( a \) and \( b \), whose effect is related to \( l_1 \) and \( c \). First, the performance of the finger is mainly dictated by ratio between \( a \) and \( c \) due to the transmission of the torque from the proximal phalanx to the distal phalanx. Second, the sum of \( b \) and \( a \) must exceed the sum of \( l_1 \) and \( c \), otherwise the distal phalanx will fail to rotate to 90°. To decouple the inherent relationship of the parameters, two new variables can be defined as

\[
 R = \frac{a}{c},
\]

(20)

\[
 P = \frac{7b}{(l_1 + c - a)}.
\]

(21)

The architecture of the under-actuated four-bar linkage is optimized with different combinations of \( P \) and \( R \), giving an overview of possible fingers.

The grasping stability is in paramount concern, which refers to positive grasping force. If the force exerted by one phalanx on the object is negative, this phalanx will slide. Considering the effect of other fingers in a robotic hand, the stable grasp is product of positive force on the terminal phalanx, which means the distal phalanx in this research. The scenario in which both \( f_1 \) and \( f_2 \) are positive is defined as a full-phalanx grasp. It is one of many stable grasp situations. As analyzed above,
\( \theta_2 \) and \( h_2 \) are the main factors determining the sign of the force. The study therefore is conducted on the plane where \( \theta_2 \) varies from \( 0 \)–\( 90^\circ \) and \( h_2 \) varies from \( 5 \)–\( 30 \text{ mm} \). The criterions of stability are very simple, established as:

(i) The area where the force on the distal phalanx, i.e. \( f_2 \) is positive should be as large as possible on the plane. The corresponding index is defined as:

\[
I_d = \frac{\text{num}(f_2 > 0)}{n},
\]

where \( n \) is the total number calculated on the plane and \( \text{num}(f_2 > 0) \) is the number of points that have positive \( f_2 \).

(ii) The full-phalanx grasp area where both force \( f_1 \) and \( f_2 \) are positive should be as large as possible. The index is defined as:

\[
I_f = \frac{\text{num}(f_1 > 0 \& f_2 > 0)}{n}.
\]

Another index is introduced to estimate the personification of the finger appearance. Large values of \( P \) and \( R \) might result in a very thick finger. Given the \( l_1 \) to be \( 40 \text{ mm} \), the thickness should not be over \( 20 \text{ mm} \). The height from the joint between the first driving bar and the under-actuation bar to the proximal phalanx is calculated as a reference of the thickness. If this distance is less than \( 10 \text{ mm} \), the appearance index \( I_a = 1 \); if not

\[
I_a = \frac{10}{H},
\]

where \( H \) is the height. The three indices are combined in order to obtain a global index

\[
I_G = I_a \sqrt{I_d I_f}.
\]

The square root is put in Eq. (25) because the area weighted by \( I_d \) and \( I_f \) partially overlap.

Figure 6 shows \( I_G \) as a function of \( P \) and \( R \).

The optimal under-actuated four-bar linkage can then be chosen at the point corresponding to the highest \( I_G : P = 1.4 \) and \( R = 1.8 \). Using Eqs. (20) and (21), \( a \) and \( b \) are calculated. All the geometric parameters are listed in Table 1.

Based on the architecture, a good grasping stability is achieved shown in Fig. 7. There are two sections. Area where \( f_1 \) and \( f_2 \) are both positive is colored by white; area where \( f_1 \) is negative but \( f_2 \) is positive is colored by light grey. In all configurations in the figure, the finger is able to execute stable grasp with at least one phalanx. It is also illustrated that the full-phalanx grasp is easier to be established when \( \theta_2 \) and \( h_2 \) are both relatively large.
Table 1. Optimal parameters.

<table>
<thead>
<tr>
<th>Name</th>
<th>$l_1$</th>
<th>$l_2$</th>
<th>$l_3$</th>
<th>$l_4$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\psi_2$</th>
</tr>
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<tbody>
<tr>
<td>Value</td>
<td>40</td>
<td>30</td>
<td>42</td>
<td>5</td>
<td>12.6</td>
<td>48.2</td>
<td>7</td>
<td>90°</td>
</tr>
</tbody>
</table>

Fig. 6. Global index $I_G$.

Fig. 7. Stability of grasp.
5. Grasp Patterns

The motion shown in Fig. 2 is the basis of COSA, however, not the only scenario of grasping. Given different distances and sizes of the object, there will be different grasping results. The multi-pattern grasp is a significant property of COSA finger. It helps the finger to grasp in an unknown environment.

5.1. Classification of grasp patterns

Deprived from the basic COSA motion, five grasp patterns in total can be summarized, which are mainly determined by the distance and the size of the object. To illustrate the typical scenario of each pattern, a cylinder is grasped by the finger in Fig. 8. Assumption is taken that equilibrium is established between the finger and forces from the opposite direction of the finger which are not shown. The patterns are classified according to the number of phalanges in contact with the object and the relationship between the two phalanges in equilibrium:

Pattern A: Only the distal phalanx contacts the cylinder and the coupling ratio between the two phalanges does not change.

Pattern B: The distal phalanx and the proximal phalanx contact the cylinder simultaneously and the coupling ratio between the two phalanges does not change.

Pattern C: The equilibrium is built in a two-phalanx grasp. The proximal phalanx contacts the cylinder first and then is blocked. The distal phalanx continues rotating with respect to the proximal phalanx until it is blocked as well, the coupling ratio changed.

Pattern D: Only the proximal phalanx contacts the cylinder. The distal phalanx rotates to its mechanical limited position, the coupling ratio changed.

Pattern E: The distal phalanx contacts the cylinder first (E₁). Afterwards the cylinder is moved by the phalanx until the cylinder is stopped by the proximal phalanx (E₂). The coupling ratio does not change meanwhile.

Fig. 8. Five grasp patterns.
In the following paragraphs, a further study is performed on the pattern A, B, C and E, respectively. The pattern D is simple in force analysis, so that it is not presented in this paper. We take assumptions that the finger grasps a cylinder which is on the same horizon with the base. In Patterns A, B and C, the cylinder can be regarded as fixed to the ground to simplify the analysis of the finger force. In Pattern E, the cylinder is movable to identify in what condition it will be held by the finger.

5.2. Pattern A

In Pattern A, only the distal phalanx grasps the cylinder. A simulation is conducted in order to throw some light on the relationship between the grasping force and the information of objects. $f_2$ is expressed as functions of two variables. One is $\text{Distance}$, referring to the distance between the center of the cylinder and the base of the finger. The other is $\text{Radius}$, the radius of the cylinder. The two parameters indicate the influence of the position and the size of the object grasped. The result is displayed in Fig. 9. It is indicated that the larger the $\text{Radius}$ is, the larger the force is. In contrast, the shorter the $\text{Distance}$ is, the larger the force is. The finger performs better on the large object than on the small object in Pattern A. From the figure, it can also be seen that $\text{Radius}$ can vary from 0–60 mm; $\text{Distance}$ is limited between 30 and 70 mm. Objects that are very near the finger cannot be grasped in Pattern A.

5.3. Pattern B

In Pattern B, the proximal and the distal phalanges contact the cylinder while the coupling ratio remains unchanged. The grasping forces are similar to those in
Pattern C, which will be analyzed later, since the forces are independent on the relation between the two phalanges. Pattern B is associated with only a specific combination of Radius and Distance. This relationship between Radius and Distance is shown in Fig. 10.

The horizontal axis represents ratio of Radius versus Distance. The vertical axis represents Distance. The Distance must exceed 34 mm and be limited within 40 mm. Two Radiuses are possible for grasp, corresponding to each Distance. The Radiuses is always less than Distance, with a maximum of 40 mm. It can be seen that Pattern B is not common in practice limited by the restrict geometry relation.

5.4. Pattern C

Pattern C is the most important one in the five patterns. The finger executes the typical COSA motion described in Fig. 2. In addition, the distribution of forces in Pattern B and Pattern E₂ is similar to that in Pattern C. The forces analysis of Pattern C can be applied for many other grasp scenarios. The simulation is divided into two steps. In the first step, the forces are described by $\theta_2$ and $h_2$ to expose the rules shared by several patterns. In the second step, the forces are expressed as a function of Radius and Distance to illustrate what object can be grasped by two phalanges in Pattern C.

The result of the first step is shown in Fig. 11. The trend of $f_1$ and $f_2$ develop in approximately opposite directions. The minimum of $f_1$ locates where $\theta_2$ and $h_2$ are smallest while the corresponding $f_2$ is the largest. $f_1$ rises with increasing $\theta_2$ and $h_2$ meanwhile $f_2$ declines. In consequence, the cost of positive $f_1$, which leads to a full-phalanx grasp, is small $f_2$.

The result of the second step is shown in Fig. 12. The range of Distance and Radius are limited. Their sum must exceed $l_1$. Opposite trends of $f_1$ and $f_2$ can be
observed from the figure as well. $f_1$ decreases as Distance gets shorter but $f_2$ increases. The forces are mainly dictated by Distance more than Radius. In conditions of large Distance, $f_1$ is negative, referring to unstable grasp by the proximal phalanx. The grasp effect equates to that in Pattern A. It can be inferred that the finger performs better by the distal phalanx with large force on the object that is further from it. But the small Distance is easier to generate full-phalanx grasp.

5.5. **Pattern E**

Pattern E includes two steps. The first step is the pre-shaping of the finger, in which the finger curls in coupled motion and form half envelop before contact with

![Fig. 11. Grasping forces as functions of $\theta_2$ and $h_2$. (a) $f_1$; (b) $f_2$.](image)
object. The second step is in fact the same with Pattern B. Pre-shaping is one of the most crucial advantages of COSA finger, which has two functions. First, if the proximal phalanx touches the object first, the pre-rotating distal phalanx might prevent the object from being pushed away. Second, if the distal phalanx touches the object first, it is likely to hold the object and push it into the finger grasp range. The finger will grasp the object in this condition where the traditional self-adaptive finger would fail.

Figure 13 compares the COSA finger and traditional self-adaptive finger when the proximal phalanx touches the object first. It is assumed that the object is on an absolutely smooth plane with no friction. In the figure, the traditional finger fails
to adapt to the object after contacting the object. This happens in practice if
the force exerted between the finger and the object is not enough to activate the
under-actuated mechanism. As the object slides, the traditional finger is not able to
process grasp. On the other hand, the pre-shaping of COSA finger improves the
possibilities of successful grasps.

In the terms of the second function, it should be found in what condition the
object can be moved by the finger. Factors influencing the stability in Pattern E are
studied as well. Stable grasps here requires the object is in force equilibrium so that
it is not allowed to slide since the object is not fixed, differing to the assumption in
Sec. 3. Shown in Fig. 14, forces on the finger are described as:

$$f_2 \cos(2\theta_2 - \pi/2) = N,$$

(26)

![Pre-shaping and coupled grasp of COSA finger.](fig14)

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Fig. 13. Pre-shaping of COSA finger in comparison with traditional self-adaptive finger. (a) Traditional self-adaptive finger; (b) pre-shaping of COSA finger.
\[ f_2 \sin(2\theta_2 - \pi/2) \geq f_r, \quad \text{(27)} \]
\[ f_r \leq \mu N. \quad \text{(28)} \]

Here, the gravity is ignored. \( \mu \) is the coefficient of maximum static friction. Only if Eq. (28) is justified strictly, the object can be pushed toward the grasp range instead of the opposite direction. Finally, the condition in which the second function works is:

\[ \theta_2 \geq \frac{\pi}{4} + \frac{1}{2} \arctan \mu. \quad \text{(29)} \]

\( \theta_2 \) is required to be over 45° and increases with increment of \( \mu \). It indicates that the finger needs to pre-shape by a larger angle to grasp the object on the plane with larger coefficient of maximum static friction. Afterwards the object is grasped by the finger, which is shown in Fig. 14(b), force equilibrium is established as:

\[ f_2 \cos(2\theta_2) + f_1 \cos(\theta_2) + f_r = 0, \quad \text{(30)} \]
\[ f_2 \sin(2\theta_2) + f_1 \sin(\theta_2) = N, \quad \text{(31)} \]
\[ f_r \leq \mu N. \quad \text{(32)} \]

A stable grasp exists if Eq. (32) is justified, otherwise the object will slide. Using Eqs. (17)–(19), Eqs. (30) and (31), Eq. (32) is modified to find the condition of stable grasp:

\[ V = (XY + h_2)(\cos \theta_2 + \mu \sin \theta_2) - Yh_1(\cos 2\theta_2 + \mu \sin 2\theta_2) \geq 0. \quad \text{(33)} \]

When \( V \) is positive, the grasp is stable. Similar to Fig. 7, \( V \) is expressed as a function of \( h_2 \) and \( \theta_2 \) in Fig. 15. \( h_1 \) is given by 10 mm and \( \mu \) is given by 0.35. It can be seen that

![Fig. 15. \( V \) as a function of \( h_2 \) and \( \theta_2 \).](image-url)
The major area is stable. Larger $h_2$ and $\theta_2$ are more likely to lead to stable grasp. More simulations are conducted to illustrate the influence of $\mu$ and $h_1$. In Fig. 16, the vertical axis is the ratio of stable area in the plane of $h_2$ and $\theta_2$. Each curve is drawn given a specific $h_1$ as $\mu$ increases from 0.1 to 0.9. It can be summarized that the ratio of stable area rises fast when $h_1$ is small. On the other hand, larger $h_1$ catalyzes the stable area and reduces the influence of $\mu$. Generally, on a rougher plane the finger is easier to grasp firmly.

6. Experiments

A prototype of two-DOF COSA finger was manufactured with its parameters according to the calculation in Sec. 4. A relatively simple test-bed was used, and the finger was controlled by the motor which can be operated by several buttons in a panel, as there was no need for precise control system. Experiments were conducted in the following way in order to validate the coupled motion and the five grasp patterns.

6.1. Experiment of coupled motion

The coupled motion of the finger was tested first. The experiment shown in Fig. 17 demonstrated that the finger flexed in coupled motion. Figure 17(a) shows the middle stage of the coupled motion. Figure 17(b) shows the final stage of the finger.
As shown in the figures, the finger could act as a coupled finger when there were no objects to grasp.

6.2. Experiment of grasp patterns

Patterns A, C and E were tested while Patterns B and D were not, on the account that if Pattern C is proved successful, Pattern D will be consequently feasible, while the success of Pattern E will surely lead to the success of Pattern B. Pattern A is shown in Fig. 18. We can see that the equilibrium was built only with the distal phalanx and the transmission ratio between the two phalanges did not change as it was supposed.

Pattern C is clearly shown by two stages in Figs. 19(a) and 19(b) in which the finger grasped a mouse. In stage 1, the proximal phalanx contacted the object and the distal phalanx rotated towards the object. In stage 2, the two phalanges grasped the object. A good adaptation was demonstrated in the experiment. Many other objects with different shapes were grasped which are not shown here, which represented the success of Pattern C.

Fig. 18. Pattern A.
It was tested that whether COSA finger can move and grasp the object in Pattern E. Experiments were conducted on an A4 print paper and on a polished aluminum board shown in Fig. 20. Patterns $E_1$ and $E_2$ are shown respectively, both of whose experiments were successful. The finger touched the marker pen by the distal phalanx first, and then it moves it toward the proximal phalanx. Finally, the marker pen was grasped firmly by the two phalanges, which demonstrated the grasp was finished in the way of Pattern E.

Fig. 19. Pattern C. (a) Stage 1; (b) stage 2.

(a)  
(b)  

Fig. 20. Pattern E. (a) Pattern $E_1$ on a paper; (b) Pattern $E_2$ on a paper; (c) Pattern $E_1$ on an aluminum board; (d) Pattern $E_2$ on an aluminum board.
7. Conclusion

In this paper, the design and the study of two-DOF COSA finger were addressed. After reviewing the traditional under-actuated hands, the principle of COSA was proposed. As a combination, COSA was designed to inherit advantages of two traditional approaches and overcome their disadvantages. The architecture of the two-DOF finger was presented and methods to design and optimize the mechanism were addressed. In order to describe the motion, the grasping stability and the personification were obtained. A static model was built to find generic grasping forces of the two phalanges. Furthermore, classification of grasp patterns were proposed and simulated to have a deeper understanding of the finger behavior. The five grasp patterns indicate the ability to accomplish multiple grasp tasks. In the last part, the coupled motion and the grasp patterns were validated by experiments on the prototype. In conclusion, the two-DOF COSA finger is successful with comprehensive capability and low complexity. It combines human-like motion inheriting from rigid coupled hands and self-adaptation from adaptive under-actuated hands. It is easy to control and to be manufactured. This research opens a new approach of under-actuated hands design. More than one type of finger can be put forward under the concept of COSA. The analysis in this paper can be applied to other COSA fingers. Further research will be conducted on coordination of multiple COSA fingers in a robot hand.

Acknowledgments

This paper was funded by the National Natural Science Foundation of China (No. 50905093) and Student Research Training (SRT) Project of Tsinghua University.

References


**Wenzeng Zhang**, born in 1975, is currently a visiting scholar in Dept. of Mechanical Engineering, Northwestern University, USA, and an associate professor in Dept. of Mechanical Engineering, Tsinghua University, China. He received his BS, and Ph.D. from Tsinghua University, China, in 1999 and 2005 respectively. His research interests are in the areas of adaptive robotic hand, welding technology and computer vision. He received Seven Best Paper Awards at international conferences on robotics and manufacturing. He has over 100 publications. He has obtained over 80 Chinese invention patents.

**Deyang Zhao**, born in 1988, is currently an undergraduate student in Dept. of Mechanical Engineering, Tsinghua University, China. His research interests include robot control, robot hand and computer vision.
Haipeng Zhou, born in 1990, is currently an undergraduate student in Dept. of Mechanical Engineering, Tsinghua University, China. His research interest is humanoid robot hands.

Zhenguo Sun, born in 1971, is currently an associate professor in Dept. of Mechanical Engineering, Tsinghua University, China. He received his BS, and Ph.D. from Tsinghua University, China, in 1992 and 1998 respectively. His research interests are in the areas of robot control, welding technology and mechatronics.

Dong Du, born in 1962, is currently a professor in Dept. of Mechanical Engineering, Tsinghua University, China. He received his BS, and Ph.D. from Tsinghua University, China, in 1986 and 1991 respectively. His research interests are in the areas of intelligent robotics and applications, and welding automation.

Qiang Chen, born in 1956, is currently a professor in Dept. of Mechanical Engineering, Tsinghua University, China. He received his BS, and Ph.D. from Tsinghua University, China, in 1982 and 1988 respectively. His research interests are in the areas of intelligent robotics and applications, and welding automation.