Recipients Maximization Multicast Scheme in IEEE 802.16j WiMAX Relay Networks

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Abstract- In this paper, we tackle an important problem in WiMAX relay networks called Multicast Recipient Maximization (MRM), which maximizes the number of multicast recipients with given budget. To deal with this problem, a novel resource allocation scheme called Dynamic Station Selection (DSS) is proposed. We prove that MRM is NP-hard, while our DSS has polynomial-time computational complexity. The simulation results show that under different situations, the performance of DSS always approximates the optimal solution. 

Index Terms—wireless relay networks, resource allocation, multicast

I. INTRODUCTION

IEEE 802.16 (WiMAX) is an emerging last mile technology for broadband wireless access [1-2]. Compared with 3G cellular networks and IEEE 802.11 Wi-Fi, it has better coverage and throughput. A typical IEEE 802.16 WiMAX network comprises a base station (BS) and multiple subscriber stations (SSs) [3]. The BS serves as a gateway to the wired network, while all SSs connect to the BS to access the Internet. However, under the IEEE 802.16j standard [4], a new type of infrastructure node called relay stations (RSs) is introduced. RSs forward data between the BS and the SSs both upward and downward to improve the system capacity and throughput.

As the capacity of mobile devices improves, many multicast applications, such as wireless IPTV, Radio over IP, Video conferencing, and etc., are developed. WiMAX technology can serve as a great platform that allows service carriers to provide such services wirelessly. However, compared with wired environment, wireless resource is relatively scarce. The available resource for each wireless service is inevitably limited. Since SSs have different bit error rates due to heterogeneous channel conditions, they may require different amount of resource. With given resource budget, BS should serve as many recipients (i.e., SSs) as possible so that user satisfaction can be maximized. When a network only consists of a BS and SSs, this maximization can be easily done by adjusting the allocated resources of the BS. However, if RSs are considered, this problem becomes much difficult because resource should be allocated among the BS and RSs.

Many existing works discuss issues of wireless multicast. For example, in [5], a reliable multicast scheme uses Code Division Multiple Access (CDMA) codes in WiMAX networks, while [6] proposes a two-level superposition coded multicast scheme (2-level SCM) to improve the channel efficiency for multicast transmission in a WiMAX network. However, they do not consider the resource allocation problem we mention above. [11] studies multicast over wireless relay networks. Two kinds of nodes, namely relay nodes and receiving nodes, are considered. However, the transmission power of each relay node is fixed such that the scheme can only choose to activate the relay node or not, resulting in limited flexibility and efficiency.

Among the related works, a class of multicast problem called Minimum-Energy Multicast (MEM) is well-studied in [7-10]. Given the channel condition of each node, the MEM problem finds the multicast tree that minimizes the total required energy for delivering the stream to a given set of subscribers. Although MEM initially looks similar to our Multicast Recipient Maximization (MRM) problem, they are substantially different. First, MEM minimizes the required resources of serving subscribers, while our MRM maximizes the number of the recipients with a budget. Second, MEM assumes that each intermediate node can forward data; however, only RSs can relay data in WiMAX relay networks. Third, each node is assumed to separately receive a copy from its sender in MEM, while MRM allows SSs to receive as long as the channel quality permits. Given these differences, we know that though MEM may be applied on adhoc networks, MRM is more suitable for providing multicast services in a wireless metropolitan area network such as WiMAX.

In this paper, we introduce a new problem called Multicast Recipient Maximization (MRM) and propose a Dynamic Station Selection (DSS) scheme. Given the budget of a multicast program and the channel quality of all nodes, MRM aims to maximize the number of recipients. We prove that MRM is NP-hard, while our DSS has polynomial-time computational complexity. The performance and efficiency of our scheme is evaluated via analysis and simulations.

The remainder of the paper is organized as follows. In Section II, we formally define the MRM problem, describe the system model, and present the DSS algorithm. In Section III, we analyze the difficulty of MRM and the computational complexity of DSS. The performance of DSS is simulated in Section IV. We then summarize our findings in Section V.
II. MULTICAST RECIPIENT MAXIMIZATION (MRM) PROBLEM

A. Network Model

Let there be $M$ RSs and $N$ SSs in a WiMAX relay network. Each SS either connects the BS directly or via a RS. The BS/RS each SS connected can be predetermined by any existing path-selecting scheme; our scheme then bases on the given routes to decide the allocation. SSs can be classified into many groups according to their associated senders. Let group 0, i.e., $S_0 = \{s_{0,0}, s_{0,1}, \ldots, s_{0,N_0}\}$ denote the set of SSs directly served by the BS, and let $S_m = \{s_{1,m}, s_{2,m}, \ldots, s_{N_m, m}\}$, $m = 1, 2, \ldots, M$ be the SSs served by the $m$th RS, where $s_{n,m}$ represents the $n$th SS in the $m$th group, and $N_m$ is the number of SSs in $S_m$.

Next we consider the radio channel. Since the channel quality of each node varies, each BS/RS needs different amounts of resources to successfully transmit the multicast stream to each node. Take a WiMAX network for example, the channel resource can be the timeslots in a TDD super frame preserved for a multicast program. The required timeslots for each node to receive the same stream may differ because they have different channel conditions, and thus need different modulation schemes and transmission rates.

Let $R_m = \{r_{1,m}, r_{2,m}, \ldots, r_{N_m, m}\}$ be the required resources of SSs in group $m$, in which $r_{n,m}$ denotes the required resource for $s_{n,m}$. To be better organized, we let all SSs in a group be placed in increasing order of $r_{n,m}$, i.e., $r_{1,m} \leq r_{2,m} \leq \ldots \leq r_{N_m, m}$ for $m = 0, 1, 2, \ldots, M$. As for RSs, let $P = \{p_1, p_2, \ldots, p_M\}$ denote the required resources for the $m$th RS to receive from BS. Fig. 1 depicts BS, RSs and SSs in a WiMAX relay network.

Since the wireless medium is naturally broadcast, if the BS multicasts a stream with resource $r_0$, all SSs in group 0 with required resource $r_{n,0} \leq r_0$ should be able to receive it. However, for SSs in groups $m > 0$ to receive the stream, two conditions have to be satisfied: 1) the RSs can receive the multicast stream from the BS (i.e., $p_m \leq r_0$); and 2) the SSs can receive the stream from the RSs (i.e., $r_{n,m} \leq r_m$), where $r_m$ denotes the resource used by the $m$th RS to relay the stream.

Therefore, the number of served SSs in group $m$ can be formulated as $D_m(r_0) \cdot N_m(r_m)$, where $N_m(r)$ represents the number of SSs that can receive the stream from their group sender with resource $r$, while $D_m(r)$ indicates whether the sender of group $m$ can receive the stream from the BS which transmits with resource $r$. Thus, for $m > 0$, $D_m(r) = \begin{cases} 1, & r > r_m \\ 0, & \text{otherwise} \end{cases}$, where $D_0(r) = 1$.

B. Problem Definition

We formally define the MRM problem as follows: given the channel conditions of all RSs and SSs, (i.e., $p_m$ and $r_m$, $m = 1, 2, \ldots, M$) MRM finds the allocated resource for the BS and RSs, (i.e., $r_0$, $r_1$, $\ldots$, $r_M$), to maximize the total number of recipients $\sum_{m=0}^{M} D_m(r_0) \cdot N_m(r_m)$, subject to $\sum_{m=0}^{M} r_m < r_{total}$, where $r_{total}$ denotes the resource budget for the service.

C. Notations

The notations used in this paper are listed as follows.

- $M$: the number of RSs
- $N$: the number of SSs
- $N_m$: the number of SSs in group $m$
- $S_m = \{s_{1,m}, s_{2,m}, \ldots, s_{N_m, m}\}$: SSs in group $m$
- $R_m = \{r_{1,m}, r_{2,m}, \ldots, r_{N_m, m}\}$: $S_m$’s required resource
- $P = \{p_1, p_2, \ldots, p_M\}$: RSs’ required resource
- $D_m(r)$: the Boolean indicator showing whether the $m$th RS can receive the stream if BS transmits with resource $r$
- $N_m(r)$: the number of served SSs in group $m$ if the group sender transmits the stream with resource $r$
- $r_m$: allocated resource for the $m$th sender
- $r_{total}$: the resource budget of the multicast stream
- $r_{res}$: the residual available resource
- $U_{max}$: the maximal allocation utility in each round
- $U_{max}$: the maximal allocation utility
- $m_{max}$: the allocation step (i.e., to include the max $m$ SSs)
- $n_{max}$: the number of SSs in group $m_{max}$ that yields the maximal average utility in each round of the algorithm
- $d_{n,m}$: the distance between the $m$th RS and its $n$th SS

D. Dynamic Station Selection (DSS) Scheme

We propose a resource allocation scheme called Dynamic Station Selection (DSS) in Fig. 2 to deal with MRM. The resource allocated to the BS and RSs (i.e., $r_m$) is initialized as 0. In each round of the do loop, the algorithm searches for all unserved SSs and selects the one that maximizes the allocation utility, which is the ratio of the “incrementally included SSs” to the “incrementally allocated resource.”
In each round, DSS calculates the average allocation utility of each group. It searches for the node that maximizes the allocation utility (i.e., the \( \text{max}_{th} \) SS of group \( \text{max}_{th} \)) and includes it into the service. The loop continues until no more additional SSs can be served, which means that either all nodes are served (i.e., \( U_{\text{max}} = 0 \)) or the resource is exhausted.

To calculate the allocation utility, three different situations are considered. For group 0, if the BS is already allocated with \( r_0 \), the utility of “including the \( n_{th} \) SSs in group 0” is \( \frac{n - N_0(r_0)}{r_{n,0} - r_0} \), as shown in Fig. 3(a). Fig. 3(b) shows the condition that \( m > 0 \) and the \( m_{th} \) RS is not yet served. In that case, the average utility of including the \( n_{th} \) SSs in group \( m \) becomes \( \frac{[N_0(p_m) - N_0(r_0)] + n}{(p_m - r_0) + r_{n,m}} \), where \( N_0(p_m) - N_0(r_0) \) refers the extra served SSs in group 0 when the \( m_{th} \) RS is included, while \( p_m - r_0 \) is the additional resource for the BS to serve the \( m_{th} \) RS. On the other hand, if \( m > 0 \) and the \( m_{th} \) RS is already served, as shown in Fig. 3(c), the utility value is \( \frac{n - N_m(r_m)}{r_{n,m} - r_m} \), where \( r_m \) is the allocated resource of the \( m_{th} \) RS in the former round.

![Figure 2. The Dynamic Station Selection Scheme](image)

**Figure 2. The Dynamic Station Selection Scheme**

We prove that MRM is NP-hard in theorem 1, and analyze the time complexity of DSS in theorem 2.

**Theorem 1 MRM is NP-hard**

**Proof:**

A well-known NP-complete problem called integral knapsack is considered. Let \( V_i \) and \( W_i \) be the value and weight of the box \( i \) respectively. The integral knapsack problem maximizes the total value of chosen boxes, i.e., \( \sum V_i \), subject to the total weight of the chosen boxes not exceeding the...
weight limit (denoted by $W$), i.e., $\sum_i W_i \leq W$, $\forall i, V_i, W_i \in Z$.

Our MRM problem can be reduced from a conventional integral knapsack problem because any box $(V_i, W_i)$ can be transformed into a group with $V_i$ SSs, $r_m = W_i$ and $p_i = 0$. So any instance (i.e., a set of boxes and the weight limitation) in the integral knapsack can be mapped to an instance of MRM (i.e., a set of groups and the resource budget) in polynomial time. It follows that the MRM is more general than the integral knapsack problem. Therefore, MRM is also NP-hard.

**Theorem 2** The complexity of DSS is polynomial-time.

**Proof:** In DSS, the complexity of the first for loop is $O(M)$. The do loop executes at most $N$ times because it includes at least one SS in each round. For each round, the complexity is $O(N)$ since it searches among all unserved nodes. Therefore, the total complexity of DSS is $O(M + N \cdot N)$. In general, the number of RSs in a relay network is far fewer than the number of SSs, so the complexity can be reduced to $O(N^2)$.

**IV. PERFORMANCE EVALUATION**

**A. Simulation Setting**

We conduct simulations to evaluate the performance. All RSs and BSs are randomly placed in each run. We put the BS at $(0,0)$ and place SSs in a square area ranging from $(-100,-100)$ to $(100,100)$. Next, we let the $x$ and $y$ coordinates of the RSs uniformly distribute in $(-75,-25)$ and $(25, 75)$ because the carrier tends to place RSs in an efficient position. Fig. 4 shows the node placements in the simulations.

A common fading model $d^{-i}$ is used to derive the transmission rate of each channel, where $d$ is the distance between the sender and the receiver, and $i$ is the attenuation factor between 2 and 4. Note that this channel setting is only for simulation convenience, and DSS can adopt different fading models as long as the value of $r_m$ is known. After placing all nodes and calculating the channel quality, we decide the route of each SS by choosing the one that minimizes total required resource. The value of $i$ is set to 2 in the following simulations. From some pilot results, we find that the value of $i$ does not have an obvious impact on system behavior and performance.

We compare DSS with two other approaches, namely OP and GD. OP is the optimal solution of the MRM problem. Since MRM is NP-hard, OP is calculated by brute force. GD is a simple greedy algorithm that only considers the next unallocated SS, and chooses the one with the maximum allocation utility in each step of the allocation.

**Figure 4. Node Placements in the simulations**

**Figure 5. Number of recipients under different budgets**

**B. Simulation I**

In the first simulation, we fix $M$ and $N$ at 5 and 100 respectively, and tune the value of $r_{total}$ to observe the number of served SSs of the three approaches. Since the network topology is randomly generated, we conduct 100 runs for each setting and average the outcome.

As shown in Fig. 5(a), the results demonstrate that the performance of DSS stays very close to OP. Also, given different budgets, DSS always outperforms GD. This is because DSS searches for the allocations with maximal average utility among all unallocated SSs, while GD only considers the first unallocated SS in each group, which is the subset of DSS’s allocation options.

To observe resource utilization more clearly, we normalize the performance difference to OP, as shown in Fig. 5(b). As...
increases, the performances of DSS and GD converge because they both include SSs until all subscribers have been served. However, the difference of DSS converges much faster because it DSS utilizes radio resources more efficiently.

The value of curves is averaged over reason is that a larger difference between OP and GD increases obviously. The performance of DSS stays close to that of OP, while the other hand, GD performs worse under high node density because the options of GD are limited.

Next, we observe the impact of the nodes number on the performance. The value of $r_{total}$ is fixed at 20,000 and the value of $N$ is tuned from 100 to 1,000. Again, each point on the curves is averaged over 100 runs. As shown in Fig. 6(a), the number of served SSs appears to be linearly proportional to the total SSs. This is because the higher density of SSs yields more served recipients.

Again, we normalize the performance difference, as shown in Fig. 6(b). The results show that as $N$ increases, the performance of DSS stays close to that of OP, while the difference between OP and GD increases obviously. The reason is that a larger $N$ offers DSS richer allocation choices, providing it more flexibility to improve performance. On the other hand, GD performs worse under high node density because the options of GD are limited.

Because MRM is NP-hard, readers may be interested to know the computation time. It takes minutes to hours to solve a single instance of OP. Since the allocation should be efficient enough to reflect the channel condition in real time, finding the optimal solution in a real environment, which may have many multicast services and numerous SSs, is impractical. The results also show that GD does not perform well under large number of nodes. Therefore, our proposed DSS, which has feasible complexity and good performance, is the only feasible choice.

V. CONCLUSIONS AND FUTURE WORK

In this work, we consider an important resource allocation problem called MRM for multicast over WiMAX relay networks. With given resource budget, it maximizes the number of recipients. We prove that MRM is NP-hard, and design a heuristic called DSS. The performance of the proposed DSS is evaluated via simulations. Under different resource budget and node density, DSS always outperforms the simple greedy approach and have nearly optimal performance. Since finding the optimal solution is time-consuming and the simple greedy approach yields a poor performance, our proposed DSS is the only feasible solution to this problem.

In the future, we will consider supporting relay networks with more hops. We will also try integrating the resource allocation scheme with scheduling algorithms and call admission control scheme to develop an integral resource allocation scheme for wireless relay networks.

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