Fuzzy Sliding Mode Control for Trajectory Tracking on Mechatronic Arms

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Abstract - This paper presents the design of fuzzy sliding mode control algorithm to achieve the specified trajectory tracking on mechatronic arms. The tracking paths are composed of a set of sequentially-operated piecewise sliding surfaces which the system’s state can follow to the equilibrium point in phase plane and from which the total tracking time can be obtained directly. It is shown that each fuzzy controller will refer to a stable closed-loop subsystem in the sense of Lyapunov stability under a common Lyapunov function, and then the overall system is also stable in the sense of Lyapunov stability. Further, the reaching phase of the trajectory which is sensitive to parameter uncertainties or disturbances can be reduced effectively and the system’s state traveling on the paths and finally to the equilibrium point be ensured. The main advantages of the proposed algorithm, in contrast to the conventional sliding-mode control design, are that the tracking time in the reaching phase no longer exists, and the tracking time the first time the system’s state hitting each sliding surface can be evaluated, and that the theoretic total tracking time and the true total one are in good agreement.

I. Introduction

Recently, fuzzy logic control (FLC), as one of the most useful approaches for collecting human knowledge and expertise, has been the focus of numerous studies to plants that are mathematically poorly modeled or the model uncertainty in the dynamics is either unknown or impossible [1], and to the problem of tracking control for nonlinear systems [2], [3]. Since the FLC depends mainly on the individual operators’ experience, it is generally inconsistent with the specified performance. Further, the inference rules will typically contain a number of subjectively as well as empirically determined parameters, and in most cases the nonlinearities existing in the dynamic system are not known a priori. In some of the above applications, fuzzy logic control with sliding mode is used in constructing a set of rules to obtain the fuzzy control parameters. But, the obtained FLC parameters and control rules may restrict the reaching phase of the sliding mode according to partial dependence of fuzzy rules, which will cause higher possibility of chattering effects or unreachable sliding surface and therefore can not achieve the tracking control purpose. Hence in this paper, an algorithm combines the advantages of the fuzzy logic control and the sliding mode control not only to reduce the required amount of fuzzy rules based on a set of sequentially-operated sliding surfaces, but to have chattering-free effects for the sliding motion and achieve the tracking control purpose. As presented in [2], fundamental problems still exist in the control of complex systems using sliding mode controllers, e.g., control chattering phenomenon and sensitivity to parameter uncertainties or noise disturbances in the reaching phase. Several theoretical methods have been developed for reducing the sensitivity to parameter uncertainties and noise disturbances in the reaching phase. In them, high gain feedback control was one of the effective method to reduce
the tracking error and reaching time [6], but it may cause unmodeled dynamics and chattering in a physical system which may further excite high frequency dynamics. In [2], a time-varying sliding surface in phase plane was proposed to reduce or eliminate the reaching phase by imposing a constraint where the initial value of the error state was zero. However in many practical applications, initial conditions of actual system may be arbitrary. Another sliding mode control design method with chattering-free is introduced in [7]. In [8], the authors introduced a time-varying sliding surface for arbitrary initial conditions. The feature is that the sliding surface is moved toward a predetermined one via rotation and/or shift by changing the magnitude of the slope and intercept of the surface in phase plane. Another way to reduce chattering effects is to combine the sliding-mode control scheme for a class of self-organizing fuzzy controller was proposed to augment the sliding-mode control scheme for a class of nonlinear system [9].

In this paper, a fuzzy sliding mode control algorithm using a sequentially-operated piecewise sliding surfaces is proposed to achieve the specified trajectory tracking on mechatronic arms. The piecewise sliding surfaces constitute the tracking paths which the system’s state can follow sequentially to the equilibrium point in phase plane and from which the total tracking time can be obtained directly. The surfaces are designed subject to the desired tracking paths accordingly and where the given initial state locates in phase plane. An analytic formulation of the fuzzy inference is presented for each sliding surface in the corresponding region of the phase plane. The fuzzy sliding mode controller with some equivalent controllers corresponding to the sliding surfaces is designed such that the hitting motion is achieved and the system’s state traveling on the tracking trajectory is ensured.

If the state is traveling into or originally is located in the neighborhood of the abscissa \(c(t)\) of the phase plane, an alternative sliding surface located nearly in parallel with the abscissa \(c(t)\) is constructed for reaching phase such that the state will move across the abscissa to this surface, and then continue the remaining tracking paths. It is shown that each fuzzy controller will refer to a stable closed-loop subsystem in the sense of Lyapunov stability under a common Lyapunov function, and then the stability of the closed-loop system is guaranteed in the sense of Lyapunov stability. Under the hitting condition, because the amount of time in the reaching phase can be made very small, we can evaluate the total tracking time for the tracking paths in which the hitting time for achieving the reaching phase and the traveling times exactly on the respective sliding surfaces. It is shown that the theoretic total tracking time and the true total one are in good agreement.

II. Fuzzy sliding mode control

Consider the dynamics of a n-joint rigid robotic manipulator [12] represented by the following n-coupled second-order nonlinear equations of the form

\[
\dot{\theta}_k = f_k(\theta, \dot{\theta}) + b_k(\theta)u_k + d_k(t), \quad k = 1, 2, \ldots, n
\]  

where \(\theta_k\) is the \(k\)th component of the vector of joint angles \(\theta = [\theta_1 \cdots \theta_n]^\top \in \mathbb{R}^n\), \(u_k\) the \(k\)th component of the vector of control inputs \(u \in \mathbb{R}^n\), \(b_k\) the \(k\)th component of the vector of control gains \(b\), and \(d_k\) the \(k\)th component of the vector of external disturbances \(d = [d_1 \ d_2 \ \cdots \ d_n]^\top\) which is unknown but bounded by the known function \(D_k(t)\), i.e., \(|d_k(t)| \leq D_k(t)\).

Defining vector \(x(t) = [\theta_1 \ \theta_2 \ \cdots \ \theta_n \ \dot{\theta}_n]^\top = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t) \ \cdots \ x_{2n-1}(t) \ x_{2n}(t)]^\top \in \mathbb{R}^{2n}\), the following equations can be obtained for the coordinates characterizing manipulator state \((k = 1, 2, \ldots, n)\):

\[
\begin{align*}
\dot{x}_{2k-1}(t) &= x_{2k}(t) \\
\dot{x}_{2k}(t) &= f_k(x) + b_k(\theta)u_k(t) + d_k(t) \\
x_0 &= x(t_0)
\end{align*}
\]  

Suppose that the nonlinear dynamics \(f_k(x)\) is not known exactly, but is estimated as a known nominal dynamics \(\hat{f}_k(x)\) with an error bounded by a known function \(F_k(x)\) as

\[
|f_k(x) - \hat{f}_k(x)| \leq F_k(x), \quad k = 1, 2, \ldots, n.
\]
Let $\bar{x}(t) = [\bar{x}_1(t) \bar{x}_2(t) \ldots \bar{x}_{2n}(t)]^\top$ be the desired position path. The control problem is to find a fuzzy sliding mode controller which will drive the error state asymptotically

$$e(t) = [e_1(t) e_2(t) \ldots e_{2n}(t)]^\top = [x_1(t) - \bar{x}_1(t) x_2(t) - \bar{x}_2(t) \ldots x_{2n}(t) - \bar{x}_{2n}(t)]^\top$$

(4)

to zero for any given initial state, $e_0 = e(t_0)$, given at an initial time $t = t_0$. For each $k$, let us define the following seven sequentially-operated sliding surfaces as shown in Fig. 1:

$$s_k(t) = e_2k(t) + \lambda_k e_{2k-1}(t) + \gamma_k, \quad i = 1, 2, \ldots, 7$$

(5)

where the slopes $\lambda_k < 0$ for $i = 1, 5$, $\lambda_k > 0$, for $i = 2, 3, 4$, and $\lambda_k = 0$ for $i = 6, 7$, and the parameters $\gamma_k > 0$ for $i = 1, 4, 6, 7$, $\gamma_k < 0$ for $i = 2, 3, 5$ and $\gamma_k = 0$, $\gamma_6 = \gamma_7$, and they are all real constants. These constants should be chosen such that when the system states are in the sliding plane (i.e., $s_k(t) = 0$, $\forall i$ and for each $k$), they will slide along these planes accordingly to the equilibrium point. For each $k$, the phase plane is then partitioned into the following six pairwise symmetric subregions except for the abscissa $e_{2k-1}$:

$$I_{k1} = \left\{ (e_{2k-1}(t), e_{2k}(t)) \mid e_{2k}(t) > 0 \cap s_{2k}(t) > 0 \right\}$$

$$I_{k2} = \left\{ (e_{2k-1}(t), e_{2k}(t)) \mid e_{2k}(t) > 0 \cap s_{2k}(t) = 0 \right\}$$

$$I_{k3} = \left\{ (e_{2k-1}(t), e_{2k}(t)) \mid e_{2k}(t) > 0 \cap s_{2k}(t) < 0 \right\}$$

$$I_{k4} = \left\{ (e_{2k-1}(t), e_{2k}(t)) \mid e_{2k}(t) < 0 \cap s_{2k}(t) < 0 \right\}$$

$$I_{k5} = \left\{ (e_{2k-1}(t), e_{2k}(t)) \mid e_{2k}(t) < 0 \cap s_{2k}(t) = 0 \right\}$$

$$I_{k6} = \left\{ (e_{2k-1}(t), e_{2k}(t)) \mid e_{2k}(t) < 0 \cap s_{2k}(t) > 0 \right\}$$

These regions are separated from one another by the straight lines $e_{2k}(t) = 0$ and $s_{2k}(t) = 0$, and each part is analyzed for closed-loop stability. Further, it is seen from (5) that each one has its corresponding sliding surface. In the sequentially-operated fuzzy sliding mode control design, for each $k$, input variables to the fuzzy logic control are characterized by system states $e_{2k-1}(t)$ and $e_{2k}(t)$, and the pth fuzzy IF-THEN rule for the controller is defined by $(p = 1, 2, \ldots, m, k = 1, 2, \ldots, n)$:

$$R_p^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is } F_p^k$$

AND $\|e_{2k}\|$ is $F_{p2}^k$ THEN $u_k(t) = u_{R_p^k}(t)$,

where $F_{pj}$ is the fuzzy set, $m$ the number of IF-THEN rules, $u_p(t)$ the singleton defined control action for the $p$th control rule, and $\| \|$ the Euclidean norm operator of $\cdot$. The membership function of $\|e_{2k}\|$ is shown in Fig. 2. In order to further visualize and make the state $(e_{2k-1}(t), e_{2k}(t))$ of the nonlinear system hit and slide along the corresponding surfaces $s_{k1}(t), i = 1, 2, \ldots, 7$, for each portion of the phase plane, the rules of the fuzzy sliding mode controller are explicitly defined by $(k = 1, 2, \ldots, n)$:

$$R_1^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is in } I_{k6}$$

AND $\|e_{2k}\|$ is $B$ THEN $u_k(t) = u_{R_1^k}(t)$.

$$R_2^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is in } I_{k2} \text{ OR in } I_{k5}$$

THEN $u_k(t) = u_{R_2^k}(t)$.

$$R_3^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is in } I_{k1}$$

AND $\|e_{2k}\|$ is $B$ THEN $u_k(t) = u_{R_3^k}(t)$.

$$R_4^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is in } I_{k3}$$

AND $\|e_{2k}\|$ is $B$ THEN $u_k(t) = u_{R_4^k}(t)$.

$$R_5^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is in } I_{k4}$$

AND $\|e_{2k}\|$ is $B$ THEN $u_k(t) = u_{R_5^k}(t)$.

$$R_6^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is in } I_{k3} \text{ OR in } I_{k4}$$

AND $\|e_{2k}\|$ is $S$ THEN $u_k(t) = u_{R_6^k}(t)$.

$R_7^k: \text{IF the state } (e_{2k-1}(t), e_{2k}(t)) \text{ is not in any region;}$ THEN $u_k(t) = u_{R_7^k}(t) = 0.$
Then from (2) and the fuzzy rules, and for non-zero respective, and

\[ \eta \]

overall fuzzy sliding mode control corresponds to the

\[ \mu \]

where \( \text{sgn} \) is a small positive constant, \( \text{sgn}(s_{ki}(t)) = +1 \), if \( s_{ki}(t) > 0 \), \( \text{sgn}(s_{ki}(t)) = -1 \), otherwise, and \( \dot{u}_{ki}(t) \) is the equivalent controller corresponding to the \( i \)th sliding condition:

\[
\dot{u}_{ki}(t) = -f_k(x) - \lambda_k e_{2k}(t) + \hat{x}_{2k-1}(t), \quad i = 1, 2, \ldots, 7. \tag{6}
\]

Under the weighted-sum defuzzification method, the overall fuzzy sliding mode control \( u_k \) is then evaluated by

\[
 u_k = \frac{\sum_{p=1}^{n} \mu_p u_{pk}}{\sum_{p=1}^{n} \mu_p}, \quad k = 1, 2, \ldots, n \tag{7}
\]

where \( \mu_p \) is the membership function value of the \( p \)th-rule. If the fuzzy rule \( p \) is fired, the degree of membership \( \mu_p \) of fuzzy rule \( p \) is nonzero.

For each \( k \) and for all \( i \), define a Lyapunov function as follows:

\[ V_{ki}(t) = \frac{1}{2} \sum_{i=1}^{n} s_{ki}(t)^2. \tag{8} \]

Then from (2) and the fuzzy rules, and for non-zero initial states \( e_{2k-1}(t_0) \) and \( e_{2k}(t_0) \), we have the following sliding condition:

\[
 V_{ki}(t) = s_{ki}(t) \left( f_k(x) + b_k(\theta)u_{pk}(t) + d_k(t) + \lambda_k e_{2k}(t) \right) \\
 - \dot{x}_{2k-1}(t) \leq |f_k(x) - \dot{f}_k(x)||s_{ki}(t)| + |d_k(t)||s_{ki}(t)| - (F_k(x) + D_k(t) + \eta_k) s_{ki}(t) \\
 \leq F_k(x)||s_{ki}(t)| + D_k(t)||s_{ki}(t)| - (F_k(x) + D_k(t) + \eta_k ||s_{ki}(t)| \\
 = -\eta_k ||s_{ki}(t)|. \tag{9} \]

Hence, the state \( (e_{2k-1}(t), e_{2k}(t)) \) can hit and slide along the corresponding sliding surface \( s_{ki}(t) \) for any given initial conditions.

Remark 1: It should be noted that all of the eight rules in the fuzzy logic control lead to stable subsystems in the sense of Lyapunov subject to the same Lyapunov function (8). Hence, the stability of the closed-loop system is guaranteed in the sense of Lyapunov when all the rules are included into the rule base of the fuzzy logic control.

Next, we will use the boundary layer technique with thickness \( \phi_{ki} \) to the control law to smooth out these drastic changes of the control input. To do this, we can replace \( \text{sgn}(s_{ki}(t)) \) with \( \text{sat}(s_{ki}(t)/\phi_{ki}) \) in (7) for each \( k \) and for all \( i \), where \( \phi_{ki} \) is the boundary layer thickness and \( \epsilon_{ki} = \phi_{ki}/\lambda_{ki} \) is the boundary layer width, and

\[
 \text{sat}(s_{ki}(t)/\phi_{ki}) = \begin{cases} 
 \text{sgn}(s_{ki}(t)/\phi_{ki}), & \text{if } |s_{ki}(t)/\phi_{ki}| \geq 1 \\
 \frac{s_{ki}(t)}{\phi_{ki}}, & \text{if } |s_{ki}(t)/\phi_{ki}| < 1.
 \end{cases} \tag{10}
\]

Then, the control input \( u_k(t) \) can be smoothed out and the phenomenon of the chattering can be reduced.

III. Evaluation of the tracking time

From (5), it is known that the sliding surface \( s_{k6}(t) \) is only used to avoid the state starting from region \( J_{k1} \) and moving along the sliding surface \( s_{k3}(t) \) won’t get to the abscissa \( e_{2k-1} \) until \( t = \infty \), but this will lead to the existence of the reaching phase. However, we can choose a small constant \( \gamma_{k6} \) such that a small amount of time to achieve the reaching phase can be obtained and thus will not affect the total tracking time in evidence. Considering the surface \( s_{k6}(t) \) first, since \( s_{k6}(t) > 0 \) before the state moves across the abscissa \( e_{2k-1} \) to hit the surface \( s_{k6}(t) \), integrating (9) from \( t = t_{f,k3} \) to \( t = t_{f,hit} \) leads to

\[
 \int_{t_{f,k3}}^{t_{f,hit}} \dot{s}_{k6} dt = \int_{t_{f,k3}}^{t_{f,hit}} -\eta_{k6} dt \\
 \Rightarrow s_{k6}(t_{f,hit}) - s_{k6}(t_{f,k3}) \leq -\eta_{k6} t_{hit}
\]

where \( t_{hit} = t_{f,hit} - t_{f,k3} \) is the total required hitting time. Because \( s_{k6}(t_{f,hit}) = 0 \), an upper bound of the hitting time can be obtained as

\[
 t_{hit} \leq \frac{s_{k6}(t_{f,k3})}{\eta_{k6}}. \tag{11}
\]
Similarly, as for the surface \( s_{k7}(t) \), since \( s_{k7}(t) < 0 \) before the state moves across the abscissa \( e_{2k-1} \) to hit the surface \( s_{k7}(t) \), an upper bound of the hitting time can be obtained as

\[
t_{h7} \leq \frac{-s_{k7}(t_{f,k4})}{\eta_{k7}}. \tag{12}
\]

For each tracking path composed of the sliding surfaces \( s_{ki}, i = 1, 2, 3, 4, 5 \), we have solutions for system's state as follows

\[
e_{2k-1}(t) = \left( e_{2k-1}(t_0) + \frac{\lambda_k}{\lambda_{ki}} \right) \exp(-\lambda_{ki}t) - \frac{\gamma_{ki}}{\lambda_{ki}} \tag{13}
\]

where \( e_{2k-1}(t_0) \) is the initial state at starting time \( t_0 \). Then, differentiating (13), we have

\[
e_{2k}(t) = -\left( \lambda_k e_{2k-1}(t_0, ki) + \gamma_{ki} \right) \exp(-\lambda_{ki}t), \quad i = 1, 2, 3, 4, 5. \tag{14}
\]

Then, for each \( k \), the tracking time \( t_{ki} \) from the starting position \( e_{2k}(t_0, ki) \) to the final position \( e_{2k}(t_f, ki) \) for the \( i \)th sliding surface \( s_{ki} \) is

\[
t_{ki} = -\frac{1}{\lambda_{ki}} \ln \left( \frac{-e_{2k}(t_f, ki)}{\lambda_{ki} e_{2k-1}(t_0, ki) + \gamma_{ki}} \right) = -\frac{1}{\lambda_{ki}} \ln \left( \frac{e_{2k}(t_f, ki)}{e_{2k}(t_0, ki)} \right), \quad i = 1, 2, 3, 4, 5. \tag{15}
\]

It is seen from (15), as the state originally in region \( I_{k1} \) approaches the neighborhood of the abscissa \( e_{2k-1} \) of the phase plane, the time \( t_{k3} \) will not approach infinity since we have replaced \( e_{2k}(t_f, k3) = 0 \) with \( e_{2k}(t_f, k3) = \gamma_{k6} \). Hence, the state will move across the abscissa \( e_{2k-1} \) to hit the surface \( s_{k6}(t) \). When the state originally is in region \( I_{k1} \), similar analysis can also be applied by replacing \( e_{2k}(t_f, k4) = 0 \) with \( e_{2k}(t_f, k4) = -\gamma_{k7} \). In this situation, the state will also move across the abscissa \( e_{2k-1} \) to hit the surface \( s_{k7}(t) \). Note that we can use the same analysis for the initial state satisfying \( e_{2k}(t_0) = 0 \) but \( e_{2k-1}(t_0) \neq 0 \). If \( e_{2k-1}(t_0) > 0 \), we can replace \( e_{2k}(t_0) = 0 \) with \( e_{2k}(t_0) = \gamma_{k6} \), and \( e_{2k}(t_0) = 0 \) with \( e_{2k}(t_0) = -\gamma_{k7} \) for \( e_{2k-1}(t_0) < 0 \).

Hence, using again the surface \( s_{k2} \) as a line of demarcation, we take \( s_{k2}(t) > 0 \) and \( s_{k2}(t) < 0 \) into account, respectively, for calculating the total tracking time.

For \( s_{k2}(t) > 0 \), we have the following descriptions for the fuzzy inference rules:

1. If the state \((e_{2k-1}(t), e_{2k}(t))\) is in \( I_{k1} \) and \( \|e_{2k}\| \) is \( B \), then the initial state satisfies \( e_{2k}(t_0) \geq \gamma_{k6} \). Hence, the state should slide along the surface \( s_{k1}(t) \), move across the abscissa \( e_{2k-1} \), slide along \( s_{k1}(t) \) and finally along \( s_{k2}(t) \) sequentially. The tracking time for \( s_{k3}(t) \) from the initial position \((e_{2k-1}(t_0), e_{2k}(t_0))\) to the position \((-\frac{\lambda_{k1}}{\lambda_{k2}}, \gamma_{k6}) \) can be directly obtained from (15) as

\[
t_{k3} = -\frac{1}{\lambda_{k3}} \ln \left( \frac{\gamma_{k6}}{e_{2k}(t_0)} \right). \tag{16}
\]

Subsequently, we use the fuzzy control law \( u_{k}(t) = u_{k1}^{\text{ref}} \) to force the state to hit the surface \( s_{k6} \) and the reaching time from (11) is

\[
t_{kr} \leq \frac{2\gamma_{k6}}{\eta_{k6}} \tag{17}
\]

since \( s_{k6}(f,k1) = e_{2k}(f,k3) + \gamma_{k6} = 2\gamma_{k6} \). The state then continues to slide on the surface \( s_{k1} \) from the starting position \((-\frac{\lambda_{k1}}{\lambda_{k2}}, -\gamma_{k6}) \) to the end position \((-\frac{\lambda_{k1}}{\lambda_{k2}}, \gamma_{k6}) \), and its total tracking time can be obtained as

\[
t_{k1} = -\frac{1}{\lambda_{k1}} \ln \left( \frac{\gamma_{k6}}{\frac{\lambda_{k1}}{\lambda_{k2}} - \frac{\gamma_{k6}}{\lambda_{k2}}} \right). \tag{18}
\]

Finally, the state will slide along the surface \( s_{k2} \) from the starting position \((-\frac{\lambda_{k1}}{\lambda_{k2}}, \frac{\lambda_{k1}}{\lambda_{k2}} - \frac{\gamma_{k6}}{\lambda_{k2}}) \) to the final position \((-\frac{\lambda_{k1}}{\lambda_{k2}}, -\gamma_{k6}) \), the intersection point with the surface \( s_{k6} \) (i.e., the neighborhood of the equilibrium point), and the tracking time can be obtained as

\[
t_{k2} = -\frac{1}{\lambda_{k2}} \ln \left( \frac{-\gamma_{k6}}{\frac{\lambda_{k1}}{\lambda_{k2}} - \frac{\gamma_{k6}}{\lambda_{k2}}} \right). \tag{19}
\]

Hence, in this situation the tracking paths have the points of inflection at \((-\frac{\lambda_{k1}}{\lambda_{k2}}, \frac{\lambda_{k1}}{\lambda_{k2}} - \frac{\gamma_{k6}}{\lambda_{k2}}) \), \((-\frac{\lambda_{k1}}{\lambda_{k2}}, -\gamma_{k6}) \), and \((-\frac{\lambda_{k1}}{\lambda_{k2}}, \frac{\lambda_{k1}}{\lambda_{k2}} - \frac{\gamma_{k6}}{\lambda_{k2}}) \), and its total tracking time can be summarized as follows:

\[
t_{track} \leq t_{k3} + t_{kr} + t_{k1} + t_{k2} \tag{20}
\]
(3) If the state \( (e_{2k-1}(t), e_{2k}(t)) \) is in \( I_{k6} \) and \( \|e_{2k}\| \) is B, and the initial state satisfying \( e_{2k}(t_0) \leq -\gamma_k \), the state will slide along the surface \( s_{k1} \) and then along \( s_{k2} \) in turn. Hence, the total tracking time for the state from the initial position to the final zero equilibrium point can be seen from (17) as
\[
t_{\text{track}} = t_{k1} + t_{k2} \tag{19}
\]
As for \( s_{k2}(t) > 0 \), the evaluation of the total tracking time for \( s_{k2}(t) < 0 \) can be obtained in a similar way.

IV. Conclusion

In this paper, we have presented a fuzzy sliding mode control algorithm using a set of sequentially-operated piecewise sliding surfaces to achieve trajectory tracking for a class nonlinear uncertain systems subject to bounded disturbances. Under the hitting condition, it is shown that the state along the tracking paths will converge to the equilibrium point by using the proposed control scheme faster than that of the conventional sliding-mode control design method, even though the reaching phase exists in the tracking paths. In addition, it is easy to evaluate the total tracking time by this algorithm but the conventional sliding-mode control design can not because the first time the system’s state hits the sliding surface is unknown. The importance of evaluation of the total tracking time lies in the fact that it can give a definite time the state traveling from the initial state to the final equilibrium point for the trajectory planning. Since the reaching phase can be made very small and based on the sequentially-operated piecewise sliding surfaces, the theoretic total tracking time and the true total one are in good agreement.

References


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