Maximizing Capacity in the SINR model in Wireless Networks with Successive Interference Cancellation

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Abstract—Successive interference cancellation (SIC) is an effective way of multipacket reception to combat interference. We consider the problem of maximizing the number of successful transmissions based on the physical model in wireless networks with SIC at the physical layer. We propose weighted simultaneity graph to characterize the sequential detection nature of SIC and the cumulative effect of multiple interfering signals. A context-aware metric, transmission cost, is defined to measure the interference of a link set. As maximizing the number of supported links is NP-hard, a greedy scheme is proposed to efficiently construct a near-optimal maximal feasible set of links. We show that the approximation performance is bounded by the transmission cost of the constructed link set. The performance of the proposed scheme is further verified by simulation.

Keywords—Transmission capacity; successive interference cancellation; physical interference model

I. INTRODUCTION

Modern wireless communication is interference-limited. Due to the broadcast nature, what arriving at a receiver is a composite signal consisting of all near-by transmissions. In general, the receiver tries to decode only one transmission by regarding all the others as interference and noise. When the arrivals of multiple transmissions overlap, collision occurs and the reception fails.

Successive interference cancellation (SIC) [1] is an effective way of multiple packet reception (MPR) to combat interference at the physical layer. With SIC, the receiver tries to detect multiple received signals using an iterative approach. In each iteration, the strongest signal is decoded, by treating the remaining signals as interference. If a required SINR (signal to interference and noise ratio) is satisfied, this signal can be decoded and removed from the received composite signal. In the subsequent iteration, the next strongest signal is decoded, and the process continues until either all the signals are decoded or a point is reached where an iteration fails.

Though significant progress has been made at the physical layer for wireless transmissions, little attention has been paid to the design of support protocols. As not all composite signals are decodable, it is indispensable to avoid harmful collisions (i.e., when the involved signals cannot be separated). In particular, as there are specific requirements to ensure the feasibility of an MPR method, it is necessary to coordinate the transmissions carefully to meet the requirements.

We consider the problem of maximizing the number of successful transmissions (refer to as transmission capacity) in the physical interference model in wireless networks with SIC at the physical layer. The aim is to choose a set of links as large as possible such that every intended receiver can successfully detect its desired signal when all links are active simultaneously. This problem is important both in its own right and also because it arises as a subproblem in many other areas of wireless networking. For example, in many link scheduling schemes, the overall schedule comprises several maximal feasible link sets [2].

Dealing with interference is one of the primary challenges in wireless communications. First, in order to extract a desired signal, all the detections in the previous iterations must be successful. That is, the detection of a weaker signal depends on that of the stronger ones. In consequence, to ensure the success of a transmission, both the far-away links (introducing weak interference) and the near-by ones (introducing strong interference) should be coordinated. Second, the aggregation effect makes it difficult to measure individual link interference. The impact of the interference of a link $L$ is determined not only by $L$, but also by the ones that transmit simultaneously with $L$. For the same link, with different set of concurrent links, the impact of its interference can be completely different. In comparison, when the interference is non-cumulative, e.g., in the protocol model, the impact of a given link is independent to the others.

To address the challenges, we make two contributions in this paper:

- We propose a weighted simultaneity graph to characterize the sequential detection nature of SIC and the cumulative effect of multiple interfering signals. Afterwards, we present a context-aware metric, transmission cost, to measure the interference of a link set.
We show that the problem of maximizing the transmission capacity is NP-hard and propose a greedy scheme to efficiently construct a near-optimal maximal set of feasible links. The approximation performance is bounded by the transmission cost of the constructed link set. We evaluate the proposed scheme by extensive simulation and show that it performs very well, e.g., the size of the link set is no less than 70% of the optimal one.

The rest of this paper is organized as follows. Section II overviews the related work and Section III describes the system model. Section IV presents the weighted simultaneity graph model and the greedy scheme. Section V presents the results of simulation. Finally, we conclude the research in Section VI. The proofs are given in Appendix.

II. RELATED WORK

Maximizing the transmission capacity has been studied in many contexts. In the literature, two main interference models have been proposed: the protocol and the physical interference models. The capacity of random networks in both the physical and the protocol models is examined in the seminal paper of Gupta and Kumar [3]. The approximation solution to approach the capacity of arbitrary networks in the protocol model has been considered in [4, 5]. For the physical model, the NP-hardness of the problem is proved in [6] and efficient approximation algorithms are given in [7] by assuming that transmitters can only broadcast at full power or not at all and in [6, 8] by choosing transmission power for each active node. In addition, recently, a distributed scheduling scheme for achieving network capacity is studied in [5].

Interference-aware protocol design in a network with MPR has only recently been considered. To deal with the MPR, the protocol model is enhanced by allowing more than one active link in an interference zone [9] and the physical model by allowing reception with a lower SINR threshold [10]. However, the models are too general to capture the sequential detection nature of SIC. Recently, in [2], the protocol model is extended to incorporate the effect of SIC and simultaneity graph is proposed to characterize the network interference. Also, in [11], topology control is studied in a multi-user MIMO network with SIC. To the best of our knowledge, this is the first work of maximizing transmission capacity based on the physical model in wireless networks with SIC.

III. SYSTEM MODEL

Consider a wireless network of $N$ stationary nodes and $n$ links. A link is denoted by $L_i$ with transmitter node $S_i$ and receiver node $R_i$, respectively, $i = 1, 2, \ldots, n$. We assume that: (i) the signal removal of SIC is perfect; (ii) each node has an omni-directional antenna, operates in the half duplex model, and is not able to transmit multiple packets simultaneously.

The physical model: To combat interference and noise, a minimum SINR is required to assure signal reception. Let $N_0$ denote the noise power and $P_{ij}$ the received signal power at the receiver node $R_i$ of link $L_i$ from the transmitter node of link $L_j$. Suppose there are $(J+1)$ links, e.g., $L_0, \ldots, L_J$ ($J \leq n - 1$). For link $L_k$ ($0 \leq k \leq J$), decoding succeeds if

$$\frac{P_{ik}}{N_0 + \sum_{0 \leq j \leq J, j \neq k} P_{kj}} \geq \beta_{ik}. \quad (1)$$

where $\beta_{ij}$ specifies the reception SINR threshold at node $R_i$ for the signal from node $S_j$. As the transmission rate and required transmission accuracy can be different among links, it is possible that $\beta_{ij} \neq \beta_{ik}$ when $j \neq k$.

The effect of SIC: Consider two links, $L_1$ and $L_2$. $L_1$ depending on $L_2$ means that the received power of $L_2$ at $R_1$ is sufficiently strong so that the signal of $L_2$ can be detected by $R_1$ in the presence of that of $L_1$. Afterwards, the signal of $L_2$ is removed to reduce the interference on $L_1$. $L_2$ is termed as a correlated link of $L_1$. The required condition is

$$\frac{P_{12}}{N_0 + P_{11}} \geq \beta_{12}. \quad (2)$$

Our interference model: Consider the reception of $L_d$. There are $J$ ($J \leq n - 1$) links active simultaneously with $L_d$ and $D$ ($D \leq J$) of them are correlated links of $L_d$. Without loss of generality, all the links are ordered with respect to the received power at $R_d$ as $L_{i_1}, \ldots, L_{i_D}$. Suppose $i_k = d$, then $P_{i_k i_1} \geq P_{i_k i_2} \geq \ldots \geq P_{i_k i_D}$ and the set of correlated links is $\{L_{i_1}, \ldots, L_{i_D}\}$. To successfully detect the signal of $L_d$, we require that

$$\frac{P_{i_k i_{x}}}{{\sum}_{(x+1)\leq j \leq D+1} P_{i_k i_j}} \geq \beta_{i_k i_{x}}, \forall x \leq D$$

$$\frac{P_{i_k i_{x}}}{{\sum}_{(x+1)\leq j \leq D+1} P_{i_k i_j}} \geq \beta_{i_k i_{x}}. \quad (3)$$

Eq. (3) indicates that, to examine whether a link can transmit successfully or not, one needs to: (i) order all links with respect to the received power at the intended receiver; (ii) for each correlated link, check whether it can be detected successfully after all the stronger signals are removed; and (iii) check whether the detection of the desired signal can tolerate the remaining aggregate interference.

IV. THE PROPOSED SOLUTION

To characterize the interference in the physical model in the network with SIC, we propose a weighted simultaneity graph and define a context-aware metric, transmission cost, to measure the interference of a link set. Afterwards, the NP-hardness of the problem of maximizing transmission capacity with SIC is shown and a greedy scheme is proposed with provable performance guarantee.

A. Weighted Simultaneity Graph

The interference is usually modeled by a network graph. A weighted conflict graph is used in [11, 12] to characterize the aggregate interference. However, it fails to capture the effect of SIC. Recently, in [2], a simultaneity graph is proposed to characterize the sequential detection nature of SIC in the protocol model. It ignores the effect of interference accumulation.

We use a weighted simultaneity graph to capture both the two effects. First, similar to the simultaneity graph,
weighted simultaneity graph deals with a link and its correlated ones together. Second, the edge weight represents the link-to-link interference. The aggregate interference is the sum of the ones together. Second, the edge weight represents the link-to-weighted simultaneity graph deals with a link and its correlated conflict graph, (b) weighted simultaneity graph.

In particular, let $SG_W = (V, E, W_V, W_E)$ denote a weighted simultaneity graph, where $V$ is the vertex set, $E$ is the set of directed edges, and $W_V$ and $W_E$ collect all the vertex weights and edge weights, respectively. There are two types of vertex in $SG_W$:

- An ordinary vertex (OV) contains a single link. For every link $L$, there is an OV $(L)$ in $SG_W$. Let $v_i = P_{ii}$ denote the weight of an OV $(L_i)$;

- For every link and each of its correlated links, a super vertex (SV) is constructed. The SV is order-aware. For example, both $(L_1L_2)$ and $(L_2L_1)$ are required if $L_1$ depends on $L_2$ and $L_2$ depends on $L_1$. Let $v_{ij} = P_{ij}$ denote the weight of a super vertex $(L_iL_j)$.

There are two types of edge in $SG_W$:

- There is an edge from $(L_2)$ to $(L_1)$ when $P_{12} < (P_{11} + N_0) \cdot \beta_{12}$, i.e., $L_2$ is not a correlated link of $L_1$. Let $e_{12}^i = P_{12}$ be the edge weight;

- There is an edge from $(L_2)$ to $(L_1L_2)$ when $P_{12} < (P_{11} + N_0) \cdot \beta_{12} \leq P_{12} \leq P_{11}$, i.e., both $L_2$ and $L_1$ are correlated links of $L_1$ but the signal of $L_3$ at $R_1$ is stronger. Let $e_{12}^i = P_{12}$ be the edge weight;

- When two vertices are disconnected in $SG_W$, the edge weight between them is set as zero.

It is clear to see that, in the $SG_W$ for the network of $n$ links, there are at most $(n^2 + n)$ vertices ($n^2$ SVs and $n$ OVs) and $O(n^3)$ edges. Let $LS$ be a link set. The relevant vertex set of $LS$, denoted by $RVS(LS)$, is defined as the subset of $V$ such that $\forall L \in LS$, $(L) \in RVS(LS)$ and $\forall (L_1L_2) \in V$, we have $(L_1L_2) \in RVS(LS)$ if $L_1 \in LS$ and $L_2 \in LS$.

An example is given in Fig. 1, where there are five links ($L_0$-$L_4$) in the network. In Fig. 1(a), the weighted conflict graph is shown. To draw the $SG_W$, suppose $P_{03} \geq \beta_{01} \cdot (P_{00} + N_0)$, $P_{01} \geq \beta_{01} \cdot (P_{00} + N_0)$, and $P_{03} > P_{01} > P_{00} > P_{02} \geq P_{04}$. First, in addition to the five OVs, two SVs ($L_0L_3$) and ($L_0L_4$) are required. Second, to represent the direct interference, two edges are established from $(L_2)$ and $(L_4)$ to $(L_0)$. Also, there is an edge between $(L_0L_3)$ and $(L_0)$ to represent the interference of $L_1$ on $L_3$ at $R_0$. The resulting weighted simultaneity graph is shown in Fig. 1(b).

Link $L (L \notin LS)$ is feasible with a link set $LS$ if the detection of $L$ is successful when $L$ transmits simultaneously with the links in $LS$. Moreover, a link set $LS$ is feasible when for every link $L' \in LS$, $L'$ is feasible with $LS - \{L'\}$.

Algorithm 1: Determine whether $L_i$ is feasible with $LS$

**Data:** $SG_W = (V, E, W_V, W_E)$: weighted simultaneity graph $LS$: a link set; $L_i$: a link not in $LS$

**Result:** TRUE if $L_i$ is feasible with $LS$, FALSE otherwise

1. Compute $\sum_{L_i \in LS} \epsilon_{iL}$, denoted by $I_0$;
2. if $I_0 > v_i / \beta_i - N_0$ then
   3. return FALSE;
4. end
5. foreach link $L_i$ in $LS$ do
   6. if $(L_iL_i) \in E$ then
      7. Compute $\sum_{L_i \in LS} \epsilon_{iL}^i$, denoted by $I_1$;
      8. if $I_0 + I_1 > v_i / \beta_i - N_0$ then
         9. return FALSE;
      10. end
   11. end
12. end
13. return TRUE;

B. Understanding the Interference

Understanding link interference is critical to the design of many network protocols. In the protocol model, the interference of a link is independent of the others and the interference relation between any two links is determined completely by themselves. In opposite, when the interference is cumulative, the impact of a link is tightly correlated to the set of links that are active concurrently. The set of links specifies the context of the link.

**Definition 1** The interfering link set of a link set $LS$, denoted by $I(LS)$, is the set of links that cannot transmit simultaneously with $LS$, i.e., $I(LS) = \{L | LS \cup \{L\} \text{ is not feasible}\}$.

Now, we present the context-aware metric, transmission cost, for an individual link and a link set. Namely, let $LS_1$ be a feasible link set. For a link $L \notin LS_1$, if $LS_1 \cup \{L\}$ is feasible, the transmission cost of $L$ is defined as follows.

**Definition 2** The transmission cost of $L$ to a link set $LS_2$ based on $LS_1$, denoted by $C_{LS_1}^{LS_2}(L)$, is the number of links in $LS_2$ that can transmit simultaneously with $LS_1$ but not within $LS_1 \cup \{L\}$. We have

$$C_{LS_1}^{LS_2}(L) = |T|$$

where $T = LS_2 \setminus (I(LS_1 \cup \{L\}) \setminus I(LS_1))$ and $|T|$ denotes the cardinality of set $T$.

In particular, $C_{LS_1}^{LS_2}(L) = 0$ when $LS_1 \cup \{L\}$ is not feasible. Let $P = L_{i_1} \rightarrow L_{i_2} \rightarrow \ldots \rightarrow L_{i_M} (M = |LS_1|)$. When $\{L_{i_1}, \ldots, L_{i_M}\}$
\(= LS_1\), we say that \(\mathcal{P}\) is a path of \(LS_1\), i.e., \(LS_1\) can be constructed from an empty set by adding \(L_i\) sequentially. Let \(S_p^1 = \emptyset\) and \(S_p^k = \{L_{i_1}, \ldots, L_{i_{k-1}}\}\) for \(2 \leq k \leq M\).

**Definition 3** The transmission cost of path \(\mathcal{P}\) of \(LS_1\) to \(LS_2\), denoted by \(C_{LS_1}^p(\mathcal{P})\), is the maximum of the transmission costs of all the links \(L_i \in LS_1\) to \(LS_2\) based on \(S_p\), i.e.,

\[
C_{LS_1}^P(\mathcal{P}) = \max_{1 \leq k \leq |LS|} C_{LS_k}^p(L_{i_k}).
\]

The transmission cost, \(C_{LS_1}^p(L_{i_k})\), measures the interference of \(L_{i_k}\) immediately after it is added into \(LS_1\) along path \(\mathcal{P}\).

**Definition 4** The transmission cost of \(LS_1\) to \(LS_2\), denoted by \(C_{LS_1}(LS_1)\), is the minimum of the transmission costs of all paths of \(LS_1\) to \(LS_2\), i.e.,

\[
C_{LS_1}(LS_1) = \min_{\mathcal{P}} C_{LS_1}^p(\mathcal{P}).
\]

where \(\Delta^p\) is the set of all paths of \(LS_1\).

For a link set of size \(M\), there are \(2^M\) different paths. An exhaustive search to find the transmission cost of a link set has exponential complexity. It is our ongoing work to find an efficient way to compute the transmission cost of a link set.

Note that the transmission cost of a link with \(LS_1 = \emptyset\) is the link interference in [12] and the opportunity cost in [11]. Particularly, in the protocol model, the transmission cost of the link interference in [12] and the opportunity cost in [11].

C. The Problem Hardness and the Greedy Policy

We first state the hardness result of the problem.

**Theorem 1** Maximizing transmission capacity based on the physical model in a wireless network with SIC is NP-hard.

Due to the NP-hardness of the link scheduling problem, we turn our attention to an approximation algorithm, to construct a link set which is a maximal feasible set of links, i.e., no more link can be added. In comparison, the optimal solution reports the maximum set, i.e., there is no other feasible set with a larger size. To measure the approximation performance, an efficiency factor is defined.

**Definition 5** For an approximation scheme \(X\), the efficiency factor, denoted by \(\alpha_X\), is the ratio of the number of links selected by the optimal algorithm to that by \(X\).

Via the transmission cost, the performance can be bounded for a scheme that finds a maximal link set.

**Theorem 2** Let \(S_X\) be the link set constructed by an approximation scheme \(X\). If \(S_X\) is maximal, then \(\alpha_X \leq C_{B_0}(S_X) + 1\), where \(B_0\) is the set of all links.

Next, we present a greedy scheme to achieve good performance. Basically, a link with the lowest transmission cost is preferred in order to lower the transmission cost of the link set to be formed. Let \(S\) be the set of links already chosen, \(B\) be a set of links such that every link in \(B\) is feasible with \(S\), and \(\mathcal{R}\) be the set of links that are not feasible with \(S\). Algorithm 2 shows the greedy scheme, which iteratively constructs the link set. In each iteration, it works as follows:

- Compute the transmission cost of every link \(L \in B\) to \(B - \{L\}\) based on \(S\);
- Choose the first link with the lowest transmission cost;
- The chosen link is moved from \(B\) to \(S\);
- Update the set of candidate links. After a new link is added into \(S\), there are some links in \(B\) that are no longer feasible with \(S\). The algorithm finds all such links and moves them from \(B\) to \(\mathcal{R}\);
- Update the weighted simultaneity graph. First, as the links in \(\mathcal{R}\) cannot be added into \(S\), all vertices such as \((L)\) \((L \in \mathcal{R})\) or \((L_1L_2)\) \((L_1 \in \mathcal{R}\) or \(L_2 \in \mathcal{R}\)) as well as the incident edges are no longer accessed by the algorithm and can be deleted from \(SG_W\). Second, reset the vertex weight to avoid any repeat computation. For \((L_i)\), let \(v_i \leftarrow v_i - \sum_{L \in S} e_i^L\), and for \((L_iL_j)\), let \(v_{ij} \leftarrow v_{ij} - \sum_{L \in S} e_{ij}^L\). Third, all edges issued from \((L)\) \((L \in S)\) are removed.

**Algorithm 2: GreedyMC - A Greedy scheme to find the approximated maximum set of feasible links**

- **Data:** \(SG_W = (V, E, W_i, W_{ij})\): weighted simultaneity graph
- **Result:** A maximal feasible link set \(S\)

\begin{align*}
1 & SG_W \leftarrow SG_W; \\
2 & S \leftarrow \emptyset; \\
3 & B \leftarrow \text{All links}; \\
4 & R \leftarrow \emptyset; \\
5 & \text{repeat} \\
6 & \quad \text{For every link } L \in B, \text{ compute } C_{S \leftarrow L}(L). \text{ Let } L_0 \text{ be the first link with the lowest transmission cost;} \\
7 & \quad \text{Move } L_0 \text{ from } B \text{ to } S; \\
8 & \quad \text{foreach link } L \in B \text{ do} \\
9 & \quad \quad \text{if set } S \cup \{L\} \text{ is not feasible then} \\
10 & \quad \quad \quad \text{Move } L \text{ from } B \text{ to } R; \\
11 & \quad \quad \text{end} \\
12 & \text{end} \\
13 & \text{Update } SG'; \\
14 & \text{until } B = \emptyset; \\
15 & \text{return } S;
\end{align*}

It is clear that the link set determined by the scheme \(GreedyMC\) is feasible and maximal. Therefore, the performance is guaranteed by Theorem 2. Below, we state the time complexity of the greedy scheme.

**Theorem 3** Given \(SG_W = (V, E, W_i, W_{ij})\), the time complexity of Algorithm 2 is at most \(O(n^3)\), where \(n\) is the number of links in network.

V. PERFORMANCE EVALUATION

We evaluate the performance of the proposed scheme by simulation in network simulator-2 (NS-2) [13]. Table I summarizes the parameter/protocol settings in simulation. Each data point is obtained by averaging the results from a number (e.g., 10) of runs. The confidence interval is 95%.

We have enhanced the functions in NS-2 to support the SINR-based reception and SIC [2]. A transmission is successful when it satisfies the condition in (3). The transmission...
Fig. 2: Throughput distribution versus the number of links in a one-hop network.

range in Table I is the maximum distance of a link such that the reception is successful in the absence of any other concurrent transmission.

The performance metric is network throughput, i.e., the number of allowed concurrent transmissions. At the beginning of a slot, each source node has a packet to transmit with a probability $\eta$. The probability is the same for all source nodes.

Fig. 3: Average throughput versus transmission probability in a network with 64 nodes at uniform grids.

Fig. 4: Average throughput versus the number of links in a network with 100 nodes at random positions.

TABLE I: Parameters in simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Protocol</th>
<th>Value/Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>64 or 100</td>
<td>1500Bytes</td>
</tr>
<tr>
<td>Transmission range</td>
<td>250m</td>
<td>Two-way ground</td>
</tr>
<tr>
<td>Transmission rate</td>
<td>2Mbps</td>
<td>TDMA</td>
</tr>
<tr>
<td>SINR threshold</td>
<td>10</td>
<td>Shortest path</td>
</tr>
</tbody>
</table>

We first evaluate the performance in single-hop scenarios. There are 100 nodes randomly positioned in a $150m \times 150m$ area. When the receiver nodes do not have the SIC capability, no simultaneous transmission is permitted. We randomly choose node pairs to construct $n$ links. The experiment is repeated 100 times at a given $n$ with different sets of chosen links. The maximal link sets reported by scheme GreedyMC are divided into four categories: (i) only one transmission; (ii) two concurrent transmissions; (iii) three concurrent transmissions; and (iv) four or more concurrent transmissions. Fig. 2 shows the distribution of throughput for different link numbers (i.e., $n$ varies from 10 to 50) in a one-hop network. First, there are a significant number of transmission opportunities resulting from SIC. When the number of links is large, e.g., 30 or more, it is highly possible to find two or more links that can transmit simultaneously. Second, when the number of links increases, GreedyMC is able to support more concurrent links. For example, when there are a total 10 links, in more than 70% cases, only one link is supported. In comparison, when the number of links is 30 or more, there are at least two links active simultaneously in more than 50% cases. The results indicate the effectiveness of SIC in supporting concurrent links and GreedyMC in exploiting the transmission opportunity.

To investigate the potential advantage of SIC in a large network, we conduct simulation of a network with 64 nodes at uniform grids in a $500m \times 500m$ area. For each active node, the receiver node is randomly chosen among the nodes within the transmission range. Fig. 3 shows the average throughput versus transmission probability ($\eta$). As a reference, we also show the results of the optimal algorithm. One can solve the problem of maximizing transmission capacity via dynamic programming with exponential complexity (please refer to [11]). Along with the increase of $\eta$, a larger number of links become active. As a result, it is possible to find more concurrent transmissions. On the other hand, the number of supported links when $\eta = 0$ is almost the same as that when $\eta = 0.8$. At this time, the network is to some extend saturated, adding more links cannot improve the overall capacity but only lowers the per-link capacity. As compared to the optimal algorithm, the greedy scheme performs very well. The size of the maximal link set found by GreedyMC is no less than 75% of the maximum size.

Finally, simulations are carried out for a 100-node network in a $1000m \times 1000m$ area. All nodes are randomly positioned. We randomly choose 20-50 nodes as the source nodes and then pick the receiver node randomly for each source node. As some of them need a multi-hop route, the total number of active links is between 28 and 78. Fig. 4 shows the average throughput for different link numbers when $\eta = 1$. In most cases, the difference between the throughput of GreedyMC
and the maximum size is less than 30%. The results indicate that the proposed scheme can effectively exploit transmission opportunities in a network with SIC.

VI. CONCLUSIONS

This paper studies the problem of maximizing the number of concurrent transmissions based on the SINR model in a wireless network with SIC. We characterize via weighted simultaneity graph the effect of aggregate interference and the link correlation introduced by SIC. We show that the problem of maximizing the number of concurrent links is NP-hard and propose a greedy scheme to find a near-optimal maximal feasible link set. Transmission cost is defined to measure the interference of a link set. The performance of an approximation scheme is bounded by the transmission cost of the link set that it constructs. Simulation results demonstrate that using our proposed scheme, the network throughput is no less than 70% of that using the optimal one, in most cases.

APPENDIX

Proof of Theorem 1: For any given link \( L_i \) and every \( j \neq i \), let the threshold \( \beta_{ij} \to \infty \). Then, \( L_i \) is unable to decode any signal other than the desired one. The problem under consideration reduces to that without SIC, which is NP-hard [6]. Our problem, as a more general case, is also NP-hard.

Proof of Theorem 2: Let \( B_0 \) be the set of all links and \( P = L_i \to \ldots \to L_{i_m} (M = |S_X|) \) be a path of \( S_X \) such that \( C_{S_X}^b(P) = C_{S_X}(S_X) \). Let \( S_p^b = \emptyset \), then we process the links in \( B_0 \) as follows: At the \( k \)-th step, add \( L_{i_k} \) to \( S_p^k \) to form \( S_p^k \), and delete all links in \( T(S_p^k) \) from \( B_0 \). As \( S_X \) is maximal, according to Definitions 2 and 3, we have the following:

- At the \( k \)-th step, there are exactly \( C_{S_X}^k(L_{i_k}) \) links removed from \( B_0 \);
- After \( |S_X| \) steps, \( B_0 \) is emptied; Otherwise, more links can be added into \( S_X \).

In summary, the size of \( B_0 \) can be expressed as:

\[
|B_0| = \sum_{1 \leq i \leq |S_X|} (C_{S_X}^i(L_{i_k}) + 1) \leq |S_X| \cdot (C_{S_X}^i(P) + 1) = |S_X| \cdot (C_{S_X}(S_X) + 1).
\]

As the number of links selected by the optimal solution is at most \(|B_0|\), we have

\[
\alpha_X \leq \frac{|B_0|}{|S_X|} \leq C_{S_X}(S_X) + 1.
\]

Proof of Theorem 3: We divide each iteration (e.g., the \( i \)-th iteration) into four steps. At the end of the \( i \)-th iteration, let \( B_i \) and \( S_i \) be the latest \( B \) and \( S \), respectively. In particular, \( B_0 \) is the set of all links and \( S_0 = \emptyset \).

- First, compute the transmission cost \( C_{S_{i-1}^j \setminus L}(L) \) for every \( L \in B_{i-1} \). The process, in the worst case, needs to access all vertices in \( RV(S_{i-1} \cup B_{i-1}) \) and the edges connecting them. Thus, the time complexity is no more than \((|V|+|E|)\).
- Second, in the foreach block (lines 8-12), it at most needs to access all vertices in \( RV(S_{i-1} \cup B_{i-1}) \) and the edges connecting them. The time complexity is also bounded by \((|V|+|E|)\).
- Third, the time complexity of updating the \( SG_{iw} \) is at most \((|V|+|E|)\).
- Finally, the other operations (i.e., line 7) needs \( O(1) \) time. Adding them together, the time complexity in one iteration is upper bounded by \( O(|V|+|E|) \). As there are in total \(|S| \) iterations, the overall time complexity of Algorithm 2 is bound by \( O((|V|+|E|) \cdot |S|) \leq O(|V|+|E|) \cdot n = O(n \cdot |E|) \leq O(n^4) \), where \( n \) is the number of links in the network.

REFERENCES