Adaptive relaying scheme for cognitive radio networks

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Abstract: Cognitive radio (CR) systems allow unlicensed secondary users to transmit on the licensed frequency bands without degrading the licensed primary transmissions. Combining CR with other emerging transmission techniques, such as user cooperation may have many benefits on both the primary and secondary transmissions. In this study, the authors propose and investigate an adaptive relay-based cooperation scheme for CR networks that improves the secondary outage performance, while respecting a primary outage probability threshold. The proposed adaptive scheme considers one multi-antenna relay node that, by selecting the antenna(s) to use, can assist either the primary, the secondary or both transmissions simultaneously. Expressions of the conditional primary outage probability for Rayleigh fading channels are derived and used to investigate the associated power allocation problem. Simulation results show that both primary and secondary outage probabilities of the proposed scheme are significantly improved and outperform non-cooperative and cooperative schemes given in the literature.

1 Introduction

The spectacular growth of wireless services over the past years demonstrates the evolving demand for wireless communications. Consequently, parts of the spectrum allocated to these services are becoming ever more crowded even though recent studies showed that most of that spectrum is largely under-used [1]. To alleviate the spectrum congestion and spectrum under-utilisation issues, cognitive radio (CR) has emerged as a promising technology that allows unlicensed users to use parts of the licensed spectrum without degrading the licensed users’ communications [2]. The licensed users, called primary users (PUs), have priority for accessing the licensed spectrum bands, whereas the unlicensed secondary users (SUs) have a lower access priority. In a CR network (CRN), in order to respect a given outage probability of the PUs, the secondary transmission system must limit its transmit power. As a consequence, when PUs require a higher quality-of-service (QoS), in terms of outage probability, the performance of the secondary system decreases.

On the other hand, user cooperation for wireless networks was developed as an efficient technique to obtain an increased spatial diversity gain. The spatial diversity is provided by several paths between the source and destination through the relay nodes (RE) [3, 4].

1.1 Related work

Recently, integrating user cooperation in CRNs has attracted attention for improving both the spectrum utilisation and the transmission performances. Cooperative diversity has been exploited to improve both PU and SUs transmissions. In [5], the authors assume that an SU acts as a relay for primary transmissions. Results show that cognitive relaying is efficient in enhancing the secondary throughput under certain network topology conditions. In [6], the authors propose a cooperation protocol, where a secondary transmitter (ST) sends the primary signal along with its own secondary signal without affecting the outage probability of the primary communication. They derived a critical geographical distance between the primary (PT) and ST. Within that distance, a fraction of the total transmit power is chosen at the ST to relay the primary signal, whereas respecting the primary outage probability threshold and improving the secondary access compared to the case with no spectrum sharing. In addition to the cooperation between SUs and PUs, cooperation among SUs has also been investigated. In [7], an adaptive cooperation diversity scheme with best-relay selection is proposed for multi-relay CRNs in order to improve the outage performance of the secondary transmissions, whereas satisfying the primary QoS. By letting the ‘best’ CR node assist the secondary transmission, the secondary outage probability can be significantly improved over the non-cooperative scheme. In [8], the secondary transmission is assisted by a group of CR REs at different locations. The authors showed that the maximal diversity order can be achieved if the REs are ‘adequately’ selected. In [9], we proposed a relaying scheme for CRN, where the RE assists simultaneously both primary and secondary transmissions each time it is able to decode correctly the primary and the secondary signals.
We showed that the secondary outage performance improves at the expense of a substantial increase on the relay’s node transmit power. We also proposed in [10], relay selection criteria for a cooperative scheme that uses two independent REs to assist simultaneously the primary and secondary transmissions. The outage probability results have shown the great potential of the relaying scheme compared with non-cooperative transmissions.

1.2 Contributions of the paper

Even though, some work has focused on exploiting user cooperation for underlay CRNs, to the best of our knowledge no work has been reported in investigating adaptive relaying schemes, where the RE decides independently when, which communication to assist and how to do it. Consequently, we propose in this paper a new opportunistic scheme (named adaptive relaying scheme), where the CR RE, that belongs to the secondary network, is able to decide when to cooperate and, if so, which transmission to cooperate with (primary, secondary or both transmissions simultaneously) depending on the channel states.

In order to make an efficient use of the proposed adaptive relaying scheme, we assume a multi-antenna CR RE. The relay uses its antennas to decode one or multiple signals simultaneously and next it uses one or two of its antennas to forward the decoded signal(s). Without loss of generality, opportunistic use of decode-and-forward (also called selective decode-and-forward [11]) has been chosen in our scheme, since it is more efficient in highly interfered and noisy wireless environments than the non-regenerative relaying techniques, such as amplify-and-forward and compress-and-forward [11, 12].

Moreover, a simple antenna selection technique is used at the relay in order to reduce the computational complexity and signalling requirements. We also assume that the receivers perform optimum combining (OC) to decode the received signals, since primary and secondary transmissions may occur simultaneously, on the same frequency band, and on a very tight geographic area. OC has been shown to be very effective in interfered wireless environments [13, 14].

The proposed relaying scheme provides better outage probability performances for both primary and secondary transmissions compared with non-cooperative and basic cooperative transmission schemes. They hence provide a better usage of the available resources, in terms of transmit power.

Table 1: Notations

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>( N )</td>
<td>number of antennas at the CR RE</td>
</tr>
<tr>
<td>( P_{\text{PT}} ), ( P_{\text{ST}} )</td>
<td>transmit power of PT, of ST</td>
</tr>
<tr>
<td>( P_{\text{max}}^{\text{RE}} ), ( P_{\text{max}}^{\text{ST}} )</td>
<td>maximal transmit power of ST, of RE</td>
</tr>
<tr>
<td>( P_{\text{RE}} )</td>
<td>transmit power of RE used by the selected antenna to relay the primary signal</td>
</tr>
<tr>
<td>( P_{\text{RE}}^{(a)} )</td>
<td>transmit power of RE used by the selected antenna to relay the secondary signal</td>
</tr>
<tr>
<td>( x_{a,p} ), ( x_b )</td>
<td>primary, secondary information signals</td>
</tr>
<tr>
<td>( y_{a}(t) )</td>
<td>received vector of signals at subslot ( t ) by node ( a )</td>
</tr>
<tr>
<td>( H_{a,b} )</td>
<td>Rayleigh fading coefficient of the channel ( a-b )</td>
</tr>
<tr>
<td>( h_{a,b} )</td>
<td>Rayleigh fading vector of the channel ( a-b ) of elements ( h_{a,b}(i) )</td>
</tr>
<tr>
<td>( h_{a,b}^{(a)} )</td>
<td>Rayleigh fading vector of the channel between the selected antenna to forward ( x_a ) at RE and ( b )</td>
</tr>
<tr>
<td>( h_{a,b}^{(b)} )</td>
<td>Rayleigh fading vector of the channel between the selected antenna to forward ( x_b ) at RE and ( b )</td>
</tr>
<tr>
<td>( d_{a,b} )</td>
<td>path-loss exponent</td>
</tr>
<tr>
<td>( d_{a,b}^{(i)} ), ( d_{a,b}^{(b)} )</td>
<td>distance between nodes ( a ) and ( b )</td>
</tr>
<tr>
<td>( n_{a,b} )</td>
<td>variance of the channel coefficient ( h_{a,b} )</td>
</tr>
<tr>
<td>( n_{\text{RE}} )</td>
<td>received AWGN vector at node ( a ), of zero mean and variance ( N_0 )</td>
</tr>
<tr>
<td>( \gamma_i = P_{\text{out}} / N_0 )</td>
<td>primary, secondary outage probability threshold</td>
</tr>
<tr>
<td>( P_{\text{out},a} )</td>
<td>transmit power of node ( a ) (considering the noise effect)</td>
</tr>
<tr>
<td>( R_i )</td>
<td>targeted data rate of transmission ( i ) (( i = p ) or ( s ))</td>
</tr>
<tr>
<td>( \text{SNR}_p ), ( \text{SNR}_b )</td>
<td>SNR at node ( p ) and ( b )</td>
</tr>
<tr>
<td>( D )</td>
<td>case indicator</td>
</tr>
</tbody>
</table>

In order to simplify the understanding of the model, we summarise in Table 1 all the notations and symbols used in the next sections.

2 System model

We assume a secondary system that coexists with a primary system as described in Fig. 1. The PT sends data to a primary destination (PD) and a ST transmits its data to a secondary destination (SD) at the same time and over the same frequency band. We assume also the presence of a secondary RE that is equipped with multiple antennas (\( N \) antennas) whereas PT, PD and ST are equipped with a single antenna.

Time is split into time-slots and each time-slot is divided into two subslots. The channels are assumed quasi-stationary, that is, they are stationary during a time-slot, but vary independently from a time-slot to another.

Fig. 1 Primary and secondary transmissions in the network
another. Channels are modelled as Rayleigh fading channels. We denote by $a_{n, b}$ the Rayleigh coefficient of the channel $a-b$ with variance $a^2_{n, b} = d_{n, b}$, where $d_{n, b}$ is the distance between nodes $a$ ($a = PT, ST$ or $RE$) and $b$ ($b = PD, SD$ or $RE$) and $\beta$ is the path-loss exponent.

We assume that $PT$ and $ST$ transmit their signals $x_p$ and $x_s$ ($E(x^2_p) = E(x^2_s) = 1$, where $E(\cdot)$ is the expected value) with powers $P_{PT}$ and $P_{ST}$, targeting to achieve data rates $R_p$, and $R_s$, respectively. We assume also that $RE$ uses the transmit power $P_{RE}$. The transmit powers of $ST$ and $RE$ are assumed limited by maximal values denoted by $P^\text{max}_{ST}$ and $P^\text{max}_{RE}$, respectively.

### 3 Proposed scheme (adaptive relaying)

The transmissions in the adaptive relaying scheme are performed over two subslots as follows.

#### 3.1 First subslot transmissions

At the first subslot, both $PT$ and $ST$ broadcast their respective signals $x_p$ and $x_s$. Hence, the received signals at $PD$, $SD$ and $RE$, during the first subslot, are expressed by

$$y_{PD}(1) = \sqrt{P_{PT}} h_{PT, PD} x_p + \sqrt{P_{ST}} h_{ST, PD} x_s + n_{PD}$$  

$$y_{SD}(1) = \sqrt{P_{ST}} h_{ST, SD} x_s + \sqrt{P_{PT}} h_{PT, SD} x_p + n_{SD}$$  

$$y_{RE}(1) = \sqrt{P_{PT}} h_{PT, RE} x_p + \sqrt{P_{ST}} h_{ST, RE} x_s + n_{RE}$$

$$= \left[ \sqrt{P_{PT}} h_{PT, RE}, \sqrt{P_{ST}} h_{ST, RE} \right] \begin{bmatrix} x_p \\ x_s \end{bmatrix} + n_{RE}$$  

$$= H_{PE} x + n_{RE}$$

where $h_{PT, RE}$ and $h_{ST, RE}$ (of dimension $N \times 1$) are the vectors of the channels between the transmitters ($PT$ and $ST$) and $RE$. $n_b$ and $n_{RE}$ are the additive white Gaussian noise (AWGN) coefficient and vector of coefficients with zero mean and variances $N_0$ and $N_0 N_a$, received at $b$ and $RE$, respectively, ($b = PD$ or SD). We assume that $P_{PT}$ is fixed and that $P_{ST}$ is calculated with respect to the primary outage probability threshold target denoted $\epsilon_i$. That means

$$P^\text{opt}_{out, p} = P \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{2P_{PT} |h_{PT, PD}|^2}{P_{ST} |h_{ST, PD}|^2 + N_0} \right) \leq R_p \right\}$$

$$= P \left\{ \frac{2P_{PT} |h_{PT, PD}|^2}{P_{ST} |h_{ST, PD}|^2 + N_0} < \Lambda_p \right\} \leq \epsilon_i$$  

where $P^\text{opt}_{out, p}$ is the outage probability of the primary system when $PT$ repeats $x_p$ over the two subslots. The use of two subslots, $\Lambda_p = 2^{2R_p} - 1$ is partly remarked and $\| \cdot \|$ is the norm operator.

The random variables (RVs) $x = |h_{PT, PD}|^2$ and $y = |h_{ST, PD}|^2$ are exponentially distributed with parameters $1/\sigma_{PT, PD}^2$ and $1/\sigma_{ST, PD}^2$, respectively. Thus, using the joint probability density function (pdf) of $x$ and $y$, (4) can be given by

$$P^\text{opt}_{out, p} = \int_0^\infty \int_0^\infty e^{-\{(\gamma_p)\} - \{(\gamma_s)\} \sigma_{PT, PD}^2} \sigma_{ST, PD}^2 \, dx \
y = 1 - 2\Lambda_p^2 \sigma_{PT, PD}^2 \sigma_{ST, PD}^2 \sigma_{PD, PD}^2 \sigma_{ST, PD}^2 \leq \epsilon_i$$

where $\gamma_p = P_{PT}/N_0$ and $\gamma_s = P_{ST}/N_0$. From (5), one can see that $P_{ST} \leq 0$ if $e^{-\{(\gamma_p)\}/(2\gamma_p \sigma_{PT, PD}^2)} \leq 1 - \epsilon_i$. Hence $P_{ST}$ should be set to zero and $ST$ waits for another transmit opportunity. In this paper, we focus on the efficiency of the transmissions using the proposed adaptive relaying scheme without further considerations of adaptive power control at $ST$ [15]. Consequently, $P_{ST}$ is controlled to achieve the maximum average power allowed for transmission. Its expression is given by

$$P_{ST} = \min \{ \max (\rho, 0), P^\text{max}_{ST} \}$$

where

$$\rho = \frac{2 \sigma_{PT, PD}^2}{\Lambda_p \sigma_{ST, PD}^2} \left( \frac{e^{-\{(\gamma_p)\}/(2\gamma_p \sigma_{PT, PD}^2)} - 1}{1 - \epsilon_i} \right)$$

$\rho$ is the required transmit power of $ST$ in order to respect the primary outage threshold, derived by solving $P^\text{opt}_{out, p} = \epsilon_i$. The use of average channel gain based power allocation approach is motivated by the following reasons. First, instantaneous channels knowledge of the primary system at the secondary system is difficult and costly in terms of signalling. To overcome the variations of the channels, a certain amount of interference is tolerated when computing the transmit power that is determined by the constraint $\epsilon_i$ at the primary outage probability [7, 16]. Consequently, power allocation is performed on longer time-scale of log-normal shadowing rather than the time-scale of Rayleigh fading, and consequently resources for signalling are saved [15]. Second, the average channel gains of the primary system are relatively stable and can be estimated at the secondary nodes [7]. Finally, information about primary data rate and primary transmit power are assumed embedded within control messages that could be overheard by the secondary nodes when $PT$ requests to send data to $PD$. Regarding $P_{PT}$ and $\Lambda_p$, we assume that these information are embedded in control messages that could be overheard by the secondary nodes when $PT$ requests to send information to $PD$.

#### 3.2 Second subslot transmissions

According to (3), $RE$ attempts to decode both signals, $x_p$ and $x_s$. Using a zero-forcing receiver [17], the signal-to-noise ratios (SNR) of the primary and secondary signals, denoted
by $\text{SNR}_p$ and $\text{SNR}_s$, are given by

$$\text{SNR}_p = \left[ \left( H_{pt}^H H_{pt} \right)^{-1} \right]_{1,1}$$

and

$$\text{SNR}_s = \left[ \left( H_{pt}^H H_{pt} \right)^{-1} \right]_{2,2}$$

where $[X]_{i,j}$ is the $i$th element on the diagonal of matrix $X$. At RE, $x_p$ (respectively $x_s$) is considered successfully decoded if and only if $\text{SNR}_p \geq \Lambda_p$ (respectively $\text{SNR}_s \geq \Lambda_s$), where $\Lambda_j = 2^{2\gamma_j} - 1$ ($j = p$ or $s$) [9]. Consequently, one out of four relaying procedures is chosen at the second time-slot. The procedures are described below:

- RE assists neither transmissions ($D = 0$);
- RE assists only the primary transmission ($D = 1$);
- RE assists only the secondary transmission ($D = 2$); and
- RE assists both primary and secondary transmissions simultaneously ($D = 3$).

where the value of the parameter $D$ indicates the occurrence of which relaying procedure is used. We define by $\alpha_i (i = 0, \ldots, 3)$ the received signal-to-interference-plus-noise ratio (SINR) at SD for the cases $D = 0, \ldots, 3$, respectively. We define also the events $A_p = \{ \text{SNR}_p \geq \Lambda_p \}$ and $A_s = \{ \text{SNR}_s \geq \Lambda_s \}$. Hence, the values of the parameter $D$ are given as follows (see equation at the bottom of the page) where $\bar{A}$ is the complementary event of $A$. The comparisons of $(\alpha_i)_{i = 0, \ldots, 3}$ indicates which relaying procedure would improve best the secondary outage probability. We detail the different cases below:

### 3.2.1 RE assists neither transmissions ($D = 0$): 
When RE is unable to decode neither the primary nor the secondary signal or when relaying is not beneficial to the secondary outage probability, the relay does not participate in the transmissions. In this case, PT and ST retransmit the same signals using the same amounts of power. Accordingly, the received SINR at SD $(\alpha_0)$ is given by (7), [9] and that at PD is obtained by inverting indexes $p$ and $s$ in (7), [9]).

### 3.2.2 RE assists only the primary transmission ($D = 1$): 
This case occurs either when (i) RE succeeds to decode the primary signal, but not the secondary one and when relaying the primary signal provides lower secondary outage probability than the repetition (i.e. $\alpha_1 > \alpha_0$) or (ii) RE succeeds to decode both the primary and secondary signals and assisting the primary transmission provides the lowest secondary outage probability (i.e. $\alpha_1 = \max_{l=0,\ldots,3}(\alpha_l)$). Hence, when the relay is able to decode the primary signal and the best choice is to assist the primary transmission, then $D = 1$. Consequently, the received signals at PD and SD, on the second subslot, are, respectively, given by

$$y_{\text{PD}}(2|D = 1) = \sqrt{p_{\text{RE}}^{(p)} h_{\text{RE}}^{(p)} x_p} + \sqrt{p_{\text{SD}}^{(p)} h_{\text{SD}}^{(p)} x_s} + n_{\text{PD}}$$

and

$$y_{\text{SD}}(2|D = 1) = \sqrt{p_{\text{RE}}^{(p)} h_{\text{RE}}^{(p)} x_p} + \sqrt{p_{\text{SD}}^{(p)} h_{\text{SD}}^{(p)} x_s} + n_{\text{SD}}$$

where $h_{\text{RE}}^{(p)}$ (respectively $h_{\text{RE}}^{(s)}$) is the coefficient of the channel between the selected antenna at RE and PD (respectively the selected antenna at RE and SD). Using (1) and (9), the overall received signal at PD is written as

$$y_{\text{PD}} = \sqrt{p_{\text{PT}}^{(p)} h_{\text{PT}}^{(p)}} x_p + \sqrt{p_{\text{SD}}^{(p)} h_{\text{SD}}^{(p)}} x_s + n_{\text{PD}}$$

and

$$y_{\text{SD}} = \sqrt{p_{\text{RE}}^{(p)} h_{\text{RE}}^{(p)} x_p} + \sqrt{p_{\text{SD}}^{(p)} h_{\text{SD}}^{(p)} x_s} + n_{\text{SD}}$$

$y_{\text{PD}}$ is then multiplied by a weight vector to maximise the SINR at the receiver PD. Using OC, the optimal weight vector is given by Winters [13]

$$w_{\text{OC}} = h_p^{-1} h_p^*$$

where $h_p = h_p^h + N_0 I_2$, $h^h$ is the conjugate transpose operator and $\{ \cdot^* \}$ denotes the conjugate operator. Finally, the received SINR at PD is expressed by Winters [13]

$$\text{SINR}_p(D = 1) = h_p^{H} R_p^{-1} h_p$$

$$= \gamma_{\text{PT}} |h_{\text{PT}}^{(p)}|^2 + \gamma_{\text{RE}} |h_{\text{RE}}^{(p)}|^2 \gamma_{\text{ST}} |h_{\text{ST}}^{(p)}|^2 + 1$$

where $P_{\text{RE}}^{(p)}$ is RE’s transmit power used to assist the primary transmission and $\gamma_{\text{RE}} = \frac{P_{\text{RE}}^{(p)}}{N_0}$. By following (12)-(13), the received SINR at SD is similarly given by Winters [13]

$$\alpha_1 = \text{SNR}_s(D = 1) = h_s^{H} R_s^{-1} h_s$$

where

$$h_s = \sqrt{p_{\text{ST}}^{(s)} h_{\text{ST}}^{(s)} \begin{bmatrix} 1 & 1 \end{bmatrix}} = h_s^{H} + N_0 I_2$$

$$h_s' = \sqrt{p_{\text{PT}}^{(s)} h_{\text{PT}}^{(s)} \begin{bmatrix} p_{\text{RE}}^{(s)} h_{\text{RE}}^{(s)} \end{bmatrix}}$$

if $A_p \cap A_s \cap \left( \alpha_1 = \max_{l=0,\ldots,3}(\alpha_l) \right)$, then $D = 3$

if $A_s \cap \left\{ A_p \cap (\alpha_2 > \alpha_0) \right\} \cup \left\{ A_p \cap (\alpha_2 = \max_{l=0,\ldots,3}(\alpha_l)) \right\}$, then $D = 2$

if $A_p \cap \left\{ A_s \cap (\alpha_1 > \alpha_0) \right\} \cup \left\{ A_s \cap (\alpha_1 = \max_{l=0,\ldots,3}(\alpha_l)) \right\}$, then $D = 1$

otherwise, $D = 0$
and $\{\cdot\}^{\dagger}$ is the transpose operator. $\alpha_1$ is kept at the matrix form, since it cannot be simplified as in (13).

3.2.3 RE assists only the secondary transmission ($D = 2$): This case occurs when (i) RE succeeds to decode only the secondary signal and when relaying the secondary signal is beneficial to the secondary transmission (i.e. $\alpha_2 > \alpha_0$) or (ii) RE succeeds to decode both signals and assisting the secondary transmission provides the lowest secondary outage probability (i.e. $\alpha_3 = \max_{a=0,...,3}(\alpha_a)$). Hence, the received signals at PD and SD, on the second subslot, are written as

$$y_2(D=2) = \sqrt{p_{pt}}h_{pt,a}x_p + \sqrt{p_{re}}h_{re,a}x_s + n_a$$  \hspace{1cm} (15)

where $h_{re,a}^{(i)}$ is the coefficient of the channel between the selected antenna at RE and destination $a$ ($a = PD$ or SD). Similarly to (13) and (14), the received SINR at PD and SD are, respectively, expressed by

$$\text{SINR}_p(D = 2) = h_p^{ii}R_p^{-1}h_p^{II}$$  \hspace{1cm} (16)

where

$$h_p = \sqrt{p_{pt}}h_{pt,pd}\begin{bmatrix} 1 & 0 
\end{bmatrix}^{\dagger}, \hspace{1cm} R_p = h_p^{II}h_p^{II} + N_0I_2$$

$$h_p^{'} = \begin{bmatrix} \sqrt{p_{st}}h_{st,pd}\sqrt{p_{re}}h_{re,pd}^{(i)} \end{bmatrix}^{\dagger}$$

while

$$\alpha_2 = \text{SINR}_s(D = 2) = \frac{\gamma_{st}|h_{st,SD}|^2 + |h_{re,SD}|^2}{\gamma_{pt}|h_{pt,SD}|^2 + 1}$$ \hspace{1cm} (17)

where $p_{re}^{(i)}$ is RE’s transmit power used to assist the secondary transmission and $\gamma_{re}^{(i)} = \frac{p_{re}^{(i)}}{N_0}$

3.2.4 RE assists both transmissions ($D = 3$): It occurs when RE succeeds to decode both primary and secondary signals and forwarding both signals provides the best secondary outage probability ($\alpha_3 = \max_{i=1,...,N}(\alpha_i)$). In this case, two antennas are selected at RE to forward simultaneously the primary and secondary signals. Using one antenna to forward both signals is not presented in this paper, since it has been treated in our previous work [9]. Thus, the received signals at PD and SD at the second subslot are written as

$$y_2(D = 3) = \sqrt{p_{re}}h_{re,a}^{(i)}x_p + \sqrt{p_{re}}h_{re,a}^{(i)}x_s + n_a$$ \hspace{1cm} (18)

where $a$ = PD or SD. Using OC, the resulting SINR at PD and at SD are given by

$$\text{SINR}_p(D = 3) = h_p^{II}R_p^{-1}h_p$$ \hspace{1cm} (19)

where

$$h_p = \begin{bmatrix} \sqrt{p_{pt}}h_{pt,PD} & \sqrt{p_{re}}h_{re,PD} \end{bmatrix}^{\dagger}, \hspace{1cm} R_p = h_p^{II}h_p^{II} + N_0I_2$$

$$h_s = \begin{bmatrix} \sqrt{p_{st}}h_{st,PD} & \sqrt{p_{re}}h_{re,PD} \end{bmatrix}^{\dagger}, \hspace{1cm} R_s = h_s^{II}h_s^{II} + N_0I_2$$

$$h_s^{'} = \begin{bmatrix} \sqrt{p_{pt}}h_{pt,PD} & \sqrt{p_{re}}h_{re,PD} \end{bmatrix}^{\dagger}$$

and

$$\alpha_3 = \text{SINR}_s(D = 3) = h_s^{II}R_s^{-1}h_s$$  \hspace{1cm} (20)

where (see equation at the bottom of the page)

4 Antenna selection and power allocation

In this section, we describe and justify the selection criteria used for our proposed adaptive relaying scheme. Then, we investigate the associated power allocation problems.

4.1 Antenna selection

Various criteria can be used to select the transmit antenna at the RE. For example, one can select the antenna that yields the largest RE–SD channel gain. Alternatively, one can select the antenna that yields the minimum interference to PD. In [18], the two selection methods are referred to as ‘maximum data gain selection and minimum data gain selection’, respectively.

In this paper, we adopt similar antenna selection criteria as in [18]. Indeed, when RE assists the secondary transmission, the selected transmit antenna at RE is selected such that

$$|\theta_{re,SD}| = \max_{i=1,...,N} |\theta_{re,SD}(i)|$$, (criteria 1) \hspace{1cm} (21)

where $\theta_{re,SD}(i)$ is the coefficient of the channel between antenna $i$ at RE and SD. Since the main objective of relaying is to improve the secondary outage probability, this selection criterion maximises the received SINR at SD and realises a diversity gain.

Whereas, if RE is assisting the primary transmission, the transmit antenna is selected such that

$$|\beta_{re,SD}| = \min_{i=1,...,N} |\beta_{re,SD}(i)|$$, (criteria 2) \hspace{1cm} (22)

In order to favour the improvement of the secondary outage performance, the interference caused by the selected antenna in this case to the secondary transmission is required to be as small as possible, while assisting the primary transmission. One can observe that in case of minimising $\theta_{re,PD}$ instead, assisting the primary system would be less beneficial for both primary and secondary transmissions, since the caused interference to the secondary transmission could be high (the interference gain is not considered in the selection criterion).

When RE assists simultaneously both transmissions, two transmit antennas are selected. The selection is performed
by combining ‘selection criteria 1 and 2’ as follows: first, selecting the antenna that assists the secondary transmission following (21) among $N$ antennas, and second, selecting the antenna that assists the primary transmission following (22) among the $N-1$ remaining antennas. The antennas are selected in this order (‘criterion 1’ then ‘criterion 2’) to provide in average better received SINR at SD.

One can note that for the presented antenna selection criteria, the transmit power of RE is not included. Indeed, the transmit power is calculated with respect to the constraint on the average primary outage probability; hence, it depends only on the average channel states and not on the instantaneous channel states (details are in Section 4.2).

Moreover, the interfering gains are not considered in the criteria expressions. Indeed, we assume that RE has knowledge of the average system channel gains (estimated as in (15)) and the instantaneous channel gains linking it to both primary and secondary systems (using pilot symbols when it is receiving, by over-hearing control messages when it is idle and/or via feedback channels) [19, 20]. This knowledge allows RE to proceed with power allocation and then with transmission procedure choice. Meanwhile, antenna selection (for both ‘criterion 1 and 2’) is performed at the SD and its decision is fed back to RE via a low rate signalling channel (In the case of full channel state availability at RE, more efficient techniques could be used to improve the transmissions’ performances such as interference cancellation using beam-forming at RE. However, with limited channel knowledge at RE, using any transmit diversity technique is expected to be somewhat less efficient than antenna selection [21]).

4.2 Power allocation

For each relaying procedure (cases $D = 0, \ldots, 3$), the transmit power at RE should be chosen in order to improve the secondary outage probability with respect to the primary outage probability threshold $\varepsilon$. Note that respecting the primary outage threshold on each case yields to satisfying $\varepsilon$ by the overall primary outage probability given by

$$P_{\text{out,p}} = \sum_{l=0}^{3} P(D = l)P_{\text{pri(out.|D = l}}} 
$$

where $P(D = l)$ is the probability of occurrence of each case (with $\sum_{l=0}^{3} P(D = l) = 1$) and $P_{\text{pri(out.|D = l}}} $ is the primary outage probability conditioned on $D = l$.

4.2.1 $D = 0$

In this case, PT and ST repeat their signals over both subslots using transmit powers $P_{PT}$ and $P_{ST}$ (given by (6)), respectively.

4.2.2 $D = 1$

By assisting the primary transmission, we aim to reduce the interference it causes to the secondary transmission (when it exists) with respect to the threshold target $\varepsilon$, or simply help the primary communication converge rapidly to $\varepsilon$ (when no secondary access is allowed). The associated power allocation problem can be formulated as it follows

$$\arg \max_{P_{\text{RE}}^{(p)}} \text{SINR}_{\text{p}}(D = 1) 
$$

s.t. \begin{equation} P_{\text{pri(out.|D = 1}}} = \varepsilon 
\end{equation}

where $P_{\text{pri(out.|D = 1}}$ denotes the primary outage probability conditioned on $D = 1$ and is expressed by

$$P_{\text{pri(out.|D = 1)}} = P\left( \text{SINR}_{\text{p}}(D = 1) < \Lambda_{p} \right) = P\left( \omega < \Lambda_{p} + \Lambda_{p}\alpha_{1} - \alpha_{2} \right) 
$$

where $\alpha_{1} = \gamma_{\text{RE}}^{(p)}|h_{\text{RE,PD}}^{(p)}|^{2}$, $\alpha_{2} = \gamma_{\text{ST}}|h_{\text{ST,PD}}|^{2}$ and $\omega_{2} = \gamma_{\text{PT}}|h_{\text{PT,PD}}|^{2}$. If $\gamma_{\text{ST}} = 0$, then the conditional primary outage probability is given straight-forward by (see (26)) where $|h_{\text{PT,PD}}|^{2}$ and $|h_{\text{RE,PD}}^{(p)}|^{2}$ are exponential RVs of parameters $1/\sigma_{\text{PT,PD}}^{2}$ and $1/\sigma_{\text{RE,PD}}^{2}$, respectively, and where $\gamma_{\text{RE},a}^{(p)} = \gamma_{\text{RE},a}^{(p)}\alpha_{PD,a}^{(p)}$ ($a = \text{PD or SD}$ and $j = p$ or $s$).

If $\gamma_{\text{ST}} > 0$, we make use of Lemma 1 given below.

**Lemma 1**: The exact closed-form expression of the conditional primary outage probability is expressed as

$$P_{\text{pri(out.|D = 1)}} = \begin{cases} \lambda_{1} + \lambda_{2}, & \text{if } \gamma_{\text{ST}} > 0 \text{ and } \tilde{\gamma}_{\text{PT,PD}} \neq \tilde{\gamma}_{\text{RE,PD}}^{(p)} \\ \lambda_{3} & \text{if } \gamma_{\text{ST}} > 0 \text{ and } \tilde{\gamma}_{\text{PT,PD}} = \tilde{\gamma}_{\text{RE,PD}}^{(p)} \end{cases} 
$$

where

$$\lambda_{1} = \frac{2^{2}\Lambda_{p}^{2} + \tilde{\gamma}_{\text{ST,PD}}^{(p)} + \tilde{\gamma}_{\text{RE,PD}}^{(p)}\alpha_{PD}}{\tilde{\gamma}_{\text{ST,PD}}^{(p)} + \tilde{\gamma}_{\text{RE,PD}}^{(p)}\alpha_{PD}} 1 - e^{-\gamma_{\text{ST}}/\tilde{\gamma}_{\text{RE,PD}}^{(p)}} \Gamma_{\text{pd}}^{(p)}/\tilde{\gamma}_{\text{RE,PD}}^{(p)} 
$$

(27)

$$\lambda_{2} = \frac{1}{\gamma_{\text{ST}}/\tilde{\gamma}_{\text{RE,PD}}^{(p)}} 
$$

(26)

$\tilde{\gamma}_{\text{ST,PD}}^{(p)}$ and $\tilde{\gamma}_{\text{RE,PD}}^{(p)}$ are exponential RVs of parameters $1/\sigma_{\text{RE,PD}}^{2}$ and $1/\sigma_{\text{ST,PD}}^{2}$, respectively, and where $\tilde{\gamma}_{\text{RE},a}^{(p)} = \gamma_{\text{RE},a}^{(p)}\alpha_{PD,a}^{(p)}$ ($a = \text{PD or SD}$ and $j = p$ or $s$).
2 and \(3\) are given by (29) and (30) respectively.

**Proof:** The proof is provided in Appendix.

Since (27) has a complicated expression, we opt for a numerical resolution to obtain the required \(P^{(p)}_{\text{RE}}\) value that satisfies \(\epsilon_t\).

4.2.3 \(D=2\):

In this case, we aim to improve the secondary outage probability. The power allocation problem is hence written as

\[
\text{arg max}_{P^{(p)}_{\text{RE}}} \text{SINR}_s(D=2) \quad \text{subject to} \quad \begin{cases} P_{\text{pre,out.}}(D=2) = \epsilon_t \\ 0 \leq P^{(p)}_{\text{RE}} \leq P^{\max}_{\text{RE}} \end{cases}
\]

with \(P_{\text{pre,out.}}(D=2)\) is the primary outage probability conditioned on \(D=2\). It can be approximated by the following expression [13]

\[
P\left(\text{SINR}_p(D=2) < \Lambda_p\right) \approx \frac{1 + 2\gamma_l}{2\gamma_l} \left(1 - e^{-\left(\Lambda_p/\gamma\right)}\right) - \frac{1}{2\gamma_l} \left(1 - e^{-\left(\left(\Lambda_p/\gamma\right)+(1+2\gamma_l)\right)}\right) \quad (32)
\]

where \(\gamma = \gamma_{PT} \sigma^2_{PT,PD} \) and \(\gamma_l = \frac{1}{2} \left(\gamma_{ST} \sigma^2_{ST,PD} + \gamma_{RE} \sigma^2_{RE,PD}\right)\).

Similarly to the previous case, a numerical resolution is opted to calculate the best value of \(P^{(p)}_{\text{RE}}\) as

4.2.4 \(D=3\):

By assisting both primary and secondary transmissions, the relaying procedure aims to improve the secondary outage performance with respect to the primary outage probability threshold. The power allocation problem is formulated as follows

\[
\text{arg max}_{P^{(p)}_{\text{RE}}} \text{SINR}_s(D=3) \quad \text{subject to} \quad \begin{cases} P_{\text{pre,out.}}(D=3) = \epsilon_t \\ 0 \leq P^{(p)}_{\text{RE}} \leq P^{\max}_{\text{RE}} \end{cases}
\]

where the exact closed-form expression of \(P_{\text{pre,out.}}(D=3)\) is given by (32) with

\[
\gamma = \frac{1}{2} \left(\gamma_{PT} \sigma^2_{PT,PD} + \gamma_{RE} \sigma^2_{RE,PD}\right)
\]

and

\[
\gamma_l = \frac{1}{2} \left(\gamma_{ST} \sigma^2_{ST,PD} + \gamma_{RE} \sigma^2_{RE,PD}\right)
\]

The last constraint in (33) is a sum constraint that reflects the limit of using CR relaying in the proposed scheme. Finally, a numerical resolution is used to obtain the optimal \(P^{(p)}_{\text{RE}}\) and \(P^{\max}_{\text{RE}}\) values.

## 5 Simulation results and discussion

We have conducted the computer simulations using the following parameters:

- We assume the network of Fig. 1, where the coordinates of PT, PD, ST, SD and RE are (0, 2), (1, 2), (0, 0), (1, 0) and \((x, y)\), respectively.
- Targeted primary and secondary spectral efficiency are \(R_p = 1 \text{ bit/s/Hz}\) and \(R_s = 0.5 \text{ bit/s/Hz}\), respectively.
- The primary outage probability threshold target is \(\epsilon_t = 0.02\) and the path-loss exponent \(\beta = 4\).
- The maximum transmit powers of ST and RE are equal to \(\gamma_{ST} = \gamma_{RE} = 30\text{dB}\).
- RE is equipped with \(N=2\) antennas (unless otherwise stated).

The primary and secondary outage probabilities, \(P_{\text{out.}}\) and \(P_{\text{out.}}\) are evaluated based on the average result of several positions of the RE such that RE’s coordinates are \((0, 0) \leq (x, y) \leq (1, 2)\).

The proposed ‘adaptive relaying scheme’ is compared with non-cooperative and cooperative reference schemes described as follows.

In the ‘non-cooperative scheme’, primary and secondary transmissions are processed simultaneously over two time slots. PT and ST repeat the same signals using the same powers over both subslots. In order to respect the primary outage probability threshold \(\epsilon_t\), ST adjusts its transmit power as given in (6).

In the ‘conventional cooperative scheme’, the RE is able to assist only the secondary transmissions, whereas primary transmissions use repetition over both subslots.

\[
\lambda_2 = \frac{\gamma_{PT,PD}^2 \left(1 - e^{-\left(\Lambda_p/\gamma_{PT,PD}\right)}\right) - \gamma_{PT,PD}^2 \gamma_{RE,PD} \left(1 - e^{-\left(\Lambda_p/\gamma_{RE,PD}\right)}\right)}{\left(\gamma_{PT,PD} + \Lambda_p \gamma_{ST,PD}\right) \left(\gamma_{RE,PD}^2 - \gamma_{PT,PD}\right)}
\]

\[
\lambda_3 = \frac{1}{\left(\gamma_{PT,PD} + \Lambda_p \gamma_{ST,PD}\right)^2} \left[\gamma_{PT,PD}^2 \left(1 - e^{-\left(\Lambda_p/\gamma_{PT,PD}\right)}\right) - \Lambda_p \gamma_{PT,PD} e^{-\left(\Lambda_p/\gamma_{PT,PD}\right)}\right] + \frac{\left(\Lambda_p \gamma_{ST,PD}\right)^2}{\left(\gamma_{PT,PD} + \Lambda_p \gamma_{ST,PD}\right)^2} \left[1 + \gamma_{PT,PD} \left(1 - e^{-\left(\Lambda_p/\gamma_{PT,PD}\right)}\right)\right]
\]

\[
\lambda_2 = \frac{1}{\left(\gamma_{PT,PD} + \Lambda_p \gamma_{ST,PD}\right)^2} \left[\gamma_{PT,PD}^2 \left(1 - e^{-\left(\Lambda_p/\gamma_{PT,PD}\right)}\right) - \Lambda_p \gamma_{PT,PD} e^{-\left(\Lambda_p/\gamma_{PT,PD}\right)}\right]
\]

\[
\lambda_3 = \frac{1}{\left(\gamma_{PT,PD} + \Lambda_p \gamma_{ST,PD}\right)^2} \left[1 + \gamma_{PT,PD} \left(1 - e^{-\left(\Lambda_p/\gamma_{PT,PD}\right)}\right)\right]
\]
Accordingly, ST controls its power using (6) at the first subslot, and RE controls its power similarly, that is given by [7] (see (34)).

In Fig. 2, we compare the primary outage probability of the ‘adaptive relaying scheme’, ‘non-cooperative scheme’ and ‘conventional cooperative scheme’ as a function of $\gamma_{PT}$. We note that the primary outage probability converges faster to the target $\varepsilon_t$ for ‘adaptive relaying scheme’ than for the other schemes (at $\gamma_{PT} \simeq 15$ dB rather than $\gamma_{PT} \simeq 18$ dB).

Indeed, ‘adaptive relaying scheme’ assists the primary transmission opportunistically, hence improving the primary outage probability. Fig. 2 indicates that as $\gamma_{PT}$ increases above 18 dB, all the schemes maintain the primary outage probability below $\varepsilon_t$.

In Fig. 3, we compare the secondary outage probability for the three schemes, used in the same network configuration. All schemes present a cutoff point at approximately $\gamma_{PT} \simeq 18.7$ dB (This exact value of the cutoff point is

$$P_{dRE}^{(s)} = \min \left[ \max \left( \frac{2P_{PT}^2 \sigma_{PT,PD}^2}{\Lambda_p \sigma_{RE,PD}^2} \left( \frac{e^{-\Lambda_p (2 \gamma_{PT} \sigma_{PT,PD})}}{1 - \varepsilon_t} - 1 \right), 0 \right), \gamma_{RE}^{\text{max}} \right]$$

(34)
obtained using ((11), [10]). Below this point, no secondary access is allowed, since the primary system does not converge to its targeted primary outage probability, or did it in the absence of interfering signals. For $\gamma_{PT} > 18.7$ dB, the secondary outage performance improves rapidly and ‘adaptive relaying scheme’ outperforms significantly all the other schemes. This is expected, since the proposed scheme works in an opportunistic manner by choosing the best relaying procedure among four different procedures in order to improve the received SINR at SD.

Fig. 4 illustrates the total average transmit power required to perform the primary transmission (called primary power) and the total transmit power provided for the secondary transmission (called secondary power) during a time-slot.

$$\gamma_{PT} = \frac{1}{\gamma_{ST}} = \frac{1}{\gamma_{RE}} = 30 \text{ dB}$$

Fig. 5 Average consumed power by primary node (PT) and by secondary nodes (ST and RE) as functions of $\gamma_{PT}$, with $N = 2$, $\varepsilon_t = 0.02$ and $\gamma_{ST} = \gamma_{RE} = 30 \text{ dB}$
For the ‘non-cooperative scheme’, $2\gamma_{PT}$ and $2\gamma_{ST}$ are the primary and secondary powers, respectively. The secondary power is greater than 0 starting from the cutoff point $\gamma_{PT} \approx 18.7$ dB. ‘Conventional cooperative scheme’ uses the same primary power as ‘non-cooperative scheme’, but slightly less secondary power, since in this case RE is able to use a smaller amount of power to relay the secondary signal. When $\gamma_{PT} \leq 15$ dB, the primary power of ‘adaptive relaying scheme’ is larger than that for the other schemes, since RE helps in forwarding the primary signal. For $\gamma_{PT} \geq 15$ dB, less primary power is provided. Indeed, as $\gamma_{PT}$ increases, less power is needed at RE to satisfy the primary outage probability target $\varepsilon_t$. Meanwhile, the secondary power of the proposed scheme is slightly lower than that of the other schemes. Indeed, RE uses less average power in order to assist the secondary or both transmissions simultaneously.

Fig. 5 presents the total average transmit power consumed by the primary node (PT) and the secondary nodes (ST and RE) during a time-slot as a function of $\gamma_{PT}$. For the conventional schemes (‘non-cooperative scheme’ and ‘conventional cooperative scheme’), the power consumed by the nodes is equal to the power required for the transmissions (depicted in Fig. 4). For ‘adaptive relaying scheme’, a lower transmit power is used by PT, since RE (SU) might assist the primary transmission on the second subslot. When $\gamma_{PT} \leq 18.7$ dB, RE uses its power to forward the primary signal, hence increasing the consumed power (ST and RE). For $\gamma_{PT} > 18.7$ dB, ‘adaptive relaying scheme’ uses less transmit power (ST and RE) than the conventional

Fig. 6 Secondary outage probability against RE’s position (adaptive relaying scheme; $N = 2$, $\varepsilon_t = 0.02$, $\gamma_{PT}^{\text{max}} = \gamma_{RE}^{\text{max}} = 30$ dB and $\gamma_{PT} = 20$ dB)

Fig. 7 Secondary outage probability against $R_p$ and $R_s$ (adaptive relaying scheme; $N = 2$, $\varepsilon_t = 0.02$, $\gamma_{ST}^{\text{max}} = \gamma_{RE}^{\text{max}} = 30$ dB and $\gamma_{PT} = 20$ dB)
schemes, since it is more likely that RE assists the secondary transmission or both transmissions using a lower power.

Fig. 6 presents the secondary outage probability of ‘adaptive relaying scheme’ against the relay position in the square zone (PT, PD, SD and ST). When RE is close to the primary nodes, the outage performance does not improve much since the occurring procedures are $D = 0$ and $D = 1$. Hence, the most important gain is obtained only from the interference reduction. As RE moves towards the secondary nodes, the outage probability decreases and reaches a first minimum for $y_{\text{RE}} \simeq 1.1$. In this region, RE is able to often decode $x_p$ and $x_s$ and hence assists simultaneously the primary and secondary transmissions. When RE becomes closer to the secondary nodes, the best outage performance is achieved in the middle region between ST and SD. Indeed, the cases $D = 2$ and $D = 3$ are predominant, which improve significantly the received SINR at SD. Hence, the most important gain is obtained from the relaying channel towards SD. We note also that the outage performance is better when RE is closer to the source nodes rather than to the destination nodes. Indeed, when RE is next to the source nodes, its decoding ability is improved.

In Fig. 7, we illustrate the secondary outage probability of ‘adaptive relaying scheme’ for different primary and secondary targeted spectral efficiencies $R_p$ and $R_s$. As $R_p$ increases, $P_{\text{out,s}}$ degrades in an exponential fashion, reaching $P_{\text{out,s}} = 1$ for $R_p = 1.2\text{bit/s/Hz}$. It indicates that no simultaneous secondary access is granted, starting from this $R_p$ value. However, as $R_s$ increases, the degradation of $P_{\text{out,s}}$ is logarithmic. Since the primary communication must satisfy its outage probability target $e_v$, the secondary communication has to be transparent to the PUs at all times. Consequently, this condition limits the desired secondary spectral efficiency.

6 Conclusions

In this paper, we have proposed and analysed a new cooperative transmission scheme for CRNs, where a multi-antenna RE selects the ‘best’ antenna(s) to assist opportunistically either the primary, the secondary or both transmissions simultaneously. By deriving the expressions of the conditional primary outage probability under the constraint of satisfying a given primary outage probability threshold, we investigate the power allocation issues. Finally, we showed through simulation results that the outage probability of both primary and secondary transmissions are significantly improved using the proposed cooperative scheme, and outperform that of non-cooperative and basic cooperative schemes presented in the literature.

7 Acknowledgment

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8 References


9 Appendix

9.1 Proof of Lemma 1

From (25), $\omega, \omega_1$ and $\omega_2$ are independent exponential RVs with parameters $1/\gamma_{\text{RE,PD}}^P$, $1/(\gamma_{\text{ST,PD}}^P)$ and $1/(\gamma_{\text{PT,PD}}^P)$, respectively. Thus, the pdfs of $\omega$ and $\phi = \omega_2 - \Lambda_p \omega_1$ are expressed by

$$f_\omega(\omega) = \frac{e^{-\omega/\gamma_{\text{RE,PD}}^P}}{\gamma_{\text{RE,PD}}^P}, \quad \omega \geq 0;$$

and

$$f_\phi(\phi) = \begin{cases} \frac{e^{-\phi/\gamma_{\text{PT,PD}}^P}}{\gamma_{\text{PT,PD}}^P + \Lambda_p \gamma_{\text{ST,PD}}^P}, & \phi \geq 0 \\ \frac{e^{\phi/\gamma_{\text{PT,PD}}^P}}{\gamma_{\text{PT,PD}}^P + \Lambda_p \gamma_{\text{ST,PD}}^P}, & \phi \leq 0 \end{cases}$$
Then, the pdf of $Z = \omega + \phi$ and $f_Z(z)$, is given by (see (35)) where $\beta_1 = \tilde{\gamma}_{PT,PD} + \Lambda_p \tilde{\gamma}_{ST,PD}$ and $\beta_2 = \tilde{\gamma}^{(p)}_{RE,PD} + \Lambda_p \tilde{\gamma}_{ST,PD}$.

Finally, $P(Z < \Lambda_p)$ is obtained using the expressions of the pdf developed in (35). This completes the proof of Lemma 1.