An analysis of two-way multi-node relay systems

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Abstract—We consider a two-way relay system where two sources exchange information with the assistance of multiple, non-cooperative, relay nodes. Specifically, we propose a capacity upper bound for the two-way relay channel and derive the achievable regions for different relay strategies, such as the amplify-and-forward (AF), the decode-and-forward (DF) and the compress-and-forward (CF). Furthermore, the upper bound and the achievable regions for the additive white Gaussian noise channel are presented. Numerical results are provided to illustrate how the capacity varies with respect to the number of relay nodes, the distance between the relays and the sources, and the transmission powers.

I. INTRODUCTION

Two-way relay systems where a relay node facilitates the information exchange between two sources has received great attentions recently, thanks to the groundbreaking development in network coding [1]. Most of the work on the two-way wireless relay channel is based on a single relay node. The achievable rate regions for different strategies were first studied in [2]. Analyses on two-way relay channels in association with physical layer network coding and lattice coding can be found in [3] and [4]. In [5], the performance of the amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) had been investigated.

In many applications, it is possible that there exist than one relay nodes for diversity or redundancy purpose. Despite the abundance of literature on one-way relay with multiple relay nodes [6]-[7], research on the two-way multi-relay channel remains limited to date. The cut-set bound of the two-way multi-relay channel was studied in [8]. The study assumes full information exchange among all relay nodes in decoding and encoding of messages, which essentially transfers the "multi-relay" model into a single relay node with multiple antennas. The problem of distributed AF beamforming for the two-way relay network consisting of two sources and multiple-relay nodes has been investigated in [9]. In [10], the optimal AF strategy for the two-way relay channel with multiple relays is considered. In both cases, phase-synchronized processing among relay nodes is required for beamforming.

In this paper, we are interested in the capacity of the wireless two-way relay channel with multiple, non-cooperative relay nodes. By non-cooperative, we assume a distributed system without phase-synchronization or message level exchange among relay nodes. Our intent is to understand the non-cooperative system behaviors, and specifically how the upper bound and the achievable rate regions vary under different relay setting and forwarding strategies. Towards this end, we derive the channel capacity upper bound based on the cut-set bound and the degraded broadcast channel [11]. In addition, the achievable regions are derived for the AF, DF and CF relay schemes. For the DF strategy, the achievable regions based on bit-level and symbol-level relay forward schemes are provided. Built upon these results, we extend the capacity analyses to the additive white Gaussian channel, obtaining their respective upper bounds and achievable regions.

The remainder of this paper is organized as follows. Section II describes the system mode for the generalized two-way multi-relay channel. In Section III, we derive the upper bound on the capacity. Section IV proposes the achievable regions for different strategies. Numerical results are presented in Section V. Finally, Section VI concludes the paper.

II. SIGNAL MODEL

In this paper, we consider a two-way multi-relay channel which consists of two sources and M relay nodes (denoted as \( r_m, m = 1, \ldots, M \)). The two sources, \( S \) and \( D \), exchange information with the help of multiple relay nodes in half-duplex mode. The relay nodes are assumed to be non-cooperative, i.e., distributed message-level information exchange or in-phase beamforming among them. The rest of the notation is defined as follow:

- \( X_S, X_D \) and \( X_m \) are the transmit signals from the source \( S \), the source \( D \), and the \( m \)-th relay node, respectively.
• $Y_S$, $Y_D$ and $Y_m$ are the receive signals of the source $S$, the source $D$, and the $m$-th relay node, respectively.
• $Z_S$, $Z_D$ and $Z_m$ are the zero mean complex Gaussian variables of $\sigma_S^2$, $\sigma_D^2$ and $\sigma_m^2$ variance at $S$, $D$, and the $m$-th relay node, respectively.
• $h_{im}$ is the channel from the $i$-th source to the $m$-th relay node, where $i = \{S, D\}$ and $j = \{1, ..., M\}$.
• $P_S$, $P_D$ and $P_m$ denote the transmit powers at the $S$, $D$, and the $m$-th relay node, respectively.
• $\omega_S$, $\omega_D$ and $\omega_m$ denote the transmit messages of $S$, $D$, and the $m$-th relay node, respectively.
• $t_1$ and $t_2$ denote the transmit time durations for the first time slot and the second time slot, so that $t_1 + t_2 = 1$.

As shown Fig. 1, the transmission of the two-way relay system is divided into two time slots. During the first time slot, $S$ and $D$ simultaneously transmit to the relay nodes. The received signal at the $m$-th relay node can be written as $Y_m = h_{Sm}X_S + h_{Dm}X_D + Z_m$. In the second time slot, all the relay nodes broadcast the signal to the sources $S$ and $D$ by applying different forward strategies. Accordingly, $Y_S = \sum_{m=1}^{M} h_{Sm}X_m + Z_S$ and $Y_D = \sum_{m=1}^{M} h_{Dm}X_m + Z_D$. For the two-way relay channel, it is worth noting that the source can use its own information a proprì to decode the received signal.

III. THE CAPACITY UPPER BOUND

In this section we first present the capacity upper bound of the two-way, non-cooperative, multiple-relay channel. Unlike the well-understood MIMO scenario, joint encoding/decoding is not feasible without significant information exchange among the relay nodes.

Theorem 1: Let $\pi_\delta(\cdot)$ denote a permutation of the relay node indices $\{1, ..., M\}$ based on the channel gain $|h_{Sm}|$, so that $|h_{S\pi_\delta(1)}| \geq |h_{S\pi_\delta(2)}| \geq \cdots \geq |h_{S\pi_\delta(M)}|$. $\pi_\delta(\cdot)$ is similarly defined for $D$. Let $\cup_{j=1}^M X_j$ be $(X_1, ..., X_M)$. The capacity upper bound for the two-way, non-cooperative, multi-relay channel is given by the union of

$$R_S \leq \min \{ \sum_{m=1}^{M} t_1 I(\{X_{\pi_\delta(m)}\}; Y_{\pi_\delta(m)}| \cup_{i>m} X_{\pi_\delta(i)}, X_D), \}
\quad t_2 I(\cup_{j=1}^M X_j; Y_D) \}$$

$$R_D \leq \min \{ \sum_{m=1}^{M} t_1 I(\{X_{\pi_\delta(m)}\}; Y_{\pi_\delta(m)}| \cup_{i>m} X_{\pi_\delta(i)}, X_S), \}
\quad t_2 I(\cup_{j=1}^M X_j; Y_S) \}$$

with some distributions $p(\cup_{j=1}^M X_j)$ $\prod_{m=1}^{M} p(X_{\pi_\delta(m)})$ for $p(X_{\pi_\delta(m)})$.

Proof: For the source $S$, the channel behaves like a broadcast channel during the first time slot. Rewrite the transmit signal at $S$ as $X_S = \sum_{m=1}^{M} X_{\pi_\delta(m)}$, which $X_{\pi_\delta(m)}$ denotes the independent signal intended to the $\pi(m)$-th relay node. Using the cut-set bound theory and the degraded channel properties, the maximum rate $R_{S1}$ for the source $S$ in the first time slot is given by

$$R_{S1} \leq t_1 I(X_S; Y_1, ..., Y_M, Y_D|X_D, X_1, ..., X_M)
\quad \text{(a)}
\quad = t_1 I(X_S; Y_1, ..., Y_M|X_D)$$

$$+ t_1 I(Y_{\pi_\delta(1)}|X_S, \pi_\delta(1), X_D, X_{\pi_\delta(2)}, ..., X_{\pi_\delta(M)}, X_D) + \cdots
\quad + t_1 I(Y_{\pi_\delta(M)}|X_S, \pi_\delta(M), X_D)$$

$$+ t_1 I(X_{\pi_\delta(M)}; Y_{\pi_\delta(M)}|X_S, \pi_\delta(M), X_D)$$

$$\quad \text{(b)}
\quad = t_1 I(X_S; Y_1, ..., Y_M|X_D)$$

$$+ \sum_{m=1}^{M} t_1 I(Y_{\pi_\delta(m)}; X_{\pi_\delta(m)}| \cup_{i>m} X_{\pi_\delta(i)}, X_D)$$

where (a) follows from the fact that the $X_m$ is the function of $Y_m$ and $Y_D$ is the function of $Y_1$, ..., $Y_M$; (b) follows from the property of the degraded broadcast channel which states that the $\pi_{S(m)}$-th relay node can always decode the messages intended for the relay nodes $\cup_{\pi_\delta(i)}$. (c) uses the definition $\cup_{i>m} X_{\pi_\delta(i)} = (X_{\pi_\delta(m+1)}, ..., X_{\pi_\delta(M)})$. Similarly, we obtain the rate $R_{D1}$ for the source $D$ in the first time slot as

$$R_{D1} \leq t_2 I(X_{\pi_\delta(1)}; Y_1, ..., Y_M|X_S, Y_1, ..., Y_M)$$

During the second time slot, the relay nodes transmit the information and the rate $R_{S2}$ from the relay nodes to $S$ is outer bounded by

$$R_{S2} \leq \max \{ t_1 I(X_1, ..., X_M; Y_D|X_S, X_D), \}
\quad \text{(a)}
\quad = t_1 I(X_1, ..., X_M; Y_D)$$

$$+ \max \{ t_1 I(X_1, ..., X_M; Y_D), \}
\quad \text{(b)}
\quad = t_1 I(X_1, ..., X_M; Y_D)$$

where (a) is the cut-set bound; (b) follows from the fact that $Y_D$ is independent of $X_S$ and $X_D$ when $\cup_{j=1}^M X_j$ is given. We then calculate the rate for the $D$ as $R_{D2} \leq t_2 I(X_{\pi_\delta(1)}; Y_1, ..., Y_M; Y_S)$. Consequently, every point $(R_S, R_D)$ of the two-way-two-relay channel is bound by

$$R_S = \min \{ R_{S1}, R_{S2} \} \quad \text{and} \quad R_D = \min \{ R_{D1}, R_{D2} \}.$$

Remark 2: Let $\tau$ denote the correlation coefficient between $X_1$ and $X_2$: $\tau = \frac{\text{Cov}(X_1, X_2)}{\sqrt{\text{Var}(X_1)\text{Var}(X_2)}}$. According to Theorem 1, the capacity upper bound in the Gaussian channel for $M = 2$ is

$$R_S \leq \min \{ \sum_{m=1}^{2} t_1 I(X_{\pi_\delta(m)}; Y_{\pi_\delta(m)}| \cup_{i>m} X_{\pi_\delta(i)}, X_D), \}
\quad \text{(a)}
\quad = t_1 I(X_{\pi_\delta(m)}; Y_{\pi_\delta(m)}| \cup_{i>m} X_{\pi_\delta(i)}, X_D)$$

$$+ \sum_{j=1}^{M} t_1 I(Y_{\pi_\delta(j)}|X_{\pi_\delta(m)}, \pi_\delta(m), X_D)$$

$$+ \sum_{j=1}^{M} t_1 I(Y_{\pi_\delta(j)}|X_{\pi_\delta(m)}, \pi_\delta(m), X_D)$$

$$\quad \text{(b)}
\quad = t_1 I(X_{\pi_\delta(m)}; Y_{\pi_\delta(m)}| \cup_{i>m} X_{\pi_\delta(i)}, X_D)$$

$$\quad \text{(c)}
\quad = t_1 I(X_{\pi_\delta(m)}; Y_{\pi_\delta(m)}| \cup_{i>m} X_{\pi_\delta(i)}, X_D)$$

When $\tau = 0$, the transmit signal $X_{\pi_\delta(m)}$ from the $\pi_\delta(m)$-th relay node only includes its own message, even though the $\pi_\delta(m)$-th relay node can also decode other $\cup_{\pi_\delta(i)}$ and $\cup_{\pi_\delta(j)}$.
messages. Here $n$ denotes the position of the $\pi_S(m)$-th relay node in $\pi_D()$.

IV. THE ACHIEVABLE REGIONS FOR DIFFERENT RELAY STRATEGIES

In this section we present the achievable rate regions for different relay forward strategies.

A. Amplify-and-Forward (AF)

With the AF strategy, the relay nodes simply amplify the received signal and then broadcast the mixed signals to the sources.

\textbf{Theorem 3:} For the two-way multi-relay channel, the amplify-forward strategy can achieve any rates up to

\[ R_S \leq \frac{1}{2} I(X_S; Y_D|X_D) \]
\[ R_D \leq \frac{1}{2} I(X_D; Y_S|X_S) \] \hspace{1cm} (5)

for any choice of $p(X_S, X_D) = p(X_S)p(X_D)$.

\textbf{Proof:} In the AF scheme, $t_1 = t_2 = \frac{1}{2}$. By taking into account that the sources are able to remove the self interference, $\frac{1}{2} I(X_S; Y_D|X_D)$ can be considered as the rate for the information transferring from $S$ to $D$. Following the same argument, we can obtain the rate for $D$ as $R_D \leq \frac{1}{2} I(X_D; Y_S|X_S)$.

For Gaussian channel, $x_m = \alpha_m y_m = \alpha_m h S_m x_S + \alpha_m h D_m x_D + \alpha_m z_m$, where $\alpha_m$ denotes the amplification factor to satisfy the power constraint at the $m$-th relay node. Let $\alpha_m = \sqrt{\frac{1}{|h S_m|^2 P_S + |h D_m|^2 P_D + \sigma_m^2}}$, the following capacity region of the AF strategy can be obtained.

\[ R_S \leq \frac{1}{2} \log(1 + \frac{\sum_{m=1}^{M} \sqrt{P_m \alpha_m |h S_m|^2 |h D_m|^2}}{|h S_m|^2 P_S + |h D_m|^2 P_D + \sigma_m^2}) \] \hspace{1cm} (6)

\[ R_D \leq \frac{1}{2} \log(1 + \frac{\sum_{m=1}^{M} \sqrt{P_m \alpha_m |h D_m h S_m|^2}}{|h S_m|^2 P_S + \sigma_m^2 + \sigma_S^2}) \] \hspace{1cm} (6)

B. Decode-and-Forward (DF)

We first analyze the decode-and-forward strategy based on the superposition coding for the degraded channel. The results will then be extended to Gaussian channel for two specific methods, i.e. bit-level DF and symbol-level DF.

Let $n$ be the position of the $\pi_S(m)$-th relay node in $\pi_D(\cdot)$, i.e., the $\pi_S(m)$ and $\pi_D(n)$ refer to the same node. Assuming that the channel strengths are known at the sources, each source can employ superposition maximization to achieve the capacity in the first time slot. Using superposition coding, each source splits the message $\omega_S$ and $\omega_D$ into $M$ independent parts $U_{\pi_S(m)}$ and $V_{\pi_D(n)}$, $m = 1, ..., M$, respectively. Upon receiving the broadcast signals from sources $S$ and $D$, $U_{\pi_S(m)}$ and $V_{\pi_D(n)}$ at rates $R_S^{\pi_S(m)}$ and $R_D^{\pi_D(n)}$ can be decoded at the $\pi_S(m)$-th relay node. Note that for the degraded broadcast channel, the $\pi_S(m)$-th relay node can also decode the messages intended for the relay nodes $\cup_{i > m} \pi_S(i)$.

Therefore, all the relay nodes can decode the signals if

\[ R_S^{\pi_S(m)} \leq t_1 I(X_S^{\pi_S(m)}; Y_S^{\pi_S(m)} \cup_{i > m} X_S^{\pi_S(i)}, X_D) \] \hspace{1cm} (7)

\[ R_D^{\pi_D(n)} \leq t_1 I(X_D^{\pi_D(n)}; Y_D^{\pi_D(n)} \cup_{j > n} X_D^{\pi_D(j)}, X_S) \] \hspace{1cm} (8)

\[ R_S^{\pi_S(m)} + R_D^{\pi_D(n)} \leq t_1 I(X_S^{\pi_S(m)}; X_D^{\pi_D(n)}; Y_S^{\pi_S(m)} \cup_{i > m} X_S^{\pi_S(i)} \cup_{j > n} X_D^{\pi_D(j)}) \] \hspace{1cm} (9)

where Inequality (9) follows from the multiple access channel constraint in the first time slot.

In the second time slot, the $\pi_S(m)$-th relay node transmits its message $X_{\pi_S(m)}$ which includes the information $\cup_{i > m} U_{\pi_S(i)}$ and $\cup_{j > n} V_{\pi_D(n)}$, using either the superposition coding or the bit-wise XOR operations. Thus, the reliable decoding of the messages at the sources $S$ and $D$ is possible if

\[ R_S^{\pi_S(m)} \leq t_2 I(X_{\pi_S(m)}; Y_D \cup_{i > m} X_{\pi_S(i)}) \] \hspace{1cm} (10)

\[ R_D^{\pi_D(n)} \leq t_2 I(X_{\pi_S(m)}; Y_S \cup_{j > n} X_{\pi_D(j)}) \] \hspace{1cm} (11)

\[ \sum_{m=1}^{M} R_S^{\pi_S(m)} \leq t_2 I(\cup_{i > m} X_{\pi_S(i)}; Y_D) \] \hspace{1cm} (12)

\[ \sum_{n=1}^{M} R_D^{\pi_D(n)} \leq t_2 I(\cup_{j > n} X_{\pi_D(j)}; Y_S) \] \hspace{1cm} (13)

where Inequalities (12) and (13) follows from the MAC constraint in the second time slot. Based on the above results, we have the following achievable region.

\textbf{Theorem 4:} For the two-way multiple-relay degraded channel, the rate region achievable through the DF scheme is given by

\[ R_S \leq \min\{\sum_{m=1}^{M} \min\{t_1 I(X_S^{\pi_S(m)}; Y_S^{\pi_S(m)} \cup_{i > m} X_S^{\pi_S(i)}, X_D), t_2 I(X_{\pi_S(m)}; Y_D \cup_{i > m} X_{\pi_S(i)})\}\} \]

\[ R_D \leq \min\{\sum_{n=1}^{M} \min\{t_1 I(X_D^{\pi_D(n)}; Y_D^{\pi_D(n)} \cup_{j > n} X_D^{\pi_D(j)}, X_S), t_2 I(X_{\pi_S(m)}; Y_S \cup_{j > n} X_{\pi_D(j)})\}\} \]

subject to the constraint

\[ R_S + R_D \leq \sum_{m=1}^{M} t_1 I(X_S^{\pi_S(m)}; X_D^{\pi_D(n)}; Y_S^{\pi_S(m)} \cup_{i > m} X_S^{\pi_S(i)} \cup_{j > n} X_D^{\pi_D(j)}) \]

with all joint distributions $p(X_S)p(X_D|x_1)p(Y_S)p(X_S|x_S)p(X_D|x_D)$.

Two different methods can be used to re-encode the messages at the relay node for the Gaussian channel, as shown in the following.
Remark 5: Bit-level DF(BLSD): When the traffics are symmetric, each relay node combines the two messages bit sequences on bit-level prior to encode using the XOR operation. Specifically, the $\pi_S(m)$-th relay node encodes the message as $\omega_{\pi_S(m)} = \omega \cup_{i=m} U_{\pi_S(i)} \cup \omega \cup V_{\pi_D(n)}$, where $\omega \cup_{i=m} U_{\pi_S(i)}$ denotes the bit messages of the transmit signals $\cup_{i=m} U_{\pi_S(i)}$. Knowing its own message, $S$ can recover $\omega \cup V_{\pi_D(n)}$ by $\omega \cup_{i=m} U_{\pi_S(i)} \cup \omega_{\pi_S(m)}$. Also, $D$ can decode $\omega \cup_{i=m} U_{\pi_S(i)}$ in a similar fashion. For simplicity, we consider the case with $M = 2$, $|h_{S1}| \geq |h_{S2}|$ and $|h_{D2}| \geq |h_{D1}|$. In the Gaussian channel, the achievable region of BLSD for the two-way two-relay channel can be expressed as

$$ R_S \leq \min\{t_1 \log(1 + |h_{S1}|^2 P_{S,U_1}), t_2 \log(1 + |h_{D1}|^2 P_{D,V_2}) + \min\{t_1 \log(1 + \frac{|h_{S2}|^2}{\sigma_1^2} P_{S,U_1}), t_2 \log(1 + \frac{|h_{D2}|^2}{\sigma_2^2} P_{D,V_2}) \} \}
$$

$$ R_D \leq \min\{t_1 \log(1 + \frac{|h_{D1}|^2}{\sigma_1^2} P_{D,V_1}), t_2 \log(1 + \frac{|h_{D2}|^2}{\sigma_2^2} P_{D,V_2}) \} $$

$$ R_S + R_D \leq t_1 \log(1 + \frac{P_{S,U_1} |h_{S1}|^2 + P_{D,V_1} |h_{D1}|^2}{\sigma_1^2 + |h_{D1}|^2 P_{D,V_2}}) + t_2 \log(1 + \frac{P_{S,U_2} |h_{S2}|^2 + P_{D,V_2} |h_{D2}|^2}{\sigma_2^2 + |h_{D2}|^2 P_{D,V_2}}) $$


as $X_{\pi_S(m)} = \sum_{i=1}^{m} \frac{\alpha_{\pi_S(m),i}}{\sqrt{P_{S,U_1}}} \sum_{i=1}^{m} \sqrt{P_{D,V_1}} V_{\pi_D(n)}$, where $\alpha_{\pi_S(m),i}$ and $\beta_{\pi_S(m),j}$ are the power coefficients that satisfy the power constraint at the $\pi_S(m)$-th relay node, i.e.,

$$ \sum_{i=m}^{M} \alpha_{\pi_S(m),i}^2 + \sum_{j=m}^{M} \beta_{\pi_S(m),j}^2 = \sum_{i=m}^{M} \alpha_{\pi_S(m),i}^2 = P_m. $$

For the Gaussian channel, the following capacity of the two-way multiple-relay channel can be achieved using SLD:

$$ C_{SFDRD} = \max_{\alpha_{\pi_S(m),i}, \beta_{\pi_S(m),i}} R_S + R_D $$

where the $R_S$ and $R_D$ are given in Equation (14)-(18). It is important to note that in theory, the source $S$ can split its power $P_S$ into $P_{S,U_1}$ and $P_{S,U_2}$ for transmission of the partial messages $U_1$ and $U_2$, respectively. Similarly for $D$, the relay node $r_1$ and the relay node $r_2$, power allocation can be performed under the total power constraints: $P_{D,V_1} + P_{D,V_2} = P_D$, $\frac{\alpha_{11}^2 + \alpha_{12}^2 + \beta_{11}^2 + \beta_{12}^2}{\alpha_{21}^2 + \beta_{21}^2 + \beta_{22}^2} = P_1$ and $\frac{\alpha_{12}^2 + \beta_{12}^2}{\alpha_{22}^2 + \beta_{22}^2} = P_2$. The optimal power allocation for Equation (15) will be provided in the full version of this paper.

C. Compress-and-Forward (CF)

For the compress-and-forward strategy, the relay node does not decode the symbols from the source $S$ and $D$, nor does it amplifies the received signal. Instead, it forwards a quantized and compressed version of its received symbols to the source $S$ and $D$ at the second time slot. By applying this scheme to the two-way multiple-relay channel, the following capacity region can be obtained.

Theorem 7: An achievable region of the two-way multiple-relay channel with the CF strategy is given by

$$ R_S \leq t_1 I(X_S; \hat{Y}_1, ..., \hat{Y}_M|X_D) $$

$$ R_D \leq t_1 I(X_D; \hat{Y}_1, ..., \hat{Y}_M|X_S) $$


subject to the constraint

\[
\begin{align*}
t_I(Y_m; \tilde{Y}_m | X_D) & \leq t_2 I(X_m; Y_D | X_m) \\
t_I(Y_m; \hat{Y}_m | X_S) & \leq t_2 I(X_m; Y_S | X_m)
\end{align*}
\]

\[
\sum_{m=1}^{M} t_I(Y_m; \hat{Y}_m | X_D) \leq t_2 I(X_1, ..., X_M; Y_D)
\]

\[
\sum_{m=1}^{M} t_I(Y_m; \hat{Y}_m | X_S) \leq t_2 I(X_1, ..., X_M; Y_S)
\]

with all joint distribution \(p(X_S)p(X_D)p(Y_S, Y_D | X_1, ..., X_M)\)

\[
\prod_{m=1}^{M} \{p(X_m)p(Y_m | X_S, X_D)p(\tilde{Y}_m | Y_m)\}.
\]

\textbf{Proof:} Due to page limitations, we skip the proof in this paper and refer interested readers to [11] and [5]-[7] for related derivations.

The following capacity region is achievable for the Gaussian channel.

Remark 8: Without loss of generality, we study the simple case with \(M = 2\). In CF, the quantization signal for the relay received signal is written as \(\hat{Y}_m = Y_m + Q_m, m = 1, 2\), where \(Q_m\) is the quantization error and the zero-mean Gaussian noise at the relay with variance \(\sigma^2_Q\), and \(Q_m\) is independent with all other random variables. Let \(\rho_s\) be the correlation coefficient between \(\tilde{Y}_1\) and \(\tilde{Y}_2\) given \(X_D\), and \(\rho_d\) is the correlation coefficient between \(\tilde{Y}_1\) and \(\tilde{Y}_2\) given \(X_S\), defined as \(\rho_s = \frac{\mathbb{E}[h_{SD}h_{D2}]}{\sqrt{(\mathbb{E}[h_{SD}^2] + \mathbb{E}[h_{D2}^2])(\mathbb{E}[h_{SD}^2] + \mathbb{E}[h_{D2}^2])}}\) and \(\rho_d = \frac{\mathbb{E}[h_{SD}h_{D2}]}{\sqrt{(\mathbb{E}[h_{SD}^2] + \mathbb{E}[h_{D2}^2])(\mathbb{E}[h_{SD}^2] + \mathbb{E}[h_{D2}^2])}}\). From the property of mutual information, we obtain

\[
R_S \leq t_1 I(X_S; \hat{Y}_1, ..., \hat{Y}_M | X_D)
\]

\[
= t_1 I(X_S; \hat{Y}_1 | X_D) + t_1 I(X_S; \hat{Y}_2 | \hat{Y}_1, X_D)
\]

\[
= t_1 \log(1 + \frac{|h_{S1}|^2 P_S}{\sigma^2_1 + \sigma^2_Q}) + t_1 \log(1 + \frac{|h_{S2}|^2 P_S}{\sigma^2_2 + \sigma^2_Q}(1 - |\rho_s|^2))
\]

Similarly,

\[
R_D \leq t_1 \log(1 + \frac{|h_{D1}|^2 P_D}{\sigma^2_1 + \sigma^2_Q}) + t_1 \log(1 + \frac{|h_{D2}|^2 P_D}{\sigma^2_2 + \sigma^2_Q}(1 - |\rho_d|^2))
\]

Hence, we obtain the achievable region for CF strategy in the following theorem.

Theorem 9: For the CF strategy, the capacity region of the Gaussian two-way two-relay channel can be given by

\[
R_S \leq t_1 \log(1 + \frac{|h_{S1}|^2 P_S}{\sigma^2_1 + \sigma^2_Q}) + t_1 \log(1 + \frac{|h_{S2}|^2 P_S}{\sigma^2_2 + \sigma^2_Q}(1 - |\rho_s|^2))
\]

\[
R_D \leq t_1 \log(1 + \frac{|h_{D1}|^2 P_D}{\sigma^2_1 + \sigma^2_Q}) + t_1 \log(1 + \frac{|h_{D2}|^2 P_D}{\sigma^2_2 + \sigma^2_Q}(1 - |\rho_d|^2))
\]

where the quantized errors need to satisfy the following constraint in order to successful transmit the quantized version.

For high SNR, the variance \(\sigma^2_Q\) of the quantization error in \(r_m\) is given in Equation (21) and (22).

V. NUMERICAL RESULTS

To compare the achievable regions for different strategies, we first consider a symmetric network with \(M = 2\) relay nodes. In all simulations, the relay system is assumed to have the equal time slots, i.e., \(t_1 = t_2 = \frac{1}{2}\). Let \(d_m\) be the distance between the source \(S\) and the \(m\)-th relay node. The channel gain is set as \(|h_{SD}|^2 = \frac{1}{d_m^2}\) [6]. The noise powers at all nodes are assumed to be the same, i.e., \(\sigma^2_s = \sigma^2_d = \sigma^2_Q = 1\).

Fig. 2 and Fig. 3 compare the sum rates obtained at both sources under two different power settings. We observe that, as the distance decreases, i.e., the relay moves towards the source, the capacity upper bound increases. When the transmit power of each relay node is higher than that of the sources, the best performance is achieved by the BLDF strategy when each relay node is close to the sources. If all the relay nodes are placed at the middle position, the AF strategy yields the best sum rate. However, from Fig. 2, the CF performs better than other strategies within certain the range of the distance.

Since the relay node does not need to decode the message in the AF strategy, we observe that the AF strategy performs well for a wide range of positions. Compared with the AF, the CF strategy has poor performance with a low transmit power at each relay node. The reason is that the AF strategy is limited under the constraints of (19), which means that the rate of compressing \(Y_m\) must be smaller than the rate of transmitting data \(X_m\). It is also seen that the BLDF outperforms the SLDF without the limitations posed by the power coefficients \(\alpha\) and \(\beta\). In addition, we note that the SLDF has a poor performance since some of the messages are considered as the interference. However, if each relay node is in the proximity of the sources, the SLDF has a good performance when the relay nodes have high transmit power.
The distance between r1 and S (Equal to the distance between r2 and D)

Capacity (bps/Hz)

PS=PD=0 dB, P1=P2=15 dB

Upper bound

AF

BLDF

SLDF

CF

Fig. 2. Sum rate vs. different strategies with the high transmit power of each relay node

The distance between r1 and S (Equal to the distance between r2 and D)

Capacity (bps/Hz)

PS=PD=15 dB, P1=P2=0 dB

Upper bound

AF

BLDF

SLDF

CF

Fig. 3. Sum rate vs. different strategies with the high transmit power of each source

The performance under different number of the relay nodes is shown in Fig. 4, where the relays are positioned as in Fig. 1. We observe that the capacity upper increases significantly from one node to two nodes, but flattens out beyond two nodes. For all the other practical non-cooperative relay schemes, the achievable sum rates increase gradually as the number of the relay nodes grows.

VI. CONCLUSIONS

In this paper, we have studied the capacity of two-way relay channel with multiple, non-cooperative, relay nodes. The capacity upper bound and the achievable regions for different relay strategies have been derived. Also presented are the capacity analyses for Gaussian channels. Numerical results have shown that the AF and the CF strategies have excellent performance when each relay node is placed at the middle position, but the BLDF strategy is the best choice if the relay nodes are in the proximity of the sources. Furthermore, it has been shown that the capacity increases slowly with the number of the relay nodes, with is in contrast with the scenarios when the relay nodes are fully cooperative.

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