Expanded Decorrelating Detector with Reduced Noise Enhancement for Multipath Frequency-Selective Fading Channels

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Abstract – A novel hybrid multiuser detection scheme that jointly uses linear and nonlinear interference suppression techniques is developed for high-speed direct-sequence code-division multiple-access (DS/CDMA) communications in multipath frequency-selective fading channels. The detector detects signals in a symbol-by-symbol fashion. Conventional decorrelating detectors suffer from the noise enhancement problem, and the problem becomes more severe for dispersive multipath channels. The proposed detector uses interference cancellation (IC) technique to reduce the rank of the expanded signal subspace and hence it enjoys the advantages of the expanded decorrelating detector in terms of complete multiple access interference (MAI) and intersymbol interference (ISI) suppression and avoids its disadvantage in terms of noise enhancement. Computer simulation shows clear superiority of the new detector to other existing methods.

Index Terms: DS-CDMA, multiuser detection, expanded decorrelating detector, interference cancellation, multipath frequency-selective fading channel
I. Introduction

Code-division multiple-access (CDMA) is the core technique in all 3G mobile communication standards. It is also used in satellite communications [1] and other wireless systems such as Wi-Fi [2] and ZigBee [3]. Undoubtedly, CDMA will continue to play a pivotal role in future generation of wireless communications. It is well known that CDMA communication systems are interference limited, that is, the system capacity is limited by the multiple access interference (MAI) [4]. In the recent decade, there have been significant efforts in developing multiuser detection technologies to overcome the MAI problem. The maximum likelihood sequence estimator is the optimum detector in terms of bit error rate (BER) performance [5], but its complexity grows exponentially with the number of users, which is prohibitive for practical implementation. Linear multiuser detectors, such as the decorrelating detector [4] [6] [7] and the minimum mean square error (MMSE) detector [8], are suboptimum solutions, but they have low complexity, which only grows linearly with the number of users. The adaptive detectors are preferable to their non-adaptive counterparts because they do not need online computation of the crosscorrelation matrix and can adapt to dynamic environment by learning from the received data. The adaptive MMSE detector introduced in [9] is derived using training sequence and the stochastic gradient descent training algorithm. This training-based method does not require users’ spreading codes, and is therefore suitable for downlink implementation. The need for training sequence is further avoided in the blind multiuser detection scheme proposed in [10], where a blind MMSE detector is developed by minimizing the mean output energy (MOE) subject to an orthogonality constraint. Its adaptive implementation is realized by using the least-mean-square (LMS) method. Another approach to blind detection is based on subspace estimation. In [11], the decorrelating detector and the MMSE detector are expressed in closed forms of the signal subspace components, and the signal subspace is tracked by using the project approximation subspace tracking deflation (PASTd) algorithm. This subspace approach is extended to the antenna array system in [12], resulting in a blind subspace-based spatio-temporal decorrelator and spatio-temporal MMSE detector. In [13] and [14], the blind detectors are expressed as variables in the signal subspace and the coefficients are adaptively derived by LMS and Kalman filtering methods, respectively. Parallel to the development of the linear detectors, there are also numerous efforts in developing interference cancellation (IC) technologies [4] [15] [16]. IC is a category of nonlinear methods that subtract the estimated interfering signals when demodulating the signal of the interested user. There are basically two classes of IC techniques, namely,
successive interference cancellation (SIC) [15] which subtracts interference in a sequential order, one user at a time; and parallel interference cancellation (PIC) [16] which subtracts the interference from all interfering users simultaneously. IC detectors are bootstrapping technologies- correct decisions will lead to correct recreation and subtraction of the interfering signals at the receiver, while wrong decisions will double the contribution of the interferer, so conventional IC methods are only suitable for high SNR environments.

In practical high-speed wireless communications, the intersymbol interference (ISI) caused by multipath propagation imposes significant concern. One way to overcome the effect of ISI is to employ multicarrier technique to transform a frequency selective fading channel to flat fading channels [17] [18] [19]. However, since the multipath signals become non-resolvable, the multicarrier approaches forgo the opportunity to make use of the effective multipath diversity gain. RAKE receiver is commonly used to solve the multipath problem [20], but it is optimum only in single-user systems. For multiuser systems, the RAKE structure can still be used but the matched filters in the front end are replaced by linear multiuser detectors [21] [22] [23]. In [24], the MOE detector in [10] is extended into multipath channel by using a Min/Max approach to simultaneously optimize the output energy and a constraint vector. The optimal constraint vector is a biased estimate of the channel vector. The estimation error is proportional to the channel noise and it causes inferior performance compared with the MMSE detector. In [25], the affect of noise is mitigated by using a Power of R (POR) method to virtually increase the SNR. The focus of this paper is the decorrelating detectors, which have been proposed in [6] [7] and [11] for multiuser detection in AWGN channel. For multipath frequency-selective fading channel, the ISI is modelled as virtual MAI in [26] and hence an expanded decorrelating detector is derived to simultaneously suppress MAI and ISI. The expanded decorrelating detectors are not limited by the channel length. Meanwhile, they maintain the desirable properties of distributed implementation with short detection delays. However, all decorrelating detectors have the disadvantage of noise enhancement, and the expanded decorrelating detector operating in a higher-rank subspace amplifies the noise even more significantly. This paper aims at solving this problem by using a hybrid detection strategy that combines linear and nonlinear methods to effectively suppress both MAI and ISI without noise enhancement. The rationale is that the received signals in neighbouring time epochs are highly correlated after convolving with multipath channel. Therefore, the detected
information bits in one time epoch can be used to improve the signal detection in the next time epoch, e.g., the interfering signals from the previous epochs are deducted from the received signal so that the quality of detection can be improved. Consequently, the decorrelating detector operates in a rank-reduced signal subspace and the noise enhancement factor is reduced and keeps static over a wide range of multipath spreads and number of user. The performance gain is especially significant for systems with long delay spread and heavy loads. Meanwhile, the rank-reduced decorrelating detector has reduced complexity compared with the conventional expanded decorrelating detectors, and the decorrelating detector mitigates the error propagation problem caused by incorrect decision feedback.

An iterative detection strategy can be further derived from this hybrid structure. In each iteration, by feeding back and forward the information from the decorrelating detector, all ISI can be deducted from the input signal so that the decorrelating detector operates in a signal subspace with further reduced rank. Performance gains are obtained compared to the non-recursive hybrid detector, but the improvement is not significant, since the latter has already approached the performance of the decorrelating detector in Gaussian channel. This novel hybrid detection structure with improved system performance is the first contribution of this paper.

The second contribution of this paper is the development of a remedy algorithm to tackle the subspace collapse problem, which could happen to the expanded decorrelating detectors when the subspace rank is deficient under certain channel conditions. This problem can severely deteriorate the performance of the detector. The proposed remedy algorithm detects the rank deficiency and prevents malfunction in the expanded decorrelating detector.

This paper is focused on the synchronous multipath channel. However, asynchronous multipath channels can be modelled similarly as synchronous channels. The only difference is the time shift of the interfering signals (both ISI and MAI) according to the relative delay of each user. With the known or estimated delay information, the proposed hybrid detection strategy can be easily extended to asynchronous channels.
II. Signal Model for Multipath Synchronous Channel

We consider a synchronous $K$-user direct-sequence (DS) CDMA system employing r-PSK/QAM modulation to transmit signal over multipath frequency-selective fading channels. The signal transmitted by the $k$th user can be written as

$$x_k(t) = \sum_{i=-\infty}^{\infty} A_k b_k(i)c_k(t-iT)$$  \hspace{1cm} (1)

where $A_k$, $b_k(i)$ and $c_k(t)$ are the amplitude, $i$th transmitted information symbol, and normalized signature waveform of the $k$th user, respectively; $T$ is the symbol interval. $b_k(i) \in \{+1,-1\}$ follows identical and independent distribution (i.i.d.), and $c_k(t)$ is defined in $[0,T]$. In DS-CDMA systems, $c_k(t)$ is of the form

$$c_k(t) = \sum_{n=0}^{N-1} c_k(n)\psi(t-NT), \quad t \in [0,T], k = 1,..., K$$  \hspace{1cm} (2)

where $N$ is the spreading gain; $(c_k(n))_{n=0}^{N-1}$ is the real-valued and normalized signature code assigned to the $k$th user. $\sum_{n=0}^{N-1} c_k^2(n) = 1$; $T_c$ is the chip interval, $NT_c = T$; $\psi(t)$ is a normalized chip waveform defined in $[0,T_c]$. $\int_0^{T_c} \psi^2(t)dt = 1$.

In the case when a spreading sequence bandwidth $W$ is much larger than the coherence channel bandwidth, the tapped delay line model is widely used for multipath frequency-selective channels (Chap.4, [14]). In this work, we adopt a tapped delay line model with tap spacing $1/W$ and tap coefficients $(h_k(l))_{l=0}^{L-1}$, where $L$ is the number of resolvable paths for each user. We consider the case when $W = 1/T_c$, $L = \lceil T_m/T_c \rceil$, where $\lceil x \rceil$ denotes the value which is obtained by rounding $x$ to the nearest integer, which is greater or equal to $x$. $T_m$ is the multipath delay spread. In literature, it is usually assumed that $L < N$, i.e., the multipath spread is less than one symbol duration. In this paper, we consider a more general and realistic scenario in high-speed CDMA communications where the delay spread can be arbitrarily long. The impulse response of the multipath channel is modeled as

$$h_k(t) = \sum_{l=0}^{L-1} h_k(l)\delta(t - \frac{l}{W})$$  \hspace{1cm} (3)

where $\delta(t)$ is the Dirac’s delta function. Transmitting the signals over the multipath channel expressed
by (3), the received signal due to the $k$th user is given by

$$y_k(t) = x_k(t)^*h_k(t) = \sum_{i=-\infty}^{\infty} A_kb_k(i)\sum_{l=0}^{L-1} h_k(l)c_i(t-iT - \frac{l}{W}) \quad (4)$$

where $\ast$ denotes the convolution operation. Finally, the total received signal is the superposition of $K$ users’ information-bearing data signals plus the AWGN, and can be expressed as

$$r(t) = \sum_{k=1}^{K} y_k(t) + v(t) \quad (5)$$

where $v(t)$ is a zero-mean white Gaussian noise.

The received signal is filtered by a chip-matched filter and then sampled at chip rate, resulting in a discrete-time format of the signal model, in which the signal component due to the $p$th chip of the $m$th symbol from the $k$th user is given by

$$y_k(m, p) = \int_{mT+pT_c}^{(m+i+1)T} y_k(t)\psi(t-mT-pT_c)dt = \sum_{i=-\infty}^{\infty} A_kb_k(i)\sum_{l=0}^{L-1} h_k(l)c_i(p+mN-l-iN) \quad (6)$$

Define $j = p + mN$, $y_k(j) = y_k(m, p)$, equation (6) can be rewritten as

$$y_k(j) = \sum_{i=-\infty}^{\infty} A_kb_k(i)\sum_{l=0}^{L-1} h_k(l)c_i(j-l-iN) = \sum_{i=-\infty}^{\infty} A_kb_k(i)s_k(j-iN) \quad (7)$$

where

$$s_k(n) = \sum_{l=0}^{L-1} h_k(l)c_k(n-l) \quad (8)$$

Comparing (7) with (1), one can see that the received signature code is the distorted version of the original signature code after convolving with the multipath channel. Finally, the total received signal is expressed in discrete-time format as

$$r(j) = \sum_{k=1}^{K} y_k(j) + v(j) \quad (9)$$

where $v(j) = \int_{jT_c}^{(j+1)T_c} v(t)\psi(t-jT_c)dt$.

At the output of the transmitter, the energy of each symbol is limited to duration $NT_c$. However, a close examination at (8) reveals that in the received signal, the energy of each symbol has been spread over an extended interval $(N + L - 1)T_c$ due to the channel convolution, and hence the original
spreading sequence \( \{c_k(n)\}_{n=0}^{N-1} \) of the length \( N \) becomes the actual received signature sequence \( \{s_k(n)\}_{n=0}^{N+L-2} \) of length \( N+L-1 \). Naturally, we employ an \((N+L-1)\)-observation window as illustrated in Figure 1. Note that this diagram just shows a special case when \( N<2N \), but in the theoretical derivation, we do not impose any constraint on the value of \( L \). To detect each information bit, \( N+L-1 \) samples are stacked to form the received signal vector, and \( L-1 \) samples of them can be reused to detect the next bit, resulting in some saving in sampling time and memory. Falling within the observation window are not only the information bits at the current time instant, but also those from the preceded instants and the succeeded instants. The received signal vector therefore contains intersymbol interference from the past as well as from the future symbols. To state this problem more clearly, we define \( P = \lceil (L-1)/N \rceil \). Then, in the processing window of length \( N+L-1 \), \( 2P+1 \) symbols of the desired user are involved, including the current symbol, \( P \) symbols before and \( P \) symbols after the current symbol. Taking account of all the \( K \) users, there are \((2P+1)K\) symbols involved in detecting each symbol from the desired user. To summarize, the \((N+L-1)\times 1\)-dimensional received signal vector for this system can be expressed as

\[
\mathbf{r}(i) = \sum_{k=1}^{K} \mathbf{y}_k(i) + \mathbf{v}(i) = \sum_{k=1}^{K} \left[ A_k \mathbf{b}_k(i) \mathbf{s}_k + \sum_{j=1}^{P} A_k \mathbf{b}_k(i-j) \mathbf{\bar{s}}_{k,j} + \sum_{j=1}^{P} A_k \mathbf{b}_k(i+j) \mathbf{\bar{\bar{s}}}_{k,j} \right] + \mathbf{v}(i)
\]

where

\[
\mathbf{y}_k(i) = A_k \mathbf{b}_k(i) \mathbf{s}_k + \sum_{j=1}^{P} A_k \mathbf{b}_k(i-j) \mathbf{\bar{s}}_{k,j} + \sum_{j=1}^{P} A_k \mathbf{b}_k(i+j) \mathbf{\bar{\bar{s}}}_{k,j}
\]

\[
\mathbf{s}_k = \begin{bmatrix} s_k(0) & \ldots & s_k(N+L-2) \end{bmatrix}^T
\]

\[
\mathbf{\bar{s}}_{k,j} = \begin{bmatrix} s_k(jN) & s_k(jN+1) & \ldots & s_k(N+L-2) & 0 & \ldots & 0 \end{bmatrix}^T, j=1...P
\]

\[
\mathbf{\bar{\bar{s}}}_{k,j} = \begin{bmatrix} 0 & \ldots & 0 & s_k(0) & s_k(1) & \ldots & s_k(N+L-2-jN) \end{bmatrix}^T, j=1...P
\]

\[
\mathbf{v}(i) = \begin{bmatrix} v(iN) & \ldots & v(iN+N+L-2) \end{bmatrix}^T
\]

where \( L \approx PN + 1 \) since \( P = \lceil (L-1)/N \rceil \). Each element of the \((N+L-1)\times 1\)-dimensional noise vector \( \mathbf{v}(i) \) has variance \( \sigma^2 I \). When \( L=1 \) (hence \( P=0 \)) and \( h_k(1) = 1, k=1...K \), \( \mathbf{\bar{s}}_{k,j} \) in (13) and \( \mathbf{\bar{\bar{s}}}_{k,j} \) in (14) are simply zero.
in (14) will be zero according to the definition of \( s_k(n) \) in (8), and (10) is simplified to the model for the single-path flat fading channel

\[
\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \mathbf{c}_k + \mathbf{v}(i)
\]

Defining

\[
\mathbf{\bar{s}}_k = \begin{bmatrix} \bar{s}_{k,1} & \cdots & \bar{s}_{k,P} \end{bmatrix}_{(N_L-1) \times P}
\]

\[
\mathbf{\bar{s}} = \begin{bmatrix} \bar{s}_1 & \bar{s}_2 & \cdots & \bar{s}_K \end{bmatrix}_{(N_L-1) \times (2P+1)K}
\]

\[
\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \cdots & \mathbf{s}_K \end{bmatrix}_{(N_L-1) \times (2P+1)K}
\]

\[
\mathbf{A} = \text{diag}\{A_1, \ldots, A_{2P+1}, \ldots, A_K\}
\]

\[
\mathbf{b}_k(i) = \begin{bmatrix} b_k(i) & b_k(i-1) & \ldots & b_k(i-P) & b_k(i+1) & \ldots & b_k(i+P) \end{bmatrix}_{(N_L-1) \times (2P+1)}, k = 1 \ldots K
\]

then (10) can be written in a more compact vector/matrix format as

\[
\mathbf{r}(i) = \mathbf{SAb}(i) + \mathbf{v}(i)
\]

Assume that \( \{\mathbf{s}_1\}_{k=1}^{K}, \{\mathbf{\bar{s}}_1\}_{k=1}^{K} \) and \( \{\mathbf{\bar{s}}\}_{k=1}^{K} \) are independent vectors, then the column vectors of \( \mathbf{S} \) spans the signal subspace \( C^{(N_L-1)\times(2P+1)K} \). Compared with the AWGN channel in which the signal subspace is \( C^{N_L \times K} \), the rank of the signal subspace for the multipath channel is \( 2PK \) higher. For the signal model in (21), the linear multiuser detection for the \( k \)th user is to do correlation and then take hard decision, i.e.,

\[
\hat{b}_k(i) = \arg \min_{b_m} \| \mathbf{d}_i^T \mathbf{r}(i) - b_m \|
\]

where \( b_m \in \{b_1, b_2, \ldots, b_P\} \), and \( \mathbf{d}_i \in \mathbb{R}^{N_L-1} \). In the case of Binary phase-shift keying (BPSK) modulation, \( \hat{b}_k(i) = \text{sgn}(\mathbf{d}_i^T \mathbf{r}(i)) \). Similar to [26], three linear multiuser detectors can be obtained as follows.
1) Linear Expanded Decorrelating Detector

Assume that all users’ spreading codes are known at the receiver, the expanded decorrelating detector which completely removes MAI and ISI for the first user is given by

\[ d_1 = S[M^{-1}]_{s,1} \]

where \( d_1 \) is a vector of size \((N+L-1)\), and \([M^{-1}]_{s,k}\) denotes the \(k\)th column of the matrix \( M^{-1} \). and

\[ M = S^T S \]

(24)

2) Expanded Subspace-Based Decorrelating Detector

In [11], blind multiuser detectors are expressed in a closed form in terms of subspace components without the knowledge of the interfering users’ spreading codes. The expanded subspace-based decorrelating detector in this subsection and the expanded subspace-based MMSE detector in the next subsection are their extension from the AWGN channel to multipath channels. In this paper, we assume that channels are known; hence the expanded subspace-based detectors are not blind. The eigenvectors and eigenvalues of the signal subspace can be obtained, e.g., by conducting eigendecomposition on the autocorrelation matrix \( E[rr^T] \). The decorrelating detector is given by [26]

\[ d_i = U_s \Lambda_s^{-1} U_s^T s_i \left[ s_i^T U_s (A_s + \sigma^2 I_s)^{-1} U_s^T s_i \right] \]

(25)

where \( U_s \) is \((N+L-1) \times (2P+1)K\) matrix, its columns are the eigenvectors of the signal subspace, and \( A_s \) is a \((2P+1)K \times (2P+1)K\) diagonal matrix, the diagonal elements of which are the corresponding eigenvalues.

3) Expanded Subspace-Based MMSE Detector

The MMSE detector expressed in closed form in terms of the expanded signal subspace components is given by [26]

\[ d_i = U_s (A_s + \sigma^2 I_s)^{-1} U_s^T s_i \left[ s_i^T U_s (A_s + \sigma^2 I_s)^{-1} U_s^T s_i \right] \]

(26)

where \( I_s \) is a \((2P+1)K \times (2P+1)K\) identity matrix.

Discussion:
1. Given that the estimated eigencomponents in (25) are accurate, the decorrelating detectors in (23) and (25) give identical performance because they are basically the same detector in different expressions. Both of them suffer from the noise enhancement problem, and the situation is exacerbated when the subspace rank increases, as will be demonstrated by the simulation results. The increase in subspace rank can be result of the increasing number of users or longer channel spread.

III. The Hybrid Multiuser Detector

In this section, we develop a hybrid multiuser detector which is designed to tackle the noise enhancement problem inherent to the decorrelating detectors introduced in the previous section. The structure of the proposed receiver is illustrated in Figure 2. The received signal \( \mathbf{r}(i) \) is pre-processed by subtracting the interfering signal \( \hat{\mathbf{I}}(i-) \) which is contributed by the symbols in the previous epochs, and then the resulting signal \( \mathbf{f}(i) \) is fed into the rank-reduced expanded decorrelating detector. Hard decision of the decorrelating detector forms the estimate of the symbol vector \( \hat{\mathbf{b}}(i) \). Finally the estimated symbol vector \( \hat{\mathbf{b}}(i) \) is sent to the ISI synthesiser to recreate the interference signals, which will be used by the interference canceller in pre-processing the next received signal.

A. Interference Synthesis and Cancellation

The item \( \sum_{k=1}^{K} \sum_{j=1}^{p} A_k b_k (i-j) \bar{s}_{k,j} \) in (10) is due to the interfering signals from the previous epochs. They can be recreated by the ISI synthesiser at time \( i \) because the bits before time \( i \) have already been detected. With the detected symbols \( \{ \hat{b}_k (i-j) \}_{j=1, P_k=1, K} \), the interference due to the previous symbols are recreated as

\[
\hat{\mathbf{I}}(i-) = \sum_{k=1}^{K} \sum_{j=1}^{p} \hat{A}_k (i-1) \hat{b}_k (i-j) \bar{s}_{k,j}
\]  

(27)

Interference cancellation results in the deflated received signal as given by

\[
\tilde{\mathbf{r}}(i) = \mathbf{r}(i) - \hat{\mathbf{I}}(i-) = \sum_{k=1}^{K} A_k b_k (i) \bar{s}_k + \sum_{j=1}^{p} \left[ A_k b_k (i-j) - \hat{A}_k (i-1) \hat{b}_k (i-j) \right] \bar{s}_{k,j} + \sum_{j=1}^{p} A_k b_k (i+j) \bar{s}_{k,j} + \mathbf{v}(i)
\]

\[
\approx \sum_{k=1}^{K} A_k b_k (i) \bar{s}_k + \sum_{j=1}^{p} A_k b_k (i+j) \bar{s}_{k,j} + \mathbf{v}(i)
\]  

(28)
B. Rank-Reduced Expanded Decorrelating Detector

The rank of the original expanded signal subspace for (10) is \((2P+1)K\). The interference cancellation reduces the subspace rank by \(PK\). Hence the remaining rank for the deflated signal in (28) is \((P+1)K\).

Define

\[
\tilde{S} = \begin{bmatrix} s_1 & \tilde{s}_k & \ldots & s_K & \tilde{s}_K \end{bmatrix}_{(N+L-1)\times(P+1)K}
\]

(29)

The rank-reduced expanded decorrelating detector for the \(k\)th user is

\[
\tilde{d}_k = \tilde{S} \tilde{M}^{-1} s_k
\]

(30)

where \(\tilde{M} = \tilde{S}^T \tilde{S}\).

C. Iterative Multiuser Detection

In Figure 2, only the intersymbol interference from the previous epochs is cancelled. In the first detection iteration, the ISI due to the symbols of the future epochs are not known and hence cannot be cancelled. After the whole frame of signals have been detected, the estimated information bits can be fed back, and then the interference due to the future epochs can be cancelled in the same way as the previous epochs. The rank of the resulting signal subspace will be further reduced to \(K\). The interference cancellation and decorrelating detection procedures can be operated on the same signal frame repeatedly, and the performance of the detection can be improved in an iterative manner. Figure 3 illustrates this idea. The deflated received signal becomes

\[
\hat{r}(i) = r(i) - \hat{I}(i-\hat{I}) - \hat{I}(i+\hat{I})
\]

(31)

where \(\hat{I}(i+\hat{I})\) is the recreated interfering signals due to the future epochs, and is defined as

\[
\hat{I}(i+\hat{I}) = \sum_{k=1}^{K} \sum_{j=1}^{P} \hat{A}_k (i-1)\hat{b}_k (i+j)\hat{s}_k
\]

(32)

For the iterative detection, (29) is changed to

\[
\tilde{S} = \begin{bmatrix} s_1 & s_2 & \ldots & s_K \end{bmatrix}_{(N+L-1)\times K}
\]

(33)

The rank-reduced decorrelating detector for the \(k\)th user is

\[
\tilde{d}_k = \tilde{S} \tilde{M}^{-1} s_k
\]

(34)

where \(\tilde{M} = \tilde{S}^T \tilde{S}\).
IV. Subspace Collapse and Remedy Algorithm

In the computation of the expanded decorrelating detectors in (23) and (30), we have assumed that all column vectors of $S$ (see (17)) and $\tilde{S}$ (see (29)) are linearly independent. However, this is not always guaranteed in practice. As an example, for zero-padded $s_{k,p}$ and $\tilde{s}_{k,p}$, $k = 1, \ldots, K$ and $p = 1, \ldots, P$, if $L-1 = (P-1)N+1$, then $\tilde{s}_{k,p} = \begin{bmatrix} s_k(PN+1) & 0 & \cdots & 0 \end{bmatrix}^T$, $\tilde{s}_{K,p} = \begin{bmatrix} s_k(PN+1) & 0 & \cdots & 0 \end{bmatrix}^T$ are correlated; and $\tilde{s}_{k,p} = \begin{bmatrix} 0 & 0 & \cdots & s_k(0) \end{bmatrix}^T$, $\tilde{s}_{K,p} = \begin{bmatrix} 0 & 0 & \cdots & s_k(0) \end{bmatrix}^T$ are also correlated. The column vectors of $S$ do not span subspace $\mathbb{R}^{(2P+1)K(N+L-1)}$, and the column vectors of $\tilde{S}$ do not span subspace $\mathbb{R}^{(P+1)K(N+L-1)}$. We say that the expanded signal subspace has collapsed. The expanded decorrelating detectors in (23) and (30) are no longer valid as an immediate consequence of subspace collapse. The conventional way to combat this problem is to use pseudo-inverse of $\tilde{S}$. Here, we propose a more efficient remedy algorithm as outlined in Table 2 to detect the subspace collapse and then compute the expanded detectors in the new subspace. The idea is to remove the column vectors that are not independent. Because column vectors in $\{s_k\}_{k=1}^K$ have the full energy of the current symbols and they are usually linearly independent, the deletion is only applied to the vectors $\{\tilde{s}_{k,p}\}_{p=1}^P$ and $\{\tilde{s}_{k,p}\}_{p=1}^P$, $k = 1 \ldots K$. Since the similar methods can be applied to the expanded detector in (23) and the rank-reduced expanded detector in (30), we only outline the algorithm for the rank-reduced expanded detector as in Table 2.

Table 2: Remedy Algorithm for the Rank-Reduced Expanded Decorrelating Detector

<table>
<thead>
<tr>
<th>Calculate $s_k$, $k = 1, \ldots, K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form matrix: $\tilde{S} = \begin{bmatrix} s_1 &amp; \cdots &amp; s_K \end{bmatrix}$</td>
</tr>
<tr>
<td>For $k = 1: K$ /<em>$K$ users</em>/</td>
</tr>
<tr>
<td>For $\tau = 1: P$ /<em>$P$ symbol duration</em>/</td>
</tr>
<tr>
<td>Form $\tilde{S}_{k,\tau}$;</td>
</tr>
<tr>
<td>If $\tilde{S}_{k,\tau}$ and the column vectors of $\tilde{S}$ are linearly independent</td>
</tr>
<tr>
<td>$\tilde{S} = \begin{bmatrix} \tilde{S} &amp; \tilde{S}_{k,\tau} \end{bmatrix}$</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>/<em>form the decorrelating detector</em>/</td>
</tr>
<tr>
<td>$\tilde{d}_i = \begin{bmatrix} \tilde{S}^T \tilde{S} \end{bmatrix}^{-1} \tilde{S}^T$</td>
</tr>
<tr>
<td>$d_i = \begin{bmatrix} \tilde{d}_i \end{bmatrix}_i$</td>
</tr>
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V. Performance Analysis

Signal-to-interference-plus-noise ratio (SINR) in the output of the linear detector for the kth user at time $i$ is defined as

$$\text{SINR}(i) = \frac{E\left[\left|d_k^T A_k b_k(i)s_k\right|^2\right]}{E\left[\left|d_k^T (r(i) - A_k b_k(i)s_k)\right|^2\right]}$$  \hspace{1cm} (35)$$

For the full-rank expanded decorrelating detector,

$$d_k^T s_k = 1$$

$$d_l^T s_k = 0, \; l \neq k$$

$$d_k^T \tilde{s} = 0, \; \forall l$$

$$d_l^T \tilde{s} = 0, \; \forall l$$

so the SINR in its output is

$$\text{SINR}_{\text{full}}(i) = \frac{A_k^2}{d_k^T d_k \sigma^2} = \frac{A_k^2}{\alpha_k \sigma^2}$$  \hspace{1cm} (36)$$

where $\alpha_k = d_k^T d_k$ is the noise enhancement factor. Compared with the SNR in the received signal which is $\frac{A_k^2}{\sigma^2}$, SINR in (35) is reduced by a factor of $\alpha_k$, or equally, the noise is amplified by $\alpha_k$ times. To gain more insight into the noise enhancement factor, we express $\alpha_k$ as

$$\alpha_k = \left[ M^{-1}\right]_{kk} S^T S \left[ M^{-1}\right]_{kk} = \left[ M^{-1}\right]_{kk} M \left[ M^{-1}\right]_{kk} = \left[ (S^T S)^{-1}\right]_{kk}$$  \hspace{1cm} (37)$$

It equals the kth main diagonal element of the matrix $(S^T S)^{-1}$. Expressing (36) in logarithm format shows the connection between the output SINR and the SNR in the received signal as follows

$$\text{SINR}_{\text{full}}(i) = \log_{10} \frac{A_k^2}{\sigma^2} - \log_{10} \alpha_k = \text{SNR} - \log_{10} \alpha_k \quad \text{(dB)}$$  \hspace{1cm} (38)$$

For the rank-reduced expanded decorrelating detector, the interference that is not completely cancelled can be modelled as Gaussian noise with variance $\sigma_I^2$. Then the SINR is given by

$$\text{SINR}_{\text{reduce}}(i) = \frac{A_k^2}{d_k^T d_k \left( \sigma^2 + \sigma_I^2 \right)} = \frac{A_k^2}{\tilde{\alpha}_k \left( \sigma^2 + \sigma_I^2 \right)}$$  \hspace{1cm} (39)$$
where $\tilde{\alpha}_k = \tilde{d}_k^T \tilde{d}_k = \left((\tilde{S}^\dagger \tilde{S})^{-1}\right)_{dd}$. We know that $\hat{I}(i-)$ converges to $\sum_{k=1}^{K} \sum_{j=1}^{P} A_k (i-1) b_j (i-j) \vec{\pi}_{kj}^t$, and $\sigma_i^2 \approx 0$, the log format of (39) can be approximated by

$$SINR_{\text{reduce}}(i) = SNR - \log_{10} \tilde{\alpha}_k \quad (dB)$$

(40)

After convergence, the SINR improvement achieved by the proposed hybrid detector over the full-rank expanded decorrelating detector is given by

$$\Delta SINR(i) = SINR_{\text{reduce}}(i) - SINR_{\text{full}}(i) = \log_{10} \alpha_i - \log_{10} \tilde{\alpha}_i \quad (dB)$$

(41)

which shows that the performance improvement is determined by the decorrelating detector and is independent of the input SNR.

VI. Numerical Results

The proposed algorithms are evaluated by computer simulation in this section. Assume that there are 10 users ($K=10$) in the synchronous DS-CDMA system unless otherwise stated. User 1 is the user of interest. Users’ transmission power is unbalanced: compared to user 1, users 2-6 are 10dB stronger, users 7-9 are 20dB stronger and user 10 is 30dB stronger, respectively. The spreading gain is $N=31$, and the spreading codes are randomly-generated, independent and normalized sequences. The multipath channel coefficients are randomly-generated and normalized and have exponential decay profile. For the blind multiuser detectors, 5000 symbols are used to estimate signal subspace. All results are averaged over 500 Monte-Carlo runs in order to yield reliable statistics. SNR values are measured by comparing the power of user 1 and the background noise. In all simulations, BPSK modulation is assumed unless otherwise stated. Channel impulse response (including amplitude) can be estimated by either a pilot aided or a decision directed channel estimation algorithm. Refer to [27] for different estimation algorithms that are suitable for estimating fading channels in DS-CDMA systems.

For simplicity, we assume perfect knowledge of the channel in our simulations.

Example 1: SINR performance. Figure 4 to Figure 6 compare the SINR performance of three multiuser detectors at different multipath channels and different input SNR levels. The investigated detectors are the expanded decorrelating detector (ExpLDD), the rank-reduced expanded decorrelating detector...
assisted by fed-forward interference cancellation (ExpLDD IC1), and the rank-reduced expanded decorrelating detector assisted by fed-forward and fed-back interference cancellation (ExpLDD IC2). The figures show that both rank-reduced expanded decorrelating detectors obtain dramatic performance improvement compared with the full-rank expanded decorrelating detector. For a given multipath channel, the SINR improvement is almost the same regardless of input SNR. This is understandable from (41): $\Delta SINR$ is only determined by the enhancement factors $\alpha_k$ and $\tilde{\alpha}_k$. On the other hand, the difference in the SINR improvement is significant in different channel conditions -- more improvement with longer channel spread, e.g., when channel spread is 25 which is within one symbol interval, the SINR performance gain is about 1.5 dB; when channel spread is 56 which is over one symbol interval, the gain is about 6 dB; when channel spread is 86 which is over two symbol intervals, the gain is about 12 dB. This improvement is due to the fact that the SINR performance of the rank-reduced decorrelating detectors almost does not change with the channel delay spread while the SINR performance of the full-rank expanded decorrelating detector degrades significantly with the channel delay spread. It is also observed that ExpLDD IC2 does not perform significantly better than ExpLDD IC1. In the following experiments, only ExpLDD IC1 will be studied.

More detailed study about the SINR-channel performance is presented in Figure 7. The ExpLDD and ExpLDD IC1 are compared and the remedy algorithms are applied to them because the expanded subspace collapses at some channel spreads. The received SNR is 20dB, and the multipath channel spreads vary from 10 to 85. The SINR for the ExpLDD IC1 remains relatively static, whereas the SINR for ExpLDD decreases as the channel delay spread elongates, and shows staircase effect when the channel spread crosses symbol intervals. The transition between stairs shows that the subspace collapse happens when the channel spread is just over integral times of the symbol interval.

In Figure 4 to Figure 7, the effect of the channel on the SINR performance is due to the change in the noise enhancement factors $\alpha_k = \mathbf{d}_k^T \mathbf{d}_k$. The channel condition affects the structure of the expanded signal subspace, which consequently affects the noise enhancement factors. For example, the rank of the full-rank signal subspace is $(2P+1)K$ and the rank of the rank-reduced signal subspace is $(P+1)K$. When $P$ increases by 1, the rank of the full-rank signal subspace increases by $2K$ and the rank of the reduced signal subspace increases by $K$. Figure 8 shows the noise enhancement factors of ExpLDD and
ExpLDD_IC1 at different channel spreads. It can be seen that the noise enhancement factor of ExpLDD_IC1 almost does not change as the channel spread increases, while the noise enhancement factor of ExpLDD increases in a staircase fashion. This coincides with the result shown in Figure 7. Note that the subspace rank is not only determined by $P$ but also $K$. Increasing $K$ could also affect the SINR performance of the ExpLDD and ExpLDD_IC1 detectors. Figure 9 verifies this idea where the channel delay spread is 56. It shows that the noise enhancement factor of ExpLDD_IC1 increases dramatically as the number of users increases, while the noise enhancement factor of ExpLDD remains static.

**Example 2:** BER performance. Figure 10 compares the bit error rate (BER) performance of several multiuser detectors at different SNR levels for the 10-user system. The compared detectors are the RAKE receiver, the expanded decorrelating detector (ExpLDD), the expanded subspace-based decorrelating detector based on (SubExpLDD), the expanded subspace-based MMSE detector (SubExpMMSE), the rank-reduced expanded decorrelating detector assisted by fed-forward interference cancellation (ExpLDD_IC1). The channel spread is set to be 56. It can be seen from the figure that the RAKE detector fails to operate. This is because that the RAKE receiver is only optimum in single-user multipath systems, but not in multiuser multipath systems. It suffers severely from MAI. The other schemes are capable of joint suppression of MAI and ISI. ExpLDD and SubExpLDD do not yield satisfactory performance due to the noise enhancement problem, which can be effectively mitigated by the SubExpMMSE detector and the proposed hybrid multiuser detector. The performance of SubExpMMSE is subject to the accurate knowledge of the subspace components and the noise. For the presented SubExpMMSE result, 5000 symbols are used to estimate the subspace components. Figure 10 shows that the proposed scheme is superior to the ExpMMSE detector, and a gap of 2 dB is observed for this 10-user system at target BER around $10^{-4}$. As will become evident in the following example, the performance gain achieved by the proposed scheme becomes more significant when the system is more heavily loaded and the channel becomes more dispersive.

**Example 3:** Capacity comparison. The system capacity, i.e., the number of users that can be supported by the system with acceptable bit error rate, is examined in Figure 11 for the ExpLDD, SubExpLDD, SubExpMMSE and the ExpLDD_IC1. The channel spread and SNR value are set to be 56 and 15 dB,
respectively. It is assumed that the transmission power of all the other users is 10 dB higher than the user of interest. It is apparent to see from the figure that the proposed scheme increases the system capacity significantly compared to the other detectors. For instance, with a target BER=10^{-4}, over 14 users can be supported by the proposed scheme, whereas only 5 users can be supported by ExpLDD and SubExpLDD and 8 users can be supported by SubExpMMSE. Compared to the other detectors, the proposed scheme is much less affected by the number of active users, which makes it more effective for heavily loaded systems. We have also measured the system capacity at different levels of channel spread. The conclusion is that the proposed hybrid multiuser detector is rather robust against the channel dispersion, whereas the performance of other detectors degrades significantly as the channel becomes more dispersive. The plot is omitted here to conserve space.

Example 4: BER performance for the QPSK modulated system. The above experiments have been conducted for BPSK modulated systems. Figure 12 compares the bit error rate (BER) performance of several multiuser detectors at different SNR levels for the 10-user system with QPSK modulation. The compared detectors are the expanded decorrelating detector (ExpLDD), the expanded subspace-based decorrelating detector based on (SubExpLDD), the expanded subspace-based MMSE detector (SubExpMMSE) and ExpLDD_IC1. The channel spread is set to be 56, and 5000 symbols are used to estimate the subspace components for the SubExpMMSE algorithm. Conclusions for the BPSK system also hold here: ExpLDD and SubExpLDD do not yield satisfactory performance due to the noise enhancement problem. The SubExpMMSE detector and the proposed hybrid multiuser detector can effectively tackle this problem. The proposed scheme outperforms the ExpMMSE detector by 2.5 dB at target BER around 10^{-3}. However, the gap tends to become smaller as SNR increases.

VI. Conclusion

A novel hybrid multiuser detection strategy for multipath frequency-selective fading channels has been developed in this paper. It has an IC pre-processing unit and an expanded decorrelating detector in the back end. The IC unit plays a crucial role in alleviating the noise enhancement problem suffered by the expanded decorrelating detector. This is achieved by cancelling ISI and hence reducing the signal subspace rank. Unlike the conventional interference cancellation techniques, the proposed IC scheme
does not suffer from the error propagation problem and it is near-far resistant. High SNR is not required either. The rank-reduced decorrelating detector enjoys the advantages of the full-rank expanded decorrelating detector, such as complete MAI and ISI suppression, and in the meantime, overcomes the noise enhancement problem. The noise enhancement factor of the rank-reduced decorrelating detector is minor compared to the full-rank decorrelating detector and it is almost irrelevant to the channel delay spread and the system loads. Compared to existing methods, dramatic performance gain in SINR, BER and capacity can be achieved with this new multiuser detection approach. The detection scheme is proposed for synchronous scenario, but it can be easily extended into asynchronous scenario provided that the delay is known or can be estimated. Combining this new multiuser detection method with advanced channel coding technique and deriving an iterative multiuser detection and decoding algorithm will be the future research topics for the authors.

REFERENCES


Figure 1: Diagram of the observation window for the synchronous multipath channel.

Figure 2: The hybrid detector including feed-forward interference cancellation, rank-reduced decorrelating detector and Kalman filter-based amplitude estimation (ExpLDD_IC1).
Figure 3: The iterative detection structure including feed-forward and feed-back interference cancellation, rank-reduced decorrelating detector and Kalman filter-based amplitude estimation (ExpLDD_IC2).

Figure 4: $(K=10, L=25)$ SINR convergence. (a) SNR=5dB; (b) SNR=10dB; (c) SNR=20dB.
Figure 5: ($K=10, L=56$) SINR convergence. (a) SNR=5dB; (b) SNR=10dB; (c) SNR=20dB.

Figure 6: ($K=10, L=86$) SINR convergence. (a) SNR=5dB; (b) SNR=10dB; (c) SNR=20dB.
Figure 7 ($K=10$, SNR=20dB): SINR versus multipath channel spread.

Figure 8 ($K=10$): Noise enhancement factor versus the multipath channel spread.
Figure 9 ($L=56$): Noise enhancement factor versus the number of users.

Figure 10: ($K=10, L = 56$) BER comparison of different multiuser detectors for BPSK system.
Figure 11: \((L = 56, \text{SNR} = 15\text{dB})\) Capacity comparison of different multiuser detectors.

Figure 12: \((K=10, L = 56)\) BER comparison of different multiuser detectors for QPSK system.