A New ARQ Method for Turbo Coded MC-CDMA

Kai Xie, Wenbo Wang
School of Telecommunication Engineering
Beijing University of Posts and Telecommunications, Beijing, 100876, P.R.China
Email: {xiekai, wbwang}@bupt.edu.cn

Abstract—This paper investigates a new ARQ method for Turbo coded MC-CDMA systems in the multipath fading environment. Some typical phenomena were observed, then a new parameter to control the ARQ protocol was defined. Encouraged by the prediction of BER in EXIT Chart, we compute a threshold for the new parameter. Simulation results show that the improved ARQ method decreases the bit error rate in less system cost and complexity.

keywords— Turbo code, MC-CDMA, EXIT Chart, ARQ, INSTD

I. INTRODUCTION

As the candidate scheme for the next generation wireless communication systems, Multi-Carrier CDMA (MC-CDMA) has a two-fold advantage. First, it applies Orthogonal Frequency Division Multiplexing (OFDM), one of the Multi-Carrier modulation techniques, to combat multipath fading. On the other hand, it applies CDMA method to enhance the bandwidth efficiency.

Turbo coding [1]–[3], seen widespread applications within wireless communications field for its impressive coding gain, has been demonstrated to work harmony with MC-CDMA in [8]. However the solutions to some questions are yet unavailable. For example, how to overcome the restrictions in performance due to deep fading in Turbo coded MC-CDMA systems?

The Automatic Repeat request (ARQ) is widely used to deal with the hostile channels, however it results in low throughput (especially at low SNRs) and impedes the high-rate transmission. To improve the throughput, the Forward Error Correction (FEC) Codes are combined with ARQ, known as Hybrid Automatic Repeat request (HARQ). HARQ schemes can be roughly classified as Type I [4], Type II [5] and Type III. Researchers also tried to employ HARQ into the MC-CDMA systems [7], in order to provide time diversity gain and to allow efficient utilization of the available bandwidth. However, inherited from the traditional ARQ method, HARQ algorithm still consumes too much system resource. While for practical purpose, the trade-off between system cost and performance improvement is always desirable. Therefore, this work targets the Turbo coded MC-CDMA system in multipath fading environment and investigates a modified ARQ method, based on Extrinsic Information Transfer (EXIT) Chart [6].

The rest of the paper is organized as follows. Section II presents the system model. Section III proposed the modified ARQ method, including a brief review of EXIT Chart. Section IV evaluates the performance improvements and compares the system cost with the conventional ARQ method. Finally, we summarize our contribution with a discussion of future work in Section V.

II. SYSTEM MODEL

Fig. 1 shows the system under study, it depicts the transmitter and receiver in downlink of the Turbo coded MC-CDMA system. The data from user 1 are first encoded by Turbo encoder, with the interleaver followed, then the coded bits are permuted to reduce errors caused by burst errors. Then each encoded and interleaved bit is spread with its user’s specific spreading code C1 vector which represents the L chips. Orthogonal Walsh-Hadamard codes are used in this work, which result in zero cross correlation. After the OFDM modulation, the symbols are sent out to multipath fading channel with symbols from other users.

The MC-CDMA together with multipath fading channel part can be regarded as a special channel. Different from the commonly addressed AWGN channel, this special channel always brings out some disaster frames related with deep fading, which means the minority of error bits. Thus, if we can extract these disaster frames and retransmit them, we can dramatically decrease the bit error rates without increasing system cost too much.

To this end, the controller module is added for ARQ at the receiver, and a new parameter INSTD was introduced. This module will enable the ARQ once the INSTD is detected lower than a certain threshold. More details about INSTD and its threshold will follow in Section III.

III. PROPOSED ARQ METHOD

The traditional ARQ methods choose retransmitted frames according to some additional error detection codes, such as Cyclic Redundancy Check (CRC) codes. Although CRC code is quite simple at implementation, it still add some additional load to the system. Further, CRC code is only able to judge whether a frame contains errors or not, rather than set a particular bit error threshold. In another word, a frame with only one error bit and a frame with hundreds of error bits are the same to the traditional ARQ method, no freedom is left but to retransmit both of them. Then could it be possible that we wisely choose and retransmit error frames who have more error bits than the requirement? Fortunately, we found the answer is positive.
A. Phenomena of Disaster Frames

Observed from the BER performance in Turbo coded MC-CDMA, we find out that there always exist a couple of special frames, containing a lot of error bits, which destroy the entire system performance at the end.

For example, assuming 5 users in the Turbo coded MC-CDMA system, QPSK modulation and MMSE Combining method is utilized. We plot the error bits in each frame vs the number of such frames at 3dB in Fig. 2, with the spread code length of 16, frame length of 2048 bits and code rate of 1/3.

Consequently, if we only retransmit these disaster frames, a significant gain in performance is possible in the price of comparatively low system consumption. Now we need to choose the disaster frames among all the frames. Clearly, two aspects need to be considered: which parameter should we choose and how to set the threshold for this parameter when selecting the disaster frames?

B. Review of EXIT Chart

EXIT Chart (initiated by S. Ten Brink in [6]) can approximately predict the BER performance by tracing the practical iterative procedure instead of the maximum likelihood algorithm, and viewing the extrinsic information exchanged between two soft-input soft-output (SISO) processors from a statistical viewpoint. Additionally, [6] demonstrated that the distribution of extrinsic information sequences between two SISO processors are Gaussian, which gives a way to derive some useful conclusions in our algorithm.

C. Proposed ARQ Method

The proposed ARQ method is based on the statistical features in Turbo codes. Fig. 3 shows the iterative decoder of a Turbo code. For each iteration, the first decoder (BCJR Decoder 1st) takes the systematic bits $S_1$ and respective parity bits $P_1$ from intrinsic information (channel observations). Then it computes soft values $D_1$ and the extrinsic information $E_1 = D_1 - A_1 - Lc \ast S_1$, where $Lc$ is the channel factor. After interleaving, $E_1$ becomes the a priori information $A_2$ for the second decoder (BCJR Decoder 2nd). Similarly, the second decoder takes channel observations $S_2$ and $P_2$, and the a priori information $A_2$ from the first decoder, then feeds back extrinsic information $E_2 = D_2 - A_2 - Lc \ast S_2$ and output $D_2, S_1, P_1, E_1, D_1, S_2, P_2, E_2$ and $D_2$ represent the log-likelihood ratios.

$|D_1|$ and $|D_2|$ denote the absolute values of detective sequence $D_1$ and $D_2$. As iteration increases, the Gaussian distributed $|D_2|$ shifts to larger values in the positive axis, which specifies more confidence on the estimated decoding values. The movements of $|D_2|$ in different iterations are
shown in Fig. 4. The more the iteration number, the higher the mean value of $|D_2|$. For the 5th iteration, all $|D_2|$ are observed to be positive and far from zero, which means all likelihood values are far from their confusing status.

We observed that $|D_2|$ in frames with less error bits increased more with iterations. In Fig. 5, the $|D_2|$ sequences for the frames with 58 error bits and 0 error bit are compared after 5 iterations. $|D_2|$ in the former mostly stay in low values, and $|D_2|$ in the latter stay in high values. This observation implies that some parameter associated with $|D_2|$ might be able to discriminate the erroneous frames. From another perspective, we can view each BCJR decoder as a node, and the exchanged information is somewhat similar as a feedback to the node. When a frame contains few errors, more information acts as positive feedback; on the contrast, when it contains too many errors, less information are useful if feedback. So when decoding, the frame with fewer errors benefits from the positive feedback more.

Simultaneously, we noticed another phenomenon: the ratio of the mean to the standard deviation of $|D_2|$ increases as iteration increases. Define $D_{out}$ as the final $|D_2|$ for hard-decision after all the iterations. Because of its Gaussian distribution, $D_{out}$ can be normalize by its mean, then we can get the normalized standard deviation NSTD. As iteration goes on, the value of NSTD decreases; equivalently, the less the value of NSTD, the less the bit error probability. Hence, we estimate the bit error rate of the frame ($P_{error}$) by Equation (1).

$$P_{error} = \frac{1}{2} \ast \text{erfc}(\frac{1}{\sqrt{2 \ast NSTD}}).$$

To avoid the excessive small value of NSTD, its reciprocal (INSTD) is chosen to be the estimated parameter. Fig. 6 shows the distribution of INSTD values for all the frames. It plots the INSTD values within every frame vs the number of frames for that INSTD value. The blank histograms are for the correct frames, and the black histograms are for the erroneous frames. Seen from Fig. 6, the error frames concentrate in the low INSTD region among the distribution; which means the lower the INSTD value, the more probably the frame contains errors. Hence, it is reasonable that we divide the whole region of INSTD values into three parts as “error region”, “mixed region” and “error free region”. By “error region” (or “error free region”), we mean the frames with INSTD in it would (or not) contain error bits in very high probability. Accordingly, we can decide to retransmit frames with INSTD in “error region”. For a frame with INSTD in “mixed region”, we can hardly judge if it contains error bits or not. Although we can not get rid of the error frames in “mixed region”, it is sufficient to remove almost all disaster frames according to the information from “error region”.

Thus far, the bound of “error region” is the key problem. If the bound is too tight, some disaster frames will still remain; if the bound is too loose, unnecessary system cost is brought. As shown in Table 1, if we set the bound to be 1.5, only 5 frames need to be retransmitted, and only 47.59% error bits belong to these frames; while if the bound is set to be 2.8, 28 frames are assumed to be retransmitted, holding 99.34% error bits. The former provides low cost, but only suggests less than 50% chance to correct the error bits; while the latter promises to correct more than 99% error bits, but requests too much price. Finally, we set the bound at 2, then with 15 frames.
retransmission, we can correct 87.3% error bits, and all these frames are the top 15 frames with most error bits. Next, we explain how to obtain this optimal threshold for INSTD.

According to the given tolerance of error bits per frame, we can get the INSTD threshold \( T_{INSTD} \), which is also the estimated bound of “error region”, by Equation (2).

\[
T_{INSTD} = \frac{1}{\sqrt{2 \cdot \text{erfcinv}(2 \cdot T_m/N_f)}},
\]

(2)

where \( T_m \) is the maximum number of error bits allowed per frame, which varies associated with the different requirements. \( N_f \) is the length of the frame and \( \text{erfcinv}(\cdot) \) is the inverse function of \( \text{erfc}(\cdot) \).

### IV. Simulation Results

Suppose a Turbo coded MC-CDMA system with totally 5 active users and the frame length is 2048 bits. If the maximum number of error bits per frame is 40, then \( T_{INSTD} \) equals to 2.0 according to Equation (2); if the maximum number of error bits per frame is 10, \( T_{INSTD} \) should be 2.5. Turbo coded MC-CDMA with and without the proposed ARQ are both simulated. No puncture is adopted in the system, QPSK modulation and MMSE combining despread method is utilized, and the Orthogonal Walsh-Hadamard codes is length of 16.

![Fig. 6. INSTD chart of Turbo coded MC-CDMA.](image)

### TABLE I

**RELATIVE STATISTICS FOR FRAMES AT “ERROR REGION”**

<table>
<thead>
<tr>
<th>INSTD</th>
<th>Number of error bits in frames</th>
<th>Percent in total error bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1.5</td>
<td>[384 195 130 88]</td>
<td>47.59%</td>
</tr>
<tr>
<td>&lt; 2.0</td>
<td>[384 195 119 144 88]</td>
<td>87.30%</td>
</tr>
<tr>
<td>&lt; 2.8</td>
<td>[384 195 119 114 88 68 68 57 47 40 39 23]</td>
<td>99.34%</td>
</tr>
</tbody>
</table>

### TABLE II

**NUMBER OF ERROR BITS IN RETRANSMITTED FRAMES AT DIFFERENT SNRs (\( T_{INSTD} = 2.0 \))**

<table>
<thead>
<tr>
<th>SNR</th>
<th>Number of error bits in retransmitted frames</th>
<th>Percent in error frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>3dB</td>
<td>[75 46 38 33 170 155 100 296 102 180]</td>
<td>40%</td>
</tr>
<tr>
<td></td>
<td>[101 83 266 45 75 119 75 35 209 33 70 176 75 130 89 210 214]</td>
<td></td>
</tr>
<tr>
<td>3.5dB</td>
<td>[17 58 53 80 36 157 50 100 62 98]</td>
<td>20%</td>
</tr>
</tbody>
</table>

![Fig. 7. BER performance comparison in Turbo coded MC-CDMA with and without the proposed ARQ.](image)

![Fig. 7. BER performance comparison in Turbo coded MC-CDMA with and without the proposed ARQ.](image)

**V. Conclusions**

Disaster frames exist in the Turbo coded MC-CDMA systems, which take the key role in the BER performance. Although in the minority of all frames, they hold the majority of error bits. Our motivation is to extract these disaster frames for retransmission, in turn to improve the system performance with little system cost.

In this paper, a new parameter INSTD is introduced to control the ARQ module, based on the analysis of the statistical...
properties for all error frames. We can estimate the sum of error bits within a frame by \( I_{\text{INSTD}} \), then extract the frames whose \( I_{\text{INSTD}} \)s are larger than \( T_{\text{INSTD}} \) for retransmission. Comparing with the traditional ARQ method, the improved ARQ method is able to give a precise measurement for error frames, such as retransmit error frames when their errors are higher than a certain value, rather than merely retransmit all error frames. Correspondingly, the new ARQ method saves system resource by less retransmissions.

The simulations prove that the proposed ARQ method in the Turbo coded MC-CDMA system can improve the system performance with low cost. Moreover, the exemption of additional error detection codes also decreases system complexity.

This scheme directly brings two aspects for future study. The straightforward extension is to utilize \( I_{\text{INSTD}} \) into HARQ Type II for Turbo coded MC-CDMA. In HARQ Type II, the receiver saves the erroneous bits from the first transmission and jointly decodes the additional parity bits from the retransmission. After the integration of the proposed scheme and HARQ Type II, we can first (1) judge an erroneous frame by its \( I_{\text{INSTD}} \) parameter (without the detection codes module) and (2) only retransmit the necessary parity bits (instead of the entire frame). Another improvement is to concatenate the entire system with Reed-Solomon (RS) codes. Since RS codes are capable of correcting all error bits less than a certain number in each block, thus if we can guarantee the errors per block at the input of the RS decoder, we can ensure almost error-free at the output of the RS decoder. Fortunately, the \( I_{\text{INSTD}} \) parameter proposed in this work can assist in this concatenated scheme.

REFERENCES