Impact of Node Mobility on Link Duration in Multi-hop Mobile Networks

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Abstract—In this paper, we study the impact of node mobility on link duration in multi-hop mobile networks. In multi-hop mobile networks, each node is free to move, and each link is established between two nodes. A link between two nodes is established when one node enters the transmission range of the other node, and the link is broken when either node leaves the transmission range of the other. The time interval during which the link remains active is referred to as the link duration. We develop an analytical framework for link duration in multi-hop mobile networks. We find that the link duration for two nodes is determined by the relative speed between the two nodes and the distance during which the link is connected, which are in turn determined by the angles between the two nodes’ velocities and the angle of one node incident to the other node’s transmission range, respectively. The analytical result is extended to model multipoint links which appear in the existing group mobility models. The accuracy of our framework is validated by simulations based on existing mobility models. The results show our model can describe accurately the link duration distribution for both types of links in multi-hop mobile networks, especially when the transmission range of each node is relatively smaller than the entire network coverage.

Index Terms—multi-hop mobile networks, link duration

I. INTRODUCTION

Mobility management in wireless networks has been an active research topic for years [1-6]. Research efforts on single hop mobile networks include the results for cellular networks [1-4] or Mobile IP networks [5-6], i.e., only the last hop to each mobile node is wireless, and communications between nodes must go through the associated base stations (BSs). Each wireless link is established only between the BS and a node in a cell. When the node roams to another cell, the link is handed over to the respective BS so as to retain the ongoing connection.

In a multi-hop mobile network, each node plays both roles of a router and an end-point, and no pre-deployed infrastructure such as BS is available for node communications. Data are relayed by intermediate nodes if the receiver node is beyond the transmission range of the sender node. As a result, a wireless link is established between two nodes. Each node is free to move arbitrarily. A link between two nodes is activated when one node enters the transmission range of the other, and the link is broken when either node leaves the transmission range of the other node. When a link on a routing path is broken, a rerouting process is initiated so as to reduce service disruption in the network.

In this paper, we study the impact of node mobility on link duration in multi-hop mobile networks. Specifically, we develop an analytical framework to evaluate link duration in such networks. In our framework, each node may move at a different speed, pause for a while, and then move again. The link duration here then corresponds to the time interval in which two nodes stay within transmission range of each other. The importance of link duration on system performance has been identified in the literature [7-8]. For example, the link duration may affect the lifetime of the routing path, which in turn determines the packet delivery ratio or per-connection throughput for an S-D pair. An analytical link duration model can also help determine the timer setting in ad hoc routing [7] or even on the design of better routing protocols to cope with link breakage caused by node mobility [9].

The framework we develop is based on the relative movement behavior of one node observed by the other node (i.e., from the perspective of the observer node). We find that the duration of a link between two nodes is determined by their relative speed and the distance traversed within the transmission range of the observer node during the link activation, which are in turn determined by the angle between the two nodes’ velocities and the angle (formed by the relative velocity) incident into the observer node’s transmission range, respectively. We also consider the link formed among a group of nodes, referred to as the multipoint link in this paper, in light of group mobility models (e.g., Reference Point Group Mobility Model [10] and Reference Velocity Group Mobility Model [11]) widely discussed in the literature for mobile ad hoc networks. The accuracy of our model is validated by simulations based on existing mobility models (e.g., random waypoint models [12], the random walk [13], and group mobility model [10-11]). We also demonstrate the usability of the derived model for different applications.

The rest of the paper is organized as follows. In Sec. II, the analytical framework for link duration of point-to-point links and multipoint links in multi-hop mobile network is developed. In Sec. III, the simulation results are provided to validate the analytical model. Finally, the paper is concluded in Sec. IV.
II. ANALYTICAL MODEL OF LINK DURATION

In this section, we develop the probability distribution function of link duration for multi-hop mobile networks. Each node is assumed to use the same transmission power and move independently. We do not assume any specific mobility model for each node in this paper. Later in the simulation section, we will validate our model based on the existing mobility models such as the random waypoint model, random walk model, and group mobility models.

A. Link Duration between Two Nodes

Consider nodes $N_i$ and $N_j$ moving in the network. Let $\bar{V}_i$ and $\bar{V}_j$ denote the velocities of node $N_i$ and node $N_j$, respectively; let $P_i^0$ denote the initial position of node $N_i$, and $P_i^t$, the position of $N_i$ after time $t$ with velocity $\bar{V}_i$. Fig. 1 shows the movements of $N_1$ and $N_2$ for time $t$. In Fig. 1 (a), the dotted circles represent the transmission ranges of the nodes centered at the circles; the solid circle represents the relationship between nodes $N_1$ and $N_2$. We are particularly interested in the relative movement behavior of $N_2$ observed by $N_1$, as shown in Fig. 1 (b), instead of the absolute viewpoints of both nodes as shown in Fig. 1 (a). From the perspective of $N_1$, $N_2$ is moving toward/away $N_1$ with relative velocity $\bar{V}_{12} = \bar{V}_2 - \bar{V}_1$, and the relative speed is $V_{12} = |\bar{V}_{12}|$.

Let $\alpha_{ij}$ denote the angle between $\bar{V}_i$ and $\bar{V}_j$ given that the link between $N_i$ and $N_j$ can be established (as shown in Fig. 1 (b)), and $\beta_{ij}$ represent the incident angle, the relative velocity $\bar{V}_{ij}$, to $N_i$’s transmission range (as shown in Fig. 1 (c)). Therefore, random variable $\alpha_{ij}$ ranges over $[0, \pi]$, and $\beta_{ij}$, over $[0, \pi/2]$. Since $N_i$ and $N_j$ can only communicate part of the time, the distance traversed by $N_j$ with relative velocity $\bar{V}_{ij}$ during which the link is activated is referred to as the active distance between $N_i$ and $N_j$, denoted by $D_{ij}$. Fig. 1(c) illustrates $D_{12}$.

In Fig. 1(b), the value of angle $\alpha_{ij}$ determines the magnitude of $V_{ij}$, i.e.,

$$V_{ij} = \sqrt{V_i^2 + V_j^2 - 2V_i V_j \cos \alpha_{ij}}.$$  \hspace{1cm} (1)

In Fig. 1(c), the value of angle $\beta_{ij}$ determines the magnitude of $D_{ij}$, i.e.,

$$D_{ij} = 2r \cos(\beta_{ij}).$$  \hspace{1cm} (2)

Since the active distance $D_{ij}$ and the relative speed $V_{ij}$ are mutually independent (which will be proved shortly in the paper), the link duration $T_{ij}$ between $N_i$ and $N_j$ can be expressed by

$$T_{ij} = \frac{D_{ij}}{V_{ij}}.$$  \hspace{1cm} (3)

Since the probability distributions of $V_{ij}$ and $D_{ij}$ are determined by those of $\alpha_{ij}$ and $\beta_{ij}$, respectively, in what follows, we first develop the probability distributions of $\alpha_{ij}$ and $\beta_{ij}$, and then those of $V_{ij}$ and $D_{ij}$, based on which the

\footnote{\alpha_{ij} is a conditional random variable defined as the angle between velocities $\bar{V}_i$ and $\bar{V}_j$ given that the link between nodes $N_i$ and $N_j$ can be activated. In other words, $\alpha_{ij}$ is not the random variable representing the angle between any two nodes in the network, as a link may not be realized between any two arbitrary nodes.}

Figure 1. The relations between two nodes in terms of velocity and position in the network, where a circle represents the transmission range of the node centered at the circle.
distribution of $T_y$ can be derived. Note that since all nodes move independently and behave statistically identically, we will omit the argument $ij$ of $\alpha_{ij}$, $\beta_{ij}$, $V_{ij}$, $D_{ij}$, and $T_y$ in the subsequent analysis.

We start with the simplest case, i.e., each node is continuously roaming and moves at the same speed (i.e., $|\vec{v}_i| = |\vec{v}_j| = v_{fix}$). Later, we will extend it to the case with nodes moving at different speeds, and discuss the impact of node pause on link duration.

**B. Probability Distribution of Link Duration for Point-to-Point Links without Pause**

1) $|\vec{v}_1| = |\vec{v}_2| = v_{fix}$

Since each node moves at the same speed $v_{fix}$, the relative speed between two nodes given that the link in between can be established is given by $V = 2v_{fix} \sin(\alpha / 2)$.

$$V = 2v_{fix} \sin(\alpha / 2). \tag{4}$$

![Figure 2. An illustration of calculating $Pr\{E_{LA} | V, T_e \}$](image)

For ease of explanation, we assume that nodes outside the transmission range of node $N_i$ are uniformly\(^2\) distributed in the network. Let $E_{LA}$ denote the event that the link between $N_i$ and $N_j$ is activated, and $T_e$, the random variable representing the elapsed time from when $N_j$ starts to move with relative speed $V$ to when the transmission range of $N_i$ is reached. Given that $N_j$ moves at relative speed $V = v$ toward $N_i$ with elapsed time $T_e = t$, the probability that the link between $N_i$ and $N_j$ can be activated is equal to the probability that $N_j$ starts moving from any point located in the shaded area as shown in Fig. 2 (a). The area of the shaded region can be obtained by $vt \cdot 2r$ as illustrated in Fig. 2 (b). Therefore, $Pr\{E_{LA} | V = v, T_e = t\} = 2vrt$, where $r$ is the transmission range of node $N_i$.

The probability $Pr\{\beta \leq b | V = v, T_e = t\}$ corresponds to the probability that node $N_j$ falls inside the striped area as shown in Fig. 2 (b), i.e., the area from which $N_j$ moving with $V = v$ and $T_e = t$ would reach $N_i$’s transmission range with an angle no more than $b$, where $0 \leq b \leq \pi / 2$. Therefore, the cumulative distribution function (cdf) of $\beta$ can be expressed by

$$Pr\{\beta \leq b | V = v, T_e = t\} = 2vrt \sin b. \tag{5}$$

From (4), we have

$$Pr\{\beta \leq b | \alpha = a, T_e = t\} = 2v_{fix} t \sin a / 2 \sin b.$$

Thus,

$$Pr\{\alpha \leq a, \beta \leq b | T_e = t\} = t \int_0^\alpha f_\alpha(x) \cdot \sin(x / 2) \sin b dx$$

\[ \approx \int_0^\alpha f_\alpha(x) \cdot \sin x / 2 \sin b dx. \tag{6} \]

where $f_\alpha(.)$ is the pdf of the random variable $\alpha$ denoting the angle between two arbitrary nodes.

Since, when the moving directions of $N_i$ and $N_j$ are assumed independent and nearly uniformly distributed from 0 to $2\pi$, the distribution of angles between two arbitrary nodes can be approximated by a uniform distribution over $[0,\pi]$, (6) can be re-expressed by

$$\int_0^\alpha \sin x / 2 \sin b dx = \frac{2}{\pi} \left[ 1 - \cos \left( \frac{a}{2} \right) \right] \sin b. \tag{7}$$

The joint probability distribution of $\alpha$ and $\beta$ in (7) is independent of the value taken by $T_e$, and can be expressed by

$$F_{\alpha, \beta}(a, b) = \Pr\{\alpha \leq a, \beta \leq b\} = \frac{2}{\pi} \left[ 1 - \cos \left( \frac{a}{2} \right) \right] \sin b \cdot K.$$

Since $F_{\alpha, \beta}(\pi, 0/2) = 1$, this yields $K = \pi / 2$. Thus,

$$F_{\alpha, \beta}(a, b) = \left[ 1 - \cos \left( \frac{a}{2} \right) \right] \sin b. \tag{8}$$

From the joint cdf $F_{\alpha, \beta}(.)$, we can further obtain

$$F_a(a) = 1 - \cos \left( \frac{a}{2} \right), \tag{9}$$

\(^2\)When this assumption does not hold, the derivation must be replaced with an integral of the grey area, instead of the area coverage itself as in the derivation.
$F_\beta(b) = \sin b$.

Thus, it follows that

$$f_\alpha(a) = \frac{dF_\alpha(a)}{da} = \frac{1}{2} \sin \left( \frac{a}{2} \right),$$

$$f_\beta(b) = \frac{dF_\beta(b)}{db} = \cos b.$$ 

From (8), (9) and (10), we have $F_{\alpha,\beta}(a,b) = F_\alpha(a) \cdot F_\beta(b).$

Thus, random variables $\alpha$ and $\beta$ are independent.

Based on the distributions of $\alpha$ and $\beta$ in (9) and (10), we can further derive the distributions of the active distance $D$ and the relative speed $V$. From (4) and (9), we obtain

$$F_V(v) = \Pr\{V \leq v\} = \Pr\left\{2\sqrt{\frac{V}{2}} \sin \left( \frac{\alpha}{2} \right) \leq v\right\}$$

$$= F_\alpha\left(\frac{v}{2\sqrt{\frac{V}{2}}} \right) = \frac{1}{2} - \sqrt{1 - \left( \frac{v}{2\sqrt{\frac{V}{2}}} \right)^2},$$

$$f_V(v) = \frac{dF_V(v)}{dv} = \frac{v}{2\sqrt{\frac{V}{2}} \sqrt{4\frac{V}{2} - v^2}}.$$ 

From (3) and (10), we obtain

$$F_D(d) = \Pr\{D \leq d\} = \Pr\{2r \cos \beta \leq d\}$$

$$= \Pr\left\{ \beta \geq \cos^{-1} \left( \frac{d}{2r} \right) \right\} = 1 - F_\beta\left(\cos^{-1}\left( \frac{d}{2r} \right) \right)$$

$$= 1 - \sqrt{1 - \left( \frac{d}{2r} \right)^2},$$

$$f_D(d) = \frac{dF_D(d)}{dd} = \frac{d}{2r\sqrt{4r^2 - d^2}}.$$ 

Substituting the derived distributions of $D$ and $V$ in (11) and (13) into (3), we obtain the distribution of $T$ accordingly, i.e.,

$$F_T(t) = \Pr\{T \leq t\} = \Pr\left\{ \frac{D}{V} \leq t \right\} = \Pr\left\{ \frac{2r \cos \beta}{2\sqrt{\frac{V}{2}} \sin \left( \frac{\alpha}{2} \right)} \leq t \right\}$$

$$= \frac{\pi}{2} \int_{-\infty}^{\infty} f_\beta(b) \cdot \Pr\left\{ \frac{2r \cos b}{2\sqrt{\frac{V}{2}} \sin \left( \frac{\alpha}{2} \right)} \leq t \right\} db$$

$$= \frac{\pi}{2} \int_{-\infty}^{\infty} f_\beta(b) \cdot \Pr\left\{ \sin \left( \frac{\alpha}{2} \right) \geq \frac{r \cos b}{\sqrt{\frac{V}{2}} t} \right\} db.$$ 

Since $\Pr\left\{ \sin \left( \frac{\alpha}{2} \right) \geq \frac{r \cos b}{\sqrt{\frac{V}{2}} t} \right\} = 1$, we can further decompose the distribution $F_T(t)$ into two cases.

Case 1: $t < \frac{r}{\sqrt{\frac{V}{2}}},$

$$F_T(t) = \frac{\pi}{2} \int_{-\infty}^{\infty} f_\beta(b) \cdot \Pr\left\{ \frac{r \cos b}{\sqrt{\frac{V}{2}} t} \right\} db$$

$$= \frac{\pi}{2} \int_{b = \cos^{-1}\left( \frac{r}{\sqrt{\frac{V}{2}} t} \right)}^{\infty} \cos b \cdot \sqrt{1 - \left( \frac{r \cos b}{\sqrt{\frac{V}{2}} t} \right)^2} \db$$

$$= \frac{\pi}{2} \int_{b = \cos^{-1}\left( \frac{r}{\sqrt{\frac{V}{2}} t} \right)}^{\infty} \cos b \cdot \sqrt{1 - \left( \frac{r \cos b}{\sqrt{\frac{V}{2}} t} \right)^2} \db$$

$$= \frac{1}{2} \cdot \frac{r^2 - \frac{V}{2} t^2}{2r\sqrt{\frac{V}{2}}} \ln \left( \frac{r + V_{fix} t}{\sqrt{V_{fix}^2 t^2 - r^2}} \right).$$

Case 2: $t > \frac{r}{\sqrt{\frac{V}{2}}},$

$$F_T(t) = \frac{\pi}{2} \int_{-\infty}^{\infty} f_\beta(b) \cdot \Pr\left\{ \sin \left( \frac{\alpha}{2} \right) \geq \frac{r \cos b}{\sqrt{\frac{V}{2}} t} \right\} db$$

$$= \frac{\pi}{2} \int_{b = \cos^{-1}\left( \frac{r}{\sqrt{\frac{V}{2}} t} \right)}^{\infty} \cos b \cdot \sqrt{1 - \left( \frac{r \cos b}{\sqrt{\frac{V}{2}} t} \right)^2} \db$$

$$= \frac{1}{2} \cdot \frac{r^2 - \frac{V}{2} t^2}{2r\sqrt{\frac{V}{2}}} \ln \left( \frac{r + V_{fix} t}{\sqrt{V_{fix}^2 t^2 - r^2}} \right).$$

From (15) and (16), we can obtain the probability distribution of link duration for point-to-point links as follows.

$$F_T(t) = \frac{1}{2} \cdot \frac{r^2 - \frac{V}{2} t^2}{2r\sqrt{\frac{V}{2}}} \ln \left( \frac{r + V_{fix} t}{\sqrt{V_{fix}^2 t^2 - r^2}} \right)$$

$$f_T(x) = \frac{dF_T(t)}{dt} = \frac{r^2 + V_{fix}^2 t^2}{2r\sqrt{\frac{V}{2}}} \ln \left( \frac{r + V_{fix} t}{\sqrt{V_{fix}^2 t^2 - r^2}} \right) - \frac{1}{2r.$$ 

2) $|V_i| = v_i$ and $|V_j| = v_j$. The relative speed between $N_i$ and $N_j$ can be expressed by
\[ V = \sqrt{v_i^2 + v_j^2 - 2v_i v_j \cos \alpha} = \sqrt{v_i^2 + v_j^2} \cdot \left(1 - \frac{2v_i v_j}{v_i^2 + v_j^2} \cos \alpha \right), \]

which can be approximated by

\[ \sqrt{v_i^2 + v_j^2} \cdot \left(1 - \frac{v_i v_j}{v_i^2 + v_j^2} \cos \alpha \right). \] (18)

Accordingly, we have

\[ \Pr \left( \alpha \leq \beta, \beta \leq b \mid T_x = t, v_i = v_j, v_j = v_j \right) = \frac{1}{\pi} \sqrt{v_i^2 + v_j^2} \cdot \left(1 - \frac{v_i v_j}{v_i^2 + v_j^2} \cos \alpha \right) \sin b dx, \]

and

\[ F_T(t) = \Pr \left( T \leq t \mid v_i = v_j, v_j = v_j \right). \]

\[ \frac{\pi}{2} \cos b \cdot \Pr \left( V > \frac{2r \cos b}{t} \right) \]

\[ \cos^{-1} \left( \frac{d(b(k_i k_j))}{2r} \right) \]

\[ \cos b \left[ \frac{1}{\pi} \left( -k_j x_i + \cos^{-1} x_j \right) \right] db, \]

where

\[ k_i = \sqrt{v_i^2 + v_j^2}, \quad k_j = \frac{v_i v_j}{v_i^2 + v_j^2}, \quad x_i = \left[ 1 - \left( \frac{k_j - \frac{2r \cos b}{t}}{k_i k_j} \right)^2 \right]^{1/2}, \quad \text{and} \]

\[ x_j = \left[ \frac{k_j - \frac{2r \cos b}{t}}{k_i k_j} \right]. \]

Denote the speed of each node by \( v_j \), which is a random variable distributed over \([ V_{min}, V_{max} ]\). Thus, we have

\[ F_T(t) = \int_{V_{min}}^{V_{max}} \int_{V_{min}}^{V_{max}} f_{I_i}(v_i) \cdot f_{I_j}(v_j) \cdot \cos^{-1} \left( \frac{d(b(k_i k_j))}{2r} \right) \cos b \left[ \frac{1}{\pi} \left( -k_j x_i + \cos^{-1} x_j \right) \right] db dv_i dv_j. \] (19)

\[ C. \textit{Probability Distribution of Link Duration with Pause} \]

We now study the impact of node pause on link duration. Suppose that each node may be either in the state of movement or in pause state. When either node enters the pause state, the link duration is determined by the moving node’s velocity.

We model link duration with two components: both nodes are in movement and either node is in pause state. The weights of the two components can be decided by the percentage of the time each node is in motion or in pause state. Let \( M \) denote the mean movement duration and \( P \) be the duration of pause time. Then, a node is in movement with a probability of \( \frac{M}{M + P} \) and in pause state with a probability of \( \frac{P}{M + P} \). Thus, the pdf of the link duration with the consideration of pause can be expressed by

\[ f_T(t) = \frac{M}{M + P} f_T(t) + \frac{P}{M + P} f_T(t), \]

where \( f_T(t) \) is the derivative of (19) (i.e., the case in which both nodes are in movement) and \( f_T(t) \) is the case that either node is in pause state during the link activation. Thus, we obtain

\[ F_{I_i}(t) = \Pr \left( \frac{2r \cos \beta}{v_i} \leq t \right) = \Pr \left( \beta \leq \cos^{-1} \left( \frac{v_i t}{2r} \right) \right), \]

leading to

\[ F_{I_i}(t) = \int_{v_{min}}^{v_{max}} f_{I_i}(v_i) \cdot \left(1 - \left( \frac{v_i t}{2r} \right)^2 \right) dv_i. \] (20)

\[ D. \textit{Link Duration of Multi-point Link} \]

Finally, we extend the derivation to capture the behavior of a multipoint link shared by multiple nodes. The derivation is similar to that of the Reference Point Group Model described in [10]. In this model, a multipoint link is formed among a group, led by a group leader. All member nodes move along with the leader according to a node mobility model, and each node moves independently by following a node mobility model. The link duration of a multipoint link is then referred to as the time interval in which all member nodes have left the transmission range of the leader node.

Suppose that there are \( m \) member nodes in a group (denoted by \( N_1, N_2, \ldots, N_m \)) led by a group leader \( N_0 \) and all member nodes in the group are initially inside the transmission range of \( N_0 \). We assume that all member nodes move independently

\[ 3 \text{ We do not consider both nodes are in pause states in this paper due to space limitations. Later in the simulation, we will show that the probability of both nodes in pause states is small especially when the transmission range of each node is relatively smaller compared to the entire network coverage.} \]
guided by their node mobility models and all nodes are continuously moving. Thus, all nodes will eventually move out of the transmission range of the leader nodes unless they move toward the same direction as the leader node. The link duration of a multipoint link corresponds to the longest link duration between each member node and the leader node, which can be expressed by

\[ T_m = \max \{ T_1, T_2, \ldots, T_m \} , \tag{20} \]

where \( T_i \) is the link duration between nodes \( N_i \) and \( N_0 \).

Let \( F_{TM}(t) \) denote the cdf of the link duration for a multipoint link shared by \( m \) nodes, and \( F_i(t) \) is the cdf of the link duration between the leader \( N_0 \) and the member node \( N_i \).

Since all nodes move mutually independently by their node mobility models and the moving direction of each node is uniformly distributed over \([0, 2\pi]\). We obtain

\[ F_{TM}(t) = \Pr [T_M < t] = \Pr [T_1 < t, T_2 < t, T_3 < t, \ldots, T_m < t] = \Pr [T_1 < t] \cdot \Pr [T_2 < t] \cdot \Pr [T_3 < t] \cdots \Pr [T_m < t] = F_1(t) \cdot F_2(t) \cdot F_3(t) \cdots F_m(t) = (F_i(t))^m , \tag{21} \]

where \( F_i(t) \) is the conditional cdf of member node \( N_i \), given \( V_{i0} \) and \( R_{i0} \).

![Figure 3. Relations between member node \( N_i \) and leader node \( N_0 \)](image)

The distribution of \( F_i(t) \) is obtained as follows. In Fig. 3, \( \theta_i \) is the angle formed by the relative velocity \( V_{i0} \) and the relative position vector \( R_{i0} \), and \( d_i = \|R_{i0}\| \). The active distance traversed by node \( N_i \) is

\[ D_i = \sqrt{r^2 - d_i^2 \sin^2(\pi - \theta_i)} + d_i \cos(\pi - \theta_i) . \]

From (3) we have

\[ F_i(t) = \Pr \left\{ \frac{D_i}{V_{i0}} \leq t \mid V_{i0} = v_0, \vec{V}_i = v_i, \vec{R}_{i0} = d_i \right\} = \Pr \left\{ \frac{\sqrt{r^2 - d_i^2 \sin^2(\pi - \theta_i)} + d_i \cos(\pi - \theta_i)}{\sqrt{v_0^2 + v_i^2 - 2v_0v_i \cos \alpha}} \leq t \right\} , \]

where \( V_{i0} \) denotes the relative speed between \( N_0 \) and \( N_i \).

Let \( F_{R_{i0}}(\cdot) \) be the cdf of \( \|R_{i0}\| \). Thus,

\[ F_i(t) = \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \left( 1 - F_{R_{i0}}(\sqrt{r^2 - v_0^2 \sin^2(\pi - \theta) + v_i \cos(\pi - \theta)}) \right) \sin \theta \ d\theta , \]

if \( t < \frac{r}{v_i} \),

\[ \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \left( 1 - F_{R_{i0}}(\sqrt{r^2 - v_0^2 \sin^2(\pi - \theta) + v_i \cos(\pi - \theta)}) \right) \sin \theta \ d\theta , \]

if \( t \geq \frac{r}{v_i} \),

where \( v = \sqrt{v_0^2 + v_i^2 - 2v_0v_i \cos \alpha} \).

III. PERFORMANCE EVALUATION

In this section, we validate our analytical results via an event-driven C++ simulator. In our simulations, 300 nodes are uniformly distributed in a unit-square area. Each node moves independently, including both speed and the moving direction, guided by a mobility model. The result in each figure is an average of 100 runs. We validate the link duration for both point-to-point links and multipoint links as follows.

A. Link Duration of Point-to-Point Links

We consider two models, described as follows.

1) The random waypoint model: each node selects a target location to move at a speed selected from a uniformly distributed interval \([V_{t_{min}}, V_{t_{max}}]\). Once the target is reached, the node pauses for a random time with probability \( p \) and then selects another target with another speed to move again.

2) The random walk model: each node selects a direction and a speed pair \([0, v]\) from uniformly distributed intervals \([0, 2\pi]\) and \([V_{t_{min}}, V_{t_{max}}]\), respectively, and then starts to move for a time \( t \) uniformly selected from \([0, t_{max}]\). Once the node has moved for \( t \) units of time, it pauses for a random time with probability \( p \) and then starts a new movement again.

1) Nodes with Fixed Speed

We first simulate the case with nodes moving at fixed speed \( v_{fix} \) = 0.01 unit per second under both mobility models (denoted by Sim-RWP and Sim-RW). The pdf of \( T \) is plotted in Fig. 4 with transmission range \( r = 0.15 \), where RWP is for the random waypoint model, and RW, for the random walk model. The figure shows that link duration has the highest probability around the value \( r / v_{fix} \) for both models, explained as follows. A larger \( T \) results from a larger \( D \) and/or smaller \( V \), which in turn results from a
larger $\beta$ and/or smaller $\alpha$, and vice versa. Let us give a more intuitive explanation with the following two facts. First, two nodes moving in a similar direction have a smaller probability to meet and form a link. Second, with a given velocity, the point at which one node enters the other node’s transmission range is uniformly distributed (denoted by $x$, $2x$, and $3x$ in Fig. 5). However, when the incident point is near the tangential line, the active distance $D$ is shortened rapidly, i.e., $D(x) > D(2x) > D(3x)$. Since $T=D/V$, the probability of large $T$ is determined by the nodes with similar moving direction, which has a smaller chance for nodes to meet. On the other hand, the case of small $T$ is determined by the low probability of a short active distance. Since the probability $T$ being small or large is low, the peak is at the point where both $\alpha$ and $\beta$ have the highest probabilities, i.e.,

$$T = 2r \cos(0) / 2v_{fix} \sin(\frac{\pi}{2}) = r / v_{fix}.$$ 

With the revised density function (21), we integrate (19) over the range $[V_{min}, V_{max}]$ to obtain the new probability density function of the link duration for point-to-point links. The curve in Fig. 6 with “Ana-non-uniform” shows the analytical result based on (21). The analytical result has an apparent accuracy improvement as compared to the uniformly distributed one.

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} f_{V_{ij}}(v)dv = 1.$$ 

$$f_{V_{ij}}(v) = \frac{1}{v \cdot \ln \frac{V_{max}}{V_{min}}}.$$  

(21)

B. Link Duration with Pause

Then, we show the results when there are pauses between movements. Here we run simulations with the random waypoint model and the speed is fixed at 0.01 units per second with different pause times. Fig. 7 shows the comparison of the simulation and analysis results with a pause time of 50 seconds. We see that there are two peaks, i.e., at $T=15$ and 30 seconds, which are contributed by the two components (i.e., both nodes are in motion, and either is in pause), respectively. In this scenario, since the transmission range of each node is relatively smaller as compared to the entire network coverage, our model can accurately capture the behavior of link duration.
C. Boundary Effect on Link Duration

To observe the boundary effect, we simulate the random walk mobility model with a fixed movement interval of 100 seconds and a fixed speed of 0.01 units per second. In this way, we can avoid having a movement ends during the link duration. From the simulation results in Fig. 8, we observe that when the ratio of transmission range to network size is below 0.2, the boundary effect is insignificant. However, when the ratio exceeds 0.25, the probability of short $T$ for nodes near the boundary may have shorter link durations, so our analysis can better describe the link duration when the node transmission range is relatively smaller compared to the entire network size.

D. Link Duration of Multipoint Links

In this simulation, there are one leader node and 10 member nodes in a unit-square network. Each member node is initially located in the transmission range of the leader node, and its distance to the leader node is uniformly selected from the range $[0, r]$ unit, where $r$ is the transmission range of the leader node. All member nodes move with a fixed speed of 0.01 units per second, and the direction to move is uniformly distributed from $[0, 2\pi]$. Fig. 9 shows that compared to the distribution of point-to-point links, multi-point links have a higher probability for long link duration. This is due to the fact that as long as at least one of the member nodes chooses a similar direction as the leader node, the link will exist for a long time.

IV. CONCLUSIONS AND FUTURE WORK

In this paper, we propose an analytical framework to model the link duration for multi-hop mobile networks. We consider both point-to-point links between nodes and multipoint links shared by a group of nodes. Each node may be in movement or in pause state. The model starts with the derivation of the distributions of two parameters $\alpha$ and $\beta$, which in turn determine the distributions of the relative speed of the two considered nodes and the active distance between the two nodes for an activated link. Then, the pdf of link duration for point-to-point links can be obtained. The derived result is then extended to model multipoint links. We validate the analytical results via simulations with three different mobility models widely used in the literature, namely, the random waypoint, the random walk, and the group movement model.

In the future, we will further derive the path lifetime based on the analytical results developed in this paper. The probability distribution of the path lifetime in mobile ad hoc networks is even more difficult to derive than link duration since it is dependent on many system parameters such as spatial distribution, node density, and path connectivity.

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REFERENCES


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