Preventive-maintenance policy for leased products under various maintenance costs

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ABSTRACT

This paper investigates the effects of preventive-maintenance cost functions on the optimal preventive-maintenance policy for a leased product with Weibull life-time distribution. During the lease period, any product failures are rectified by minimal repairs and may incur a penalty to the lessor, if the time duration for performing a minimal repair exceeds a pre-specified time limit. To reduce repair costs and possible penalty, preventive-maintenance actions are scheduled in the lease contract. The objective of this paper is to derive the optimal preventive-maintenance schedule and maintenance degrees such that the expected total cost is minimized. Some structural properties of the optimal policy are obtained and an efficient algorithm is provided to search for the optimal policy. For the cases where the preventive maintenance cost is a constant or a linearly increasing function, the effects of the preventive-maintenance cost function on the optimal policy are investigated in detail both theoretically and numerically.

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1. Introduction

This paper derives the optimal preventive-maintenance policy for a leased product with Weibull life-time distribution and investigates the effects of preventive-maintenance cost function on the optimal policy. Due to the increase in complexity of products/systems and rapid advances in technological innovation, performing maintenance actions for such complex products now requires expensive equipments and special professional technicians, which is not economical for a company to keep. Therefore, there is a trend to lease products instead of owning them (Desai & Purohit, 1998; Fang & Huang, 2008). For a leased product, the maintenance actions are usually provided by the lessor (the one who owns the product) and specified in a lease contract to ensure that the product could fulfill its intended performance requested by the lessee (the one leasing the product).

In general, there are two types of maintenance considered in a lease contract – corrective maintenance (CM) and preventive-maintenance (PM). CM actions are employed to rectify failed products back to operational status, and PM actions are used to improve the operational status of a product to avoid failures (Barlow & Hunter, 1960; Valdez-Flores & Feldman, 1989). The articles in Dekker (1996), Dekker and Scarf (1998), Pieskalla and Voelker (1976), Sherif and Smith (1981) and Valdez-Flores and Feldman (1989) are excellent reviews of maintenance models for products subject to stochastic failures. In developing a maintenance model, minimal repair is the most commonly used corrective maintenance action to restore a failed product (Nakagawa, 1981; Nakagawa & Kowada, 1983), since the failure rate of the product remains unchanged after performing a minimal repair. Various maintenance models involving minimal repair can be found in Boland and Proshan (1982), Nakagawa (1981), Nakagawa and Kowada (1983) and Sheu (1991).

For a leased product, minimal repairs are carried out by the lessor to restore a failed product back to its operational condition. When the time required to perform a minimal repair exceeds a pre-specified amount of time, there is a penalty to the lessor to compensate the loss to the lessee. Therefore, there is a need for the lessor to provide some remedial measures to avoid the costs of minimal repairs and penalty incurred by product failures.

To reduce the number of product failures and possible penalties within the lease period, PM actions are widely employed since the cost for carrying out a planned PM action is usually less than the cost incurred by a product failure. Various PM models have been proposed for different situations such as perfect or imperfect PM (Barlow & Hunter, 1960; Jack & Dagpunar, 1994; Pham & Wang, 1996), and periodical or sequential PM (Chun, 1992; Jack & Dagpunar, 1994; Yeh & Lo, 2001). For the imperfect PM, the maintenance degree for each PM action is characterized by age-reduction or failure rate-reduction methods (Barlow & Hunter, 1960; Pham & Wang, 1996). Using the age-reduction method, the age of the product after taking a PM action becomes a certain amount of time younger than before. On the other hand, using the failure rate-reduction method, the failure rate of the product is reduced by a certain amount after a PM action.

In this paper, the age-reduction method is adopted since it is easily measured and implemented in practice. Using the
age-reduction method, a mathematical cost model is developed and the optimal PM policy is derived such that the expected total cost in the lease period is minimized. Furthermore, the effects of the PM cost function on the optimal PM policy are investigated in detail.

The remainder of this paper is organized as follows. The mathematical model is developed in Section 2 for the case when the failure rate of the product remains the same as that just before failure. It is well-known that the product is operational but the failure rate of the product re- mains the same as that just before failure. It is well-known that the failure rate function of a Weibull distribution with scale parameter $\lambda > 0$ and shape parameter $\beta > 1$, increases if $\beta > 1$, and is a constant if $\beta = 1$. If a product fails during the lease period, minimal repair is performed immediately with a fixed repair cost $C_m$ and a random amount of repair time $t_m$, which follows a general cumulative distribution function $G(t_m)$. If the repair time exceeds a predetermined value, then there is a penalty $C > 0$ to the lessor. That is, a possible penalty of $C_i$ to the lessor may occur with probability $G(t_i)$ for each failure, when the repair time is longer than $\tau$.

In addition to minimal repairs at failures, there are n PM actions to be specified in the lease contract. That is, the lessor will perform n PM actions sequentially at time epochs $T_i$ (where $0 < T_1 < T_2 < \ldots < T_n < L$) with maintenance degrees $x_i > 0$, where $\sum_{i=1}^{n} x_i = 1$. After performing the ith PM action at time $T_i$ with degree $x_i$, the age of the product is $x_i$ units of time younger than before. The purpose of performing PM actions is to reduce the total number of product failures and the resulting costs of minimal repairs and penalty within the lease period. However, performing a PM action also incurs a certain amount of cost. Let $C_p(x)$ be the cost for carrying out a PM action with maintenance degree $x$. In general, the cost of a PM action is a non-negative and non-decreasing function of maintenance degree $x$; that is, $C_p(x) \geq 0$.

Under minimal repairs, the failure process of the product can be represented by a non-homogeneous Poisson process (NHPP) with failure intensity $h(t)$ when PM actions are not performed (Pham & Wang, 1996; Pieskalla & Voelker, 1976). Without PM actions, the expected number of failures of the product within the interval $[0,t]$ becomes $\int_0^t h(t)dt = \lambda t^\beta$. When PM actions are taken at time epochs $T_i$, the failure process of the product in each interval $[T_{i-1},T_i)$ is still an NHPP after the ith PM action, but the failure intensity becomes $h(t - \sum_{j=1}^{i-1} x_j)$ for $t \in [T_{i-1},T_i)$. Assuming that the times required for carrying out the minimal repair and PM actions are negligible compared to the leased period $L$. Then, under the sequential PM scheme, the expected number of failures during the lease period becomes

$$A(L) = \lambda \sum_{i=1}^{n} \left[ T_{i-1} - \sum_{j=1}^{i-1} x_j \right]^\beta - \left[ T_i - \sum_{j=1}^{i} x_j \right]^\beta + \lambda \int_0^L H(x) \left[ \sum_{i=1}^n x_i^\beta + \left( L - \sum_{i=1}^n x_i \right)^\beta \right] dx,$$

where $T_0 = 0$ and $T_{n+1} = L$.

As a result, the expected total cost to the lessor under the aforementioned lease contract includes the minimal repair cost, penalty cost, and PM cost. To simplify the notations, let $X = (x_1,x_2,\ldots,x_n)$ and $T = (T_1,T_2,\ldots,T_n)$. Then, the expected total cost to the lessor under PM policy $(n,X,T)$ becomes

$$C(n,X,T) = \sum_{i=1}^{n} C_p(x_i) + \left[ C_m + C_i G(T) \right] \left( \sum_{i=1}^{n} x_i^\beta + \left( L - \sum_{i=1}^{n} x_i \right)^\beta \right).$$

with $T_0 = 0$ and $T_{n+1} = L$. Our objective here is to find an optimal PM policy $(n^*,X^*,T^*)$ for the lessor such that the expected total cost in Eq. (2) is minimized under the constraints of $T_i \geq \sum_{j=1}^{i} x_j$ for $i = 1,2,\ldots,n$ and investigate the effects of $C_p(x)$ on the optimal PM policy $(n^*,X^*,T^*)$.

### 3. Optimal PM policy

Obviously, when $\beta < 1$, the failure rate function of the product is non-increasing in $t$; that is, $h(t_1) > h(t_2)$ for all $0 \leq t_1 < t_2$. Hence, reducing the age of the product by PM actions is not desired in this case. That is, any policy with $n^* = 0$ is optimal when $\beta < 1$ and the resulting expected total cost is $C(0,X,T) = \left[ C_m + C_i G(T) \right] (\lambda L)^\beta$. On the other hand, PM actions may be required to reduce the age of the product to avoid failures in the lease period when $\beta > 1$. Therefore, in the following discussion, we will focus on this case. Based on the objective function (2), we first derive some structural properties of the optimal PM policy for a lease contract and then investigate the optimal PM policy in detail under various PM cost functions.

To investigate the optimal maintenance schedule $T^*$ when $n$ and $X$ are pre-specified, we take the first partial derivative of Eq. (2) with respect to $T_i$ as follows:

$$\frac{\partial C(n,X,T)}{\partial T_i} = \beta \lambda \left[ C_m + C_i G(T) \right] \left[ T_i - \sum_{j=1}^{i-1} x_j \right]^\beta \left\{ - \left[ T_i - \sum_{j=1}^{i-1} x_j \right]^\beta + \left[ T_i - \sum_{j=1}^{i-2} x_j \right]^\beta \right\}.$$

From Eq. (3), the following theorem holds.

**Theorem 1.** Given any $n > 0$ and $X > 0$, the optimal maintenance schedule $T^*$ is $T_i^* = \sum_{j=1}^{i} x_j$ for $i = 1,2,\ldots,n$, when $\beta > 1$.

**Proof.** When $\beta > 1$ and $X > 0$, from Eq. (3), we have $\frac{\partial C(n,X,T)}{\partial T_i} > 0$ for all $i = 1,2,\ldots,n$, since $\left[ T_i - \sum_{j=1}^{i-1} x_j \right]^\beta > \left[ T_i - \sum_{j=1}^{i-1} x_j \right]^\beta - 1$. This implies that $C(n,X,T)$ is an increasing function of $T_i$. Hence, under the constraints $T_i \geq \sum_{j=1}^{i} x_j$ for $i = 1,2,\ldots,n$, it is clear that $T_i^* = \sum_{j=1}^{i} x_j$ when $\beta > 1$. □

**Theorem 1** shows that PM actions should be scheduled at the time epochs $x_1,x_1+x_2,x_1+x_2+x_3,\ldots,x_1+x_2+\cdots+x_n$ when $\beta > 1$. In other words, PM actions are performed to restore the product back to its original condition if the maintenance degrees are pre-specified. Applying this optimal PM schedule, the objective function becomes

$$C(n,X) = \sum_{i=1}^{n} C_p(x_i) + \lambda \left[ C_m + C_i G(T) \right] \left[ \sum_{i=1}^{n} x_i^\beta + \left( L - \sum_{i=1}^{n} x_i \right)^\beta \right].$$

for $T_0 = 0$ and $T_{n+1} = L$. Our objective here is to find an optimal PM policy $(n^*,X^*,T^*)$ for the lessor such that the expected total cost in Eq. (2) is minimized under the constraints of $T_i \geq \sum_{j=1}^{i} x_j$ for $i = 1,2,\ldots,n$ and investigate the effects of $C_p(x)$ on the optimal PM policy $(n^*,X^*,T^*)$.
Clearly, given any $n > 0$, Eq. (4) plays an important role in determining the optimal maintenance degree $x_t$. Taking the first partial derivative of Eq. (4) with respect to $x_i$ for $i = 1, 2, \ldots, n$, we have

$$\frac{\partial C(n, x)}{\partial x_i} = C'_p(x_i) + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] \left[ x_i^{\beta - 1} - \left( L - \sum_{i=1}^{n} x_i \right)^{\beta - 1} \right].$$

(5)

Letting Eq. (5) equal to zero, we have the following result.

**Theorem 2.** When $\beta > 1$, the optimal maintenance degree $x^\star$ is $x^\star_1 = x^\star_2 = \cdots = x^\star_n$ for any $n > 0$.

**Proof.** Letting $\frac{\partial C(n, x)}{\partial x_i} = 0$ in Eq. (5), we have $C'_p(x_i) + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] (x_i^\beta - 1) = \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] (L - \sum_{i=1}^{n} x_i)^{\beta - 1}$ for all $i$ and $j$. Since $C'_p(x) + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] x^\beta$ is a strictly increasing function of $x$ when $\beta > 1$, we have $x^\star_1 = x^\star_2 = \cdots = x^\star_n$. □

**Theorem 2** shows that the optimal maintenance degree for each PM action should be equal when $\beta > 1$. This result is not surprising since the product is restored after each PM action and the maintenance degree should be the same if it is worthy to perform a PM action during the rest of the lease period.

Using Theorems 1 and 2, the number of decision variables now reduces from $2n + 1$ to 2, which are the number of PM actions $n$ and the common maintenance degree $x$. The maintenance scheme becomes performing PM at time epochs $x_1, x_2, \ldots, x_n$ with the same maintenance degree $x$. Under this maintenance scheme, the expected total cost is reduced to

$$C(n, x) = n C_p(x) + x^\beta \left[ C_m + C_t G_t(\tau) \right] \left[ nx^\beta + (L - nx)^{\beta - 1} \right].$$

(6)

Note that the time interval between successive PM actions is the same, but it may be different from the time interval between the last PM action and the end of the lease period. The rest of the optimization problem is to find $n(x, x^\star)$ such that Eq. (6) is minimized. Relaxing the integer restriction of $n$ and taking the first partial derivatives of Eq. (6) with respect to $x$ and $n$, we have

$$\frac{\partial C(n, x)}{\partial x} = n \left[ C'_p(x) + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] (x^\beta - 1) - (L - nx)^{\beta - 1} \right]$$

(7)

and

$$\frac{\partial C(n, x)}{\partial n} = C'_p(x) + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] (x^\beta - n(L - nx)^{\beta - 1}).$$

(8)

As we can see from Eqs. (7) and (8), the PM cost function $C_p(x)$ plays an important role in finding the optimal policy $(n, x^\star)$. Therefore, the effects of $C_p(x)$ on the optimal PM policies are investigated in the following subsections.

### 3.1 General PM cost function

When the PM cost is a general function of the maintenance degree, the derivation of the optimal policy $(n^\star, x^\star)$ is not straightforward. In this case, setting Eq. (7) equal to zero, we have

$$\beta x^\beta \left[ C_m + C_t G_t(\tau) \right] (L - nx)^{\beta - 1} = C'_p(x) + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] x^{\beta - 1},$$

(9)

for any $n > 0$. In other words, the optimal maintenance degree for any $n > 0$ is the solution of Eq. (9). The following theorem show that there exists an unique optimal maintenance degree corresponding to any $n > 0$.

**Theorem 3.** When $\beta > 1$ and $C'_p(x) > 0$ for all $x > 0$, if $C'_p(0) > \beta x^\beta L^{\beta - 1} \left[ C_m + C_t G_t(\tau) \right]$ then $(n^\star, x^\star) = (0,0)$. Otherwise, there exists an unique $x^\star \in (0,L/n]$ minimizing $C(n, x)$ for any $n > 0$.

**Proof.** Taking the second derivative of Eq. (7) with respect to $x$, we have

$$\frac{\partial^2 C(n, x)}{\partial x^2} = \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] x^{\beta - 1} + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] (L - nx)^{\beta - 1} > 0$$

for all $x \in (0,L/n]$ when $C'_p(x) \geq 0$. This means that $C(n, x)$ is a strictly increasing function of $x$. If $C'_p(0) > \beta x^\beta L^{\beta - 1} \left[ C_m + C_t G_t(\tau) \right]$, then $C(n,x)$ is an increasing function of $x$ for any $n > 0$. Hence, the optimal maintenance degree is $x^\star = 0$ and $(n^\star, x^\star) = (0,0)$. On the other hand, if $C'_p(0) \leq \beta x^\beta L^{\beta - 1} \left[ C_m + C_t G_t(\tau) \right]$, then $C(n,x)$ changes its sign exactly once in the interval $(0,L/n]$. Therefore, there exists an unique solution $x \in (0,L/n]$ such that $\frac{\partial C(n, x)}{\partial x} = 0$ for any $n > 0$. □

**Theorem 3** shows that the optimal maintenance degree is unique when the number of PM actions is pre-specified. Now, substituting the result of Eq. (9) into Eq. (8) and setting Eqs. (7) and (8) equal to zero, we have

$$\hat{x} = \frac{1}{\beta - 1} \left[ C'_p(\hat{x}) + \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] \right]^{1/\beta},$$

(10)

and

$$\hat{n} = \frac{L}{x} - \left[ \beta x^\beta \left[ C_m + C_t G_t(\tau) \right] x^{\beta - 1} + 1 \right]^{1/(\beta - 1)}.$$
can be easily checked that 
\[ \frac{3}{2} \frac{C_{n}(x)}{C_{n}^{*}} = n \beta (\beta - 1) \beta^{2} C_{m} + C_{c} \left( \frac{1}{\beta} \right)^{\beta - 2} > 0 \] 
for all \( x \leq L/n \) when \( \beta > 1 \).

Theorem 4 indicates that PM actions should be performed periodically with maintenance degree \( \frac{1}{\beta} \) when the PM cost is a constant. Using this result, the expected total cost becomes a function of \( n \). Replacing \( x \) by \( \frac{1}{\beta} \), we have
\[
C \left( \frac{n - L}{n + 1} \right) = na + \left( \frac{C_{n} + C_{c}}{C_{n}} \right) (n + 1)^{1-\beta}  \tag{12}
\]
and
\[
\frac{\partial C}{\partial n} \left( \frac{n - L}{n + 1} \right) = a - (\beta - 1) (\beta L)^{\beta} \left[ \frac{C_{n} + C_{c}}{C_{n}} \right] (n + 1)^{1-\beta}  \tag{13}
\]
It is clear that
\[
\frac{\partial \phi}{\partial n} \left( \frac{n - L}{n + 1} \right) = \beta (\beta - 1) (\beta L)^{\beta} \left[ \frac{C_{n} + C_{c}}{C_{n}} \right] (n + 1)^{1-\beta} > 0
\]
for all \( n \geq 0 \) when \( \beta > 1 \). Therefore, setting Eq. (13) equal to zero, a closed-form solution \( n \) can be obtained as
\[
n = \frac{L \left[ \frac{a}{(\beta - 1) (\frac{C_{n} + C_{c}}{C_{n}})} \right]^{1/\beta}}{\beta - 1} - 1.  \tag{14}
\]
Eq. (14) indicates that the optimal number of PM actions decreases as the PM cost increases, but it increases as the minimal repair cost and penalty cost increase. This result is consistent with Eqs. (10) and (11) in different forms. Since \( C_{n}(x) = 0 \), we have \( n = \frac{L}{\beta - 1} \) from Eq. (11), which means \( \beta = \frac{1}{\beta - 1} \). Furthermore, the result in Eq. (10) shows that \( x = \left( \frac{\beta - 1}{\beta} \right)^{1/\beta} \) and hence \( n \) is the same as the result given in Eq. (14).

Again, the optimal integer \( n^{*} \) can be easily obtained as either \( \lceil n \rceil \) or \( \lfloor n \rfloor \), whichever results in a smaller expected total cost in Eq. (12) and the corresponding optimal maintenance degree is when the PM cost is a constant.

3.3. Linearly increasing PM cost function

Consider the case when PM cost is linearly dependent on the maintenance degree, say \( C_{n}(x) = ax + bx > 0 \) for \( x > 0 \), where \( a > 0 \) and \( b > 0 \) represent the fixed cost and the marginal cost of a PM action with degree \( x \), respectively. In this case, \( x \) in Eq. (10) becomes \( x = \left( \frac{\beta - 1}{\beta} \right)^{1/\beta} \) and the corresponding number of maintenance actions \( n \) is \( n = \frac{L}{\beta - 1} - \left( \frac{b}{(\beta - 1) (\frac{C_{n} + C_{c}}{C_{n}})} \right)^{1/\beta - 1} \) as given in Eq. (11).

Note that the optimal maintenance degree under linearly increasing maintenance cost function is the same as the maintenance degree under constant maintenance cost. But, the optimal number of PM actions in this case is less than or equal to the optimal number in the case of constant maintenance cost and the differences increase as the marginal cost \( b \) increases. Given \( x \), the number of PM actions should be \( \frac{L}{\beta - 1} \) if PM actions are carried out periodically over the whole lease period (case of constant cost function). However, the above result shows that it may not be worthy to perform PM actions near the end of the lease period when the PM cost increases linearly in maintenance degree. These results are reasonable since the number of preventive-maintenance actions will decrease as the associated cost increases. Furthermore, the result of \( x \) shows that the optimal maintenance degree increases as the fixed PM cost increases, but it decreases as the minimal repair cost and penalty cost increase.

Now, the optimal integer \( n^{*} \) can be easily obtained as either \( \lceil n \rceil \) or \( \lfloor n \rfloor \). However, their corresponding maintenance degrees should be re-evaluated using steps 4 through 6 of the search procedure given in Section 3.1.

### 4. Numerical examples

In this section, the performance of the optimal policy is evaluated through some numerical examples. Consider that a new product is leased for a period \( L = 10 \) units of time. The life-time distribution of this product follows a Weibull distribution with a scale parameter \( \lambda = 0.1 \) and a shape parameter \( \beta = 1.5 > 1 \). Obviously, the mean life of the product is \( \mu = \frac{1}{\lambda F} \left( \frac{L}{1 + 1} \right) = 9.027 \). Within the lease period, each failure of the product incurs an expected cost \( C_{n} + C_{c} \left( \frac{1}{\beta} \right) = 100 \). If PM actions are not performed, then the expected total cost to the lessor is \( C(0,0) = C_{n} + C_{c} \left( \frac{1}{\beta} \right) = 100 \), which provides an upper bound for the following analysis. The performance of PM actions is evaluated by \( A_{C} = C(n^{*},x^{*}) - C(0,0) \times 100/C(0,0) \), which is the percentage of cost reduction if PM actions are carried out. In the following discussion, we will investigate the effects of the PM cost function on the \( A_{C} \) and the optimal PM policy.

When the PM cost is a fixed constant \( C_{n}(x) = a > 0 \), the relationship between \( x \) and \( x^{*} = \frac{x}{\frac{1}{\beta}} \) is obtained. Hence, we can focus on the effects of the fixed cost \( a \) on the optimal number \( n^{*} \). Using the results given in Section 3.2, the optimal PM policies and the percentage of cost reductions for various values of \( a \) are summarized in Table 1.

For example, if \( a = 1 \), the optimal PM policy is \( (n^{*},x^{*}) = (13,0.71) \). That is, there is a total of 13 PM actions during the lease period and

<table>
<thead>
<tr>
<th>( a )</th>
<th>( n^{*} )</th>
<th>( x^{*} )</th>
<th>( C(n^{<em>},x^{</em>}) )</th>
<th>( A_{C} )</th>
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![Fig. 1. Optimal number of PM actions when \( C_{n}(x) = a \).](image-url)
Furthermore, the expected total cost increases as the fixed PM actions. The relationship between and as the fixed cost increases. When, the PM cost becomes too high to perform any PM actions.

Table 2

<table>
<thead>
<tr>
<th>b</th>
<th>n*</th>
<th>x*</th>
<th>C(n*,x*)</th>
<th>Δx%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13</td>
<td>0.71</td>
<td>39.73</td>
<td>60.27</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>0.74</td>
<td>48.78</td>
<td>51.22</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>0.76</td>
<td>57.41</td>
<td>42.59</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>0.71</td>
<td>65.49</td>
<td>34.51</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.71</td>
<td>72.93</td>
<td>27.07</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>0.71</td>
<td>79.67</td>
<td>20.33</td>
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<tr>
<td>6</td>
<td>7</td>
<td>0.77</td>
<td>85.60</td>
<td>14.40</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.75</td>
<td>90.61</td>
<td>9.39</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>0.72</td>
<td>94.66</td>
<td>5.34</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0.77</td>
<td>97.64</td>
<td>2.36</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>0.69</td>
<td>99.46</td>
<td>0.54</td>
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<td>0</td>
<td>0</td>
<td>100</td>
<td>0.00</td>
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Table 2 shows that the optimal number of PM actions decreases as the fixed cost a increases. When a ≥ 30, it is not worthy to perform PM actions. The relationship between and is illustrated in Fig. 1. Furthermore, the expected total cost increases as the fixed cost a increases and hence the Δ% decreases. This result shows that PM actions can achieve a significant reduction in cost when the PM cost is relatively lower than the minimal repair cost and the penalty cost.

Now, let the PM fixed cost a = 1 and the PM cost be a linearly increasing function of maintenance degree, . The effects of the marginal cost b on the optimal policy (n*,x*) and the expected total cost C(n*,x*) are summarized in Table 2.

Table 2 shows that when b = 0, the optimal policy is (n*,x*) = (13,0.71) which is the same as in the case of constant PM cost. As the marginal cost b increases, the optimal number of PM actions decreases. For example, when (a,b) = (1,6), the optimal policy becomes (n*,x*) = (7,0.77). That is, if the marginal PM cost b = 6, then the optimal number of PM actions reduces to 7, and these PM actions should be carried out at time epochs 0.77, 1.54, 2.31, 3.08, 3.85, 4.63, and 5.39. Note that after the 7th PM action, it is not worthwhile to perform any PM actions during the rest of the lease period (10 – 5.39 = 4.61 units of time) when the PM cost is a linearly increasing function of maintenance degree.

Fig. 2 illustrates the relationship between the optimal number of PM actions and the marginal PM cost b. Since is an integer, it is a step-function of b and decreases as b increases. When b ≥ 11, the PM cost becomes too high to perform any PM actions. Furthermore, for the same n*, we found that the corresponding optimal maintenance degree x* also decreases as b increases. However, the variation of the x* is relatively small (less than 0.15) in these numerical examples.

5. Conclusion

In this paper, we derive the optimal PM policy for a leased product with Weibull life-time distribution. Under the age-reduction maintenance scheme, we focus on investigating the effects of the PM cost function on the optimal PM policy. Given a general PM cost function, we show that there exists an unique optimal policy such that the expected total cost during the lease period is minimized. Under the optimal policy, PM actions are carried out at time epochs x*,2x*,3x*,…,n*x* with same maintenance degree x*. An efficient algorithm is provided in this paper to search for the optimal policy (n*,x*) when b > 1.

Furthermore, when the PM cost function is constant, = a > 0, we found that the optimal policy is a periodical PM policy. That is, PM actions should be performed at time epochs with same maintenance degree . And, the optimal number of PM actions n* decreases as the PM cost increases. However, when the PM cost linearly increases as the maintenance degree increases, i.e., = a + bx, we found that the PM actions are performed periodically at the beginning of the lease period and it is not worthwhile to perform any PM actions near the end of the lease period. The numerical results also show that n* decreases as the marginal cost b increases.

Acknowledgements

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References


