Using a three-dimensional particle-tracking model to estimate the residence time and age of water in a tidal estuary

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A B S T R A C T

A three-dimensional hydrodynamic model that includes a Lagrangian particle-tracking simulation was applied to the Danshuei River estuarine system in northern Taiwan. The model’s accuracy was validated with data from 1999; the results from the model agreed well with empirical observations of water surface elevation, tidal currents, and salinity. The validated model was then used to investigate the residence time and water age in response to different levels of freshwater discharge. A regression analysis of the model results revealed that an exponential equation best explained the correlation between residence time and freshwater input. We found that the residence times during the low and high freshwater discharge episodes were 4.4 and 2.5 days, respectively. The water age during the low–flow periods was greater than that during the high–flow periods. The modelled residence time and water age values without density-induced circulation were higher than those with density-induced circulation, which indicates that density-induced estuarine circulation may play a significant role in the estuary.

1. Introduction

Many estuaries and coastal seas have become increasingly eutrophic over the past few decades as a result of increased anthropogenic nutrient inputs (Carpenter et al., 1998; Nixon, 1995; Kemp et al., 2005). Both the volume of the nutrients discharged into estuaries and the nutrient retention time have contributed to the eutrophic conditions currently observed in estuaries. It has been suggested that retention time is a key parameter in the control of nutrient budgets (Boynton et al., 1995). Characteristics of the processes that transport a dissolved substance not only depend on flood and ebb tidal flows, but also on the low-frequency residual flow that is influenced by interactions between baroclinic currents, river flows, wind, and the non-linear rectification of periodic tides in an estuary. Thus, it is difficult to separate and quantify the influences of different mechanisms on long-term transport (Shen and Wang, 2007).

The residence time is the average time that a dissolved or suspended material resides in the estuary before it is transported into the open sea. It is a conventional parameter that represents the timescales of physical transport processes and is often compared with the timescales of biogeochemical processes. Studies have used estuarine residence times to assess nutrient exports and imports (Dettmann, 2001) and to quantify the possible effects of changes in nutrient loads on nutrient concentration levels (Kelly, 1997), estimate chlorophyll concentrations (Monbet, 1992), and estimate primary production (Jorgensen and Richardson, 1996). These studies have demonstrated the need for a standard measure of residence time to allow for comparisons between estuary types.

The water age of a particle is defined as the time that has elapsed since the particle entered the river or estuary (Takeoka, 1984). Age represents the time it takes for a dissolved or suspended material at any location to be transported from its source to its current location (Delhez et al., 1999). Therefore, the concept of the age of a water parcel has been used as a timescale to quantify the transport of pollutants in lagoons, estuaries, and oceans (Takeoka, 1984; Delhez et al., 1999; Deleersnijder et al., 2001; Monsen et al., 2002; Shen and Haas, 2004; Shen and Lin, 2006; Shen and Wang, 2007; Chen, 2007; Gong et al., 2009). Shen and Haas (2004) and Shen and Lin (2006) studied the age of water in the tidal York and James rivers, respectively, and found that water age was strongly dependent on the upstream freshwater discharge.

Many practical estuarine management cases have used sophisticated models to accurately estimate important transport timescales. For example, Delhez et al. (1999) introduced an age theory based on the advection–diffusion of a tracer and provided a general methodology for computing age using numerical models. Huang and Spaulding (2002) and Huang (2007) used a three-dimensional hydrodynamic model to calculate salinity transport and then used the fraction of freshwater in the system to compute the residence...
time. Deleersnijder et al. (2001) successfully simulated the age of technetium-99 released from the La Hague nuclear fuel reprocessing plant in the English Channel using a three-dimensional model. Beckers et al. (2001) studied the isotropy of the age field in a homogeneous environment, regardless of the values of the diffusion and velocity or the discharge function. They presented a simple method with a standard stationary tracer calculation that evaluates the concentration distribution function and the age of water without having to solve additional differential equations. Shen and Haas (2004) used a three-dimensional hydrodynamic-eutrophication model to simulate the transport of conservative tracers and then to estimate the mean ages of the tracers at different times and in different locations. Dabrowski and Hartnett (2008) used a three-dimensional barotropic and baroclinic numerical model to simulate travel and residence times in the eastern Irish Sea.

Particle-tracking models have been widely used to simulate the dispersion of passive tracers (Gomez-Gesteira et al., 1999; Chen et al., 2002; Blumberg et al., 2004; Stentchev and Korotenko, 2005; Suh, 2006; Thonon et al., 2007; Gong et al., 2008), larvae dispersal (Murray and Gillibrand, 2006; Tilburg et al., 2007), radionuclides (Perianez and Elliott, 2002; Nakano and Povinec, 2003; Perianez and Pascual-Granged, 2008), oil spills (Korotenko et al., 2004; Perianez, 2004; Wang et al., 2005), and even contaminated milk (Elliott et al., 2001) in both estuaries and coastal waters. Burwell et al. (1999) used three-dimensional Eulerian and Lagrangian models to calculate the residence time in Tampa Bay, Florida. Chen (2007) presented a laterally averaged two-dimensional trajectory model for narrow rivers and estuaries to calculate transport timescales in the Alafia River estuary. Oliveira et al. (2006) and Dias et al. (2009) used a three-dimensional barotropic and baroclinic numerical model in conjunction with a two-dimensional particle-tracking model, VELApart, to compute residence times for passive tracers in a lagoon in Portugal. The advantage of Lagrangian methods is that they are suitable for simulating the transit time between point sources of pollution and parameters, such as estimated water age, at a number of different locations.

In the present study, a three-dimensional high-resolution hydrodynamic model with a Lagrangian particle-tracking approach was developed to estimate transport times in the Danshuei River estuarine system of northern Taiwan. Model validation was conducted using tidal elevation and salinity measurements from 1999. The model was then applied to estimate the estuarine residence times and the age of water in the estuarine system.

2. Description of study site

The Danshuei River, located on the outskirts of Taipei, is the largest estuary in northern Taiwan. The Danshuei River is formed by the confluence of the Tahan Stream, the Hsintien Stream, and the Keelung River (Fig. 1). The Tahan Stream joins the Hsintien Stream at Wan-Hwa. The river then combines with the Keelung River at the Kuan-Du. The segment of the estuary from Wan-Hwa to the estuary's mouth is called the Danshuei River. The downstream portions of all three tributaries are influenced by the tide and are subject to seawater intrusion. The upriver reaches are affected by daily variations in freshwater discharge rates. The length of the main stem is 159 km, and the mean annual discharge is 210 m$^3$/s (Liu et al., 2007a, 2007b). The drainage area encompasses 2728 km$^2$, with a total channel length of 327.6 km. The major portion of the estuarine system, upriver of Kuan-Du (Fig. 1), lies within the Taipei basin, whereas the downriver stretch is confined by high mountains on both sides. There is no significant wind-induced current because of the presence of these mountains and the narrowness of the river (Liu et al., 2007b). Except for occasional storm surges induced by hurricanes, the major forcing mechanisms of the barotropic flow include the astronomical tide at the mouth of the river and the discharge of the river at the upriver end. Semi-diurnal tides are the principal tidal constituents, with a mean tidal range of 2.22 m and a spring tidal range of 3.1 m (Hsu et al., 1999). The average river discharges are 62.1, 72.7, and 26.1 m$^3$/s in the Tahan Stream, the Hsintien Stream, and the Keelung River, respectively. In addition to barotropic flow forced by tide and river discharges, the baroclinic flow forced by seawater intrusion is another important transport mechanism in the Danshuei River estuarine system (Liu et al., 2004).

Six-million people, over a quarter of Taiwan’s entire population, live in the catchment area of the Danshuei River system. The population density in the most densely populated region of the Taipei metropolitan area, which is near the Danshuei River estuary, is as high as 9684 people/km$^2$. The Danshuei River estuary receives untreated domestic discharge as well as both treated and untreated industrial effluent from its tributaries; it is heavily polluted by trace metals and organic materials (Jeng and Han, 1994). Approximately 20.72 m$^3$/s of domestic sewage, which is largely untreated, enters the Danshuei River system and, along with industrial waste effluent, contributes to hypoxia in the upper estuary. Diatoms are usually the most abundant taxonomic group in the estuary throughout the year, whereas the abundance of chlorophytes and cyanobacteria is higher in the upriver regions (Wu and Chou, 2003).

The estuarine hydrodynamic processes of the Danshuei River system involve tidal flow in the presence of complex geometry and bathymetry, which significantly impact flow and mass transport around estuary structures and dredge spoils (Wu, 1997). Determining the estuarine residence time and the age of water particles in this complicated and partially mixed estuarine system requires the use of three-dimensional hydrodynamic models that can effectively handle flows induced by tides, freshwater discharges, and density gradients.

3. Model description

3.1. Governing equations

The numeric modelling of ocean circulation at scales ranging from estuaries to ocean basins is becoming a mature field. A plethora of model codes for ocean circulation models are available, many of which are open-source. Most modern oceanic and estuarine circulation codes solve for some form of the three-dimensional...
Navier–Stokes equations and can be complemented with conservation equations for a given water volume and salt concentration. Common codes use either structured (POM, Blumberg and Mellor, 1987; HEM-3D, Hamrick, 1996) or unstructured (UnTRIM, Casulli and Walters, 2000; FVCOM, Chen and Liu, 2003; ELCIRC, Zhang et al., 2004) grids and are typically based on finite differences, finite elements, or hybrid approaches involving finite volumes.

In this paper, a three-dimensional, semi-implicit Eulerian–Lagrangian finite-element model (SELFE, Zhang and Baptista, 2008) was implemented to simulate the Danshuei River estuarine system and its adjacent coastal sea. SELFE solves the Reynolds-stress averaged Navier–Stokes equations that use conservation laws for mass, momentum, and salt, under the hydrostatic and the Boussinesq approximations to determine the free-surface elevation, three-dimensional water velocity, and salinity according to the following equations:

\[
\nabla \cdot \vec{U} + \frac{\partial \vec{W}}{\partial z} = 0
\]

(a) Bathymetry contour map of the Danshuei River estuarine system and its adjacent coastal sea. SELFE uses the Generic Length Scale (GLS) turbulence closure approach proposed by Umlauf and Buchard (2003), which has the advantage of incorporating most of the 2.5-equation closure model. The model calculates the advection term in the momentum and transport equations using an Eulerian–Lagrangian methodology. A detailed description of the turbulence closure model, the vertical boundary conditions for the momentum equation, and the numerical solution methods can be found in Zhang and Baptista (2008).

### 3.2. Lagrangian particle tracking

In this study, a Lagrangian particle-tracking technique was adopted to study the pathways of particulate pollutant dispersion in the Danshuei River estuarine system and the adjacent coastal ocean. Lagrangian models are a natural and suitable set of tools for investigating and modelling the transport pathways, mixing, and estuary-ocean exchange of dissolved pollutants (Burwell et al., 1999; Bilgili et al., 2005).

The SELFE model was used to generate the relevant hydrodynamic fields that drive the Lagrangian particle motion. Lagrangian particle tracking was combined with a random-walk model of horizontal eddy diffusion that enabled a particle-based statistical treatment of turbulent mixing. The position of a particle is described by

\[
\vec{x}(t + \Delta t) = \vec{x}(t) + \vec{u} \cdot \Delta t + \vec{z}_0 \sqrt{2 K \Delta t}
\]

where \(\vec{x}\) is the particle’s position, \(\Delta t\) is the time step, \(\vec{u}\) is the velocity vector, \(\vec{K}\) is the eddy diffusion tensor, which can be

| Table 1: Amplitudes and phases used for the model simulation at the coastal sea boundaries. |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Constituent (period) | East boundary | Northeast boundary | West boundary | Northwest boundary |
| Amplitude (m) | Phase (deg) | Amplitude (m) | Phase (deg) | Amplitude (m) | Phase (deg) | Amplitude (m) | Phase (deg) |
| M2 (12.42 h) | 0.47 | 172.37 | 0.6711 | 171.33 | 0.1236 | 181.63 | 1.3716 | 180.42 |
| S2 (12 h) | 0.12 | 331.75 | 0.1866 | 332.25 | 0.3452 | 354.5 | 0.3987 | 355.2 |
| N2 (12.9 h) | 0.10 | 262.74 | 0.1370 | 262.35 | 0.2252 | 281.373 | 0.2526 | 279.41 |
| K1 (23.93 h) | 0.21 | 232.33 | 0.2223 | 234.2 | 0.2121 | 246.06 | 0.2275 | 246.69 |
| O1 (25.82 h) | 0.17 | 67.54 | 0.170 | 67.05 | 0.1748 | 75.02 | 0.1866 | 74.40 |
determined through the hydrodynamic model, and $z_n$ is a random vector normally distributed with an average value of zero and unit standard deviation, which was based upon Press et al. (1994).

Owing to the relatively large time step used in the hydrodynamic model, the time step was divided into multiple sub-steps to track particles and meet the Courant–Friedrichs–Levy (CFL) condition. During every sub-step, the movement of each particle due to advection was calculated as described in Blanton (1993). Combined with a quick-search algorithm embedded in SELFE to compute the next cell for the particle, the velocity given the particle's position was obtained by linear interpolation if the particle was in a triangle cell.

### 3.3. Model implementation

In this study, bathymetric data for the coastal sea and Danshuei River estuarine system were obtained from the National Center for

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**Fig. 3.** (a) Freshwater discharge inputs from three tributaries from August 1–16, 1999, and a comparison of water surface elevations at the following locations: (b) the Danshuei River mouth, (c) the Taipei Bridge, (d) the Hsin-Hai Bridge, (e) Chung-Cheng Bridge, and (f) the Ta-Chih Bridge.
Fig. 4. Comparison of computed longitudinal velocity and time series data measured at (a) the Kuan-Du Bridge, (b) the Taipei Bridge, (c) the Hsin-Hai Bridge, (d) the Chung-Cheng Bridge, and (e) the Pa-Ling Bridge.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Kuan-Du Bridge</th>
<th>Taipei Bridge</th>
<th>Hsin-Hai Bridge</th>
<th>Chung-Cheng Bridge</th>
<th>Pa-Ling Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean absolute error (m/s)</td>
<td>Root-mean-square error (m/s)</td>
<td>Mean absolute error (m/s)</td>
<td>Root-mean-square error (m/s)</td>
<td>Mean absolute error (m/s)</td>
</tr>
<tr>
<td>Surface</td>
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<td>0.342</td>
<td>0.162</td>
<td>0.189</td>
<td>0.061</td>
</tr>
<tr>
<td>Bottom</td>
<td>0.084</td>
<td>0.099</td>
<td>0.070</td>
<td>0.088</td>
<td>0.044</td>
</tr>
</tbody>
</table>
Ocean Research and Water Resources Agency, Taiwan. The greatest depth within the study area was 110 m (below the mean sea level) near the northeast corner of the model in the coastal sea (Fig. 2a). The model mesh for the Danshuei River estuarine system and its adjacent coastal sea consisted of 5119 polygons (Fig. 2b). Higher resolution grids were used in the Danshuei River estuary, and coarser grids were used in the coastal sea. For the hybrid “SZ” vertical grid, ten z-level and ten evenly spaced S-levels were used. For this model grid, a large time step ($\Delta t = 120$ s) was used in simulations with no signs of numerical instability.

4. Model validation

To determine the model’s accuracy for practical applications, a large set of observational data was used to validate the model and
to verify its ability to predict the water surface elevation and salinity in this study.

4.1. Water surface elevation

The model validation of water surface elevation was conducted using daily freshwater discharge values from upriver regions of the Tahan Stream, the Hsintien Stream, and the Keelung River in 1999. Jan et al. (2001) reported that a five-constituent tide (i.e., $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$) is sufficient to represent the tidal components in the Taiwan Strait. Therefore, a five-constituent tide was adopted here as the forcing function at the coastal sea boundaries. Table 1 shows the amplitudes and phases used in the model simulation. Before running a 1-year simulation, the model was run for a 15-day period to reach an equilibrium state. The equilibrium conditions were kept for the rest of the simulation. After running a 1-year simulation, the model was run for another 1-year period to verify its stability. The model reflected the large dynamic variations in salinity throughout the tidal cycle.

4.2. Tidal current

An intensive survey of the currents was conducted at five transects on April 16, 1999 to evaluate the model's ability to calculate tidal currents in the estuary. The currents were measured by trained technicians on boats every half an hour for 13 daylight hours. The velocity data were obtained using handheld Price current meters that measured the current's magnitude but not its direction. The data were recorded by hand, and the flow direction was labeled visually as either “ebb” or “flood”. We encountered some uncertainty in determining the current's direction during slack tides. To compare the measured and computed velocities, the computed velocity in the horizontal plane was converted to the velocity along the channel. Fig. 4a–e compares the time series data for the velocity in the horizontal plane with the velocity along the channel (i.e., the longitudinal velocity) on April 16, 1999. The intra-tidal variations of the velocity on that day were accurately simulated with minor deviations from the observational data. We conclude that the flood tidal current was weaker and of shorter duration than that of the ebb tidal current in the estuary, which illustrates the tidal asymmetry caused by the interaction between the incoming tide and the river flow. The mean absolute errors and root-mean-square errors between the computed and measured velocities are shown in Table 2.

4.3. Salinity distribution

The salinity distribution reflects the combined results of all processes. In turn, it controls the density circulation and modifies the mixing processes. In the present study, an intensive salinity data collection approach at five stations was adopted to validate the model. The salinities at the open boundaries with the coastal sea were set to 35 ppt. Salinity values at the upstream boundaries were set to zero due to freshwater discharge from the boundaries. Fig. 5 presents a comparison of the time series salinities between the computed and measured data at the Kuan-Du Bridge, the Taipei Bridge, the Hsin-Hai Bridge, the Chung-Cheng Bridge, and the Pa-Ling Bridge on April 16, 1999. Table 3 shows the mean absolute errors and root-mean-square errors of differences between the computed and measured salinities. Overall, the SELFIE model reflected the large dynamic variations in salinity throughout the tidal cycle.

5. Model applications and discussion

The validated three-dimensional hydrodynamic model was then applied to estimate the estuarine residence time and the water age distribution in the Danshuei River estuarine system using the Lagrangian particle-tracking model.

5.1. Estuarine residence time

The freshwater flow discharges with $Q_{10}$, $Q_{15}$, $Q_{20}$, $Q_{25}$, $Q_{30}$, $Q_{35}$, and $Q_{40}$ are specified at the upstream boundary conditions and shown in Table 4. For the case of the $Q_{40}$ flow (the flow that is equaled or exceeded 90% of time), the freshwater discharge values at the tidal limits of three major tributaries, the Tahan Stream,

<table>
<thead>
<tr>
<th>Freshwater discharge</th>
<th>Tahan Stream</th>
<th>Hsin-Tien Stream</th>
<th>Keelung River</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{20}$</td>
<td>89.35</td>
<td>148.80</td>
<td>62.50</td>
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<tr>
<td>$Q_{15}$</td>
<td>62.44</td>
<td>110.59</td>
<td>45.20</td>
</tr>
<tr>
<td>$Q_{20}$</td>
<td>43.70</td>
<td>88.77</td>
<td>33.50</td>
</tr>
<tr>
<td>$Q_{30}$</td>
<td>25.59</td>
<td>60.90</td>
<td>20.50</td>
</tr>
<tr>
<td>$Q_{35}$</td>
<td>20.50</td>
<td>51.52</td>
<td>16.50</td>
</tr>
<tr>
<td>$Q_{40}$</td>
<td>16.51</td>
<td>53.44</td>
<td>13.57</td>
</tr>
<tr>
<td>$Q_{50}$</td>
<td>10.93</td>
<td>31.50</td>
<td>9.15</td>
</tr>
<tr>
<td>$Q_{60}$</td>
<td>7.10</td>
<td>23.57</td>
<td>6.20</td>
</tr>
<tr>
<td>$Q_{75}$</td>
<td>3.36</td>
<td>14.23</td>
<td>3.33</td>
</tr>
<tr>
<td>$Q_{100}$</td>
<td>2.01</td>
<td>5.86</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Unit: m$^3$/s.

Table 3
Mean absolute errors and root-mean-square errors of differences between computed and measured salinities.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Station</th>
<th>Kuan-Du Bridge</th>
<th>Taipei Bridge</th>
<th>Hsin-Hai Bridge</th>
<th>Chung-Cheng Bridge</th>
<th>Pa-Ling Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean absolute error (ppt)</td>
<td>Root-mean-square error (ppt)</td>
<td>Mean absolute error (ppt)</td>
<td>Root-mean-square error (ppt)</td>
<td>Mean absolute error (ppt)</td>
<td>Root-mean-square error (ppt)</td>
</tr>
<tr>
<td>Surface</td>
<td>1.295</td>
<td>1.570</td>
<td>0.587</td>
<td>0.675</td>
<td>0.289</td>
<td>0.348</td>
</tr>
<tr>
<td>Bottom</td>
<td>0.922</td>
<td>1.129</td>
<td>0.966</td>
<td>1.253</td>
<td>0.267</td>
<td>0.325</td>
</tr>
</tbody>
</table>
the Hsintien Stream, and the Keelung River, were 2.01, 5.86, and 1.17 m³/s, respectively. Five tidal constituents (M₂, S₂, N₂, K₁, and O₁) were specified to generate time series of surface elevations as open boundary conditions at the coastal sea for model simulation purposes. A constant salinity of 35 ppt at the open boundaries was used for model simulations. A total of 4000 virtual particles were evenly distributed within the main Danshuei River estuary, from the Danshuei River mouth to the mouth of the Hsintien Stream (shown in the first panel of Fig. 6), at time zero with the water stage set to zero. To evenly distribute particles in the estuary at time zero, each grid cell was assigned one particle.

Particle distributions 24, 48, and 57.9 h after starting the model with high freshwater discharge, Q₁₀, are shown in Fig. 6. This figure presents the particle distributions in the surface and bottom layers. Miller and McPherson (1991) defined the estuarine residence time as the time required to flush out a certain percentage (e.g., 67%, 95%, or 100%) of conservative particles that were evenly distributed at time zero. If the percentage is set to 100%, the definition of estuarine residence time is similar to the hydraulic retention time required for all of the particles in the estuary to be flushed out. Makarynskyy et al. (2007) and Zigic et al. (2009) adopted an estuarine residence time of 67%, which is defined as the time required for 67% of the originally evenly distributed particles to be flushed out of the system. The estuarine residence time during the high freshwater discharge scenario was 59.7 h.

Fig. 7 presents snapshots of particle distributions at 0, 24, 72, and 106.6 h after the starting time for freshwater discharge, Q₉₀. The simulated results revealed that the particles took a much longer time to be flushed out of the main Danshuei River estuary under the low...
freshwater discharge condition. We found that the estuarine residence time during the extremely low freshwater discharge ($Q_{90}$) scenario was 106.6 h. Figs. 6 and 7 show that particles that were originally located in the freshwater reaches had a tendency to move into the top layer of the water column as they were transported to the saline reaches further downstream. This phenomenon is caused by the estuarine circulation in which the upstream flow occurs in the estuary through the top layer, whereas saline water enters the estuary from the bottom layer. The effects of estuarine circulation on residence time have also been reported by Shen and Haas (2004) and Wang et al. (2004).

Fig. 8 presents the estuarine residence time against the freshwater discharge, with and without salinity effects. The model results showed that the estuarine residence time decreased as freshwater inputs increased. The residence time without density-induced circulation was higher than that with density-induced circulation, which indicates that density-induced estuarine circulation may play a significant role in the mixing characteristics of the estuary. A general relationship between the estuarine residence time and the freshwater input would be helpful to explain the physical and hydrological processes in the estuary. A regression of the estuarine residence time ($T_r (h)$) versus freshwater input ($Q (m^3/s)$) was therefore performed for the model simulation described below.

The estuarine residence time was calculated using Eqs. (7) and (8) with and without density-induced circulation, respectively:

$$T_r = 102.57 e^{0.002Q}, \quad R^2 = 0.95$$

$$T_r = 194.41 e^{0.004Q}, \quad R^2 = 0.93$$

where $R^2$ is the coefficient of determination.
Interestingly, the calculated residence time results were similar to the results reported by Wang et al. (2004) that were based on a vertical (laterally averaged) two-dimensional hydrodynamic model. The relatively short residence time is likely to be one of the limiting factors that causes low phytoplankton biomass in the Danshuei River estuary. Similar observations have been documented by Monbet (1992), Balls (1994), and Kelly (1997).

The Danshuei River estuarine system is a highly polluted estuary. At some monitoring stations, the mean dissolved oxygen in the surface water dropped to 0.5 mg/L, and mean total organic carbon loadings rose to 500 μM. Similarly, the mean nitrogen loading rose to 570 μM, and the mean concentration of total dissolved copper increased to 60 μg/L (Sun and Peng, 2001). A short residence time is beneficial for the removal of pollutants. Because of excessive pollutant loads, water quality is generally poor throughout the Danshuei River estuary, in spite of the short residence time. A significant proportion of the pollutants are exported, which impacts coastal waters outside the estuary.

### 5.2. Age of water

Consistent with its definition in this study, the age of water reflects the timescales associated with material that is transported to the downstream estuary. The water age distribution in the Danshuei River estuarine system indicates the rate of mass transport under different hydrological conditions. The freshwater discharges with $Q_{20}$, $Q_{50}$, $Q_{60}$, $Q_{60}$, and $Q_{60}$, as shown in Table 4, were specified at the three major upstream boundaries. To calculate the age of freshwater entering the estuary from the upstream boundaries, 1000 particles were released at the Cheng-Ling Bridge (Tahan Stream), the Hsui-Lang Bridge (Hsintien Stream), and the Wu-Tu (Keelung River) for each model run. The locations of these particles were then recorded at each time step. Chen (2007) used the median locations of the particles to calculate the age of water. The median location is defined as the place where 50% of the particles are upstream of the location and 50% are downstream. We adopted the same approach to estimate the age of water in the Danshuei River estuarine system. Gong et al. (2009) applied the three-dimensional hydrodynamic Environmental Fluid Dynamics Code (EFDC) to investigate the effect of wind on the age of water in the tidal Rappahannock River. They found that the local wind profiles had a significant impact on the distribution of the age of water because this factor changed the vertical mixing and estuarine circulation in the estuary. However, the influence of wind was not included in the model simulation because there is no significant wind-induced current in the estuarine systems as a result of the shelter provided by the mountains in the Taipei basin (Liu et al., 2007b).

Fig. 9a–c shows the plots of the calculated ages of water at different locations along the Danshuei River: the Tahan Stream, the Hsintien Stream, and the Keelung River, respectively. Curves were plotted to present the relationship between the age of water and the location for six freshwater discharge scenarios. The model results indicated that as the river discharge values increased, the age of water distribution (i.e., the transport time) was reduced significantly. The age of water in the Keelung River was higher than that of the Danshuei River as well as that of the Tahan Stream and the Hsintien Stream. The Hsintien is the shortest (10.5 km; Fig. 9b) of the three major tributaries, and therefore the Hsintien Stream exhibited the smallest water age in the estuaries. At a downstream cross section, for example at the Danshuei River mouth, the Hsintien Stream mouth, or the Keelung River mouth, the age of water was much greater with a small freshwater discharge than with a large freshwater discharge. A relatively short age of water was observed at the upstream reaches (i.e., near the Cheng-Ling Bridge, the Hsui-Lang Bridge, and Wu-Tu) because the waterways are shallow, which results in a high flux of water. The age of water was significantly influenced by fluxes of water. For the $Q_{60}$ freshwater discharge, the ages of water were approximately 320, 100, and 485 h at the mouths of the Tahan Stream, the Hsintien Stream, and the Keelung River, respectively (Fig. 9a–c).

To study the impact of density-induced estuarine circulation on the age of water, we compared model experiments with and without salinity under the condition of $Q_{60}$ freshwater discharge. Fig. 10 presents the model results of water age with and without density-induced circulation from the Danshuei River to the Tahan Stream, the Hsintien Stream, and the Keelung River. The age of water was greater without the inclusion of density-induced circulation when compared to the case with density-induced circulation included in the estuarine system. The experimental results show that density-induced circulation has a significant impact on the transport timescales in the estuary.

### 6. Conclusions

A three-dimensional hydrodynamic model, SELFE, with Lagrangian particle tracking, was used to investigate the transport timescales in the Danshuei River estuarine system. The hydrodynamic model was validated with water surface elevation, tidal current,
and salinity data measured in 1999. Overall, the model simulation results agreed with the empirical observations.

The validated model was then used to calculate the estuarine residence times and ages of water in response to different freshwater discharges in the river system. The estuarine residence time, which was calculated using the particle tracking model, varied between 2.5 and 4.4 days depending on the freshwater discharge at upstream boundaries. The residence time decreased as the upstream freshwater input increased. An empirical relationship between the estuarine residence time and the freshwater discharge in the Danshuei River estuarine system has been estimated as $T_r = 102.53e^{-0.002Q}$ with density-induced circulation. The age of water in the river system depends on both the locations and the freshwater discharges at the upstream boundaries. Because the fluxes of water were relatively small at the upstream reaches, the age of water was always small for different freshwater discharge

![Fig. 9](image-url)

Fig. 9. Water ages calculated at the following locations under various freshwater discharge inputs from upstream boundaries: (a) the Danshuei River-Tahan Stream, (b) the Hsintien Stream, and (c) the Keelung River (▲ represents the location from which the particles were released).
The age of water increased significantly at the downstream reaches when the upstream freshwater discharge decreased. At the mouth of the Danshuei River, the age of water varied between approximately 13.3 days during $Q_{60}$ flow to less than 4 days during high flow ($Q_{10}$). The water ages were approximately 320, 100, and 485 h at the mouths of the Tahan Stream, the Hsintien Stream, and the Keelung River, respectively, for $Q_{60}$ freshwater discharge. The calculated age with density-induced circulation was lower than that...
without density-induced circulation. The simulated results provide useful information for understanding transport processes and can be used to estimate the dissipative capacity in the estuary.

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